

# Astrophysics

## Compulsory Home Exercises. Problem Set 2.

Return by Wednesday, February 11, 2026.

Please, write down **every step in your line of thinking** and state assumptions etc.  
A sole answer is not enough.

### Problem 2.1

What is the energy released by the nuclear reactions of carbon burning (fusion of 2 carbon nuclei)?  
Give the answer in **MeV** and **ergs per gram**.

### Problem 2.2

Calculate the mean molecular weight  $\mu$  for

- 1) the completely ionized stellar interior, where we have 45% hydrogen, 52% helium, and 3% heavy elements by mass,
- 2) completely ionized hydrogen,
- 3) completely ionized helium,
- 4) **neutral** gas at the solar interior abundance, 73% hydrogen, 25% helium, and 2% heavy elements by mass.

### Problem 2.3

Prove that for the case when  $Z$  is negligible, the mean molecular weight per **electron**,  $\mu_e = \frac{\rho}{n_e m_H}$ , can be approximately expressed as

$$\mu_e \approx \frac{2}{1 + X}$$

### Problem 2.4

Does a lower Gamow energy  $E_G$  increase or decrease the probability of penetration?

### Problem 2.5

- a) Calculate the Gamow energy  $E_G$  (in kiloelectronvolts, **KeV**) for the collision of two  $\alpha$ -particles (helium-4 nuclei,  ${}^4_2\text{He}^{2+}$ ) and find the penetration probability  $P_{\text{pen}}$  for the typical kinetic energy of particles in the Sun's core,  $E \sim 1$  keV. Compare the results with the case of two protons. Explain the result.
- b) What temperature is required to have the probability of penetration of two  $\alpha$ -particles similar to that of two protons in the Sun's core?

### Problem 2.6

Prove that according to the virial theorem, the mean temperature of a star can be expressed as

$$\bar{T} \propto M^{2/3} \rho^{1/3}$$