Astrophysics

Compulsory Home Exercises. Problem Set 4.

Return by Wednesday, April 1, 2026.
Please, write down **every step in your line of thinking** and state assumptions etc.
A sole answer is not enough.

1. Let's approximate the directional (μ) dependence of the specific intensity using the first-order Taylor expansion,

$$I(\tau, \mu) = I_0(\tau) + I_1(\tau)\, \mu$$

where I_0 and I_1 depend on the vertical optical depth τ but not on μ.

Assuming a grey atmosphere,

a. Derive expressions for the flux F, mean intensity J, and radiation pressure P_{rad}, in terms of I_0 and I_1.

b. Show that the radiation field obeys the Eddington approximation and

$$P_{rad} = \frac{4\pi}{3c} J$$

c. Within the Eddington approximation, the solution of the radiative transfer equation is

$$S = \frac{3}{4\pi}\left(\tau + \frac{2}{3}\right) F$$

Use this solution to find an expression for I_0 as a function of I_1 and τ.

d. Show that if $\tau \gg 1$ then $I_0 \gg I_1$ (this result justifies the use of the first-order Taylor expansion at large optical depths, showing that the radiation field becomes isotropic).

2. Assuming a grey atmosphere and using the Eddington approximation and an expression for the source function S as a function of vertical optical depth τ and flux F (Lecture 18),

a. Calculate the upward specific intensity $I(\tau, \mu)$ as a function of F, τ_v, and direction $\mu > 0$, using the formal solution to RTE (Lecture 17)

$$I_\lambda(\tau_{\lambda,\mathrm{v}}, \mu > 0) = \int_{\tau_{\lambda,\mathrm{v}}}^{\infty} S_\lambda \mathrm{e}^{(\tau_{\lambda,\mathrm{v}} - t)/\mu} \frac{\mathrm{d}t}{\mu},$$

b. Using your expression for the upward specific intensity $I(\tau, \mu)$, evaluate the upward component of the radiative flux,

$$F^+ = 2\pi \int_0^1 I(\mu)\, \mu\, d\mu$$

c. Find the corresponding downward component of the flux, F^-.