ISM: Ionized regions

SOURCE OF IONIZATION
IONIZATION AND RECOMBINATION
STRÖMGREN SPHERES

The non-equilibrium ISM

The ISM is generally not in thermodynamic equilibrium due to its low density and consequent low collision rates and low optical depths. Collisions between particles may result in radiative decay rather than redistributing internal energies. Photons can escape the system, or enter from external sources. Whereas the distribution of velocities remains generally Maxwellian, and described by a kinetic temperature on scales greater than a mean free path, the distribution of energy levels may be significantly different from the Boltzmann distribution. This can be formulated either as departure coefficients,

$$b_i = \frac{n_{i,actual}}{n_{i,LTE}}$$

or, more commonly, by defining an excitation temperature, T_{ex}, such that

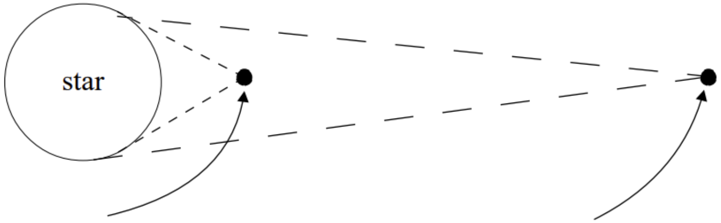
$$\frac{n_u}{n_l} = \frac{g_u}{g_l} e^{-(E_u - E_l)/kT_{ex}}$$

Note that although T_{ex} has units of K, it is not a physical temperature and may not be equal to the kinetic temperature. It is a function of the energy level and parameterizes how far the distribution of states is from Boltzmann.

Radiation field

Interstellar radiation field is characterized by a huge discrepancy between the frequency-integrated radiation density and spectral composition.

Compare photons close to and far from an extended source like a star:



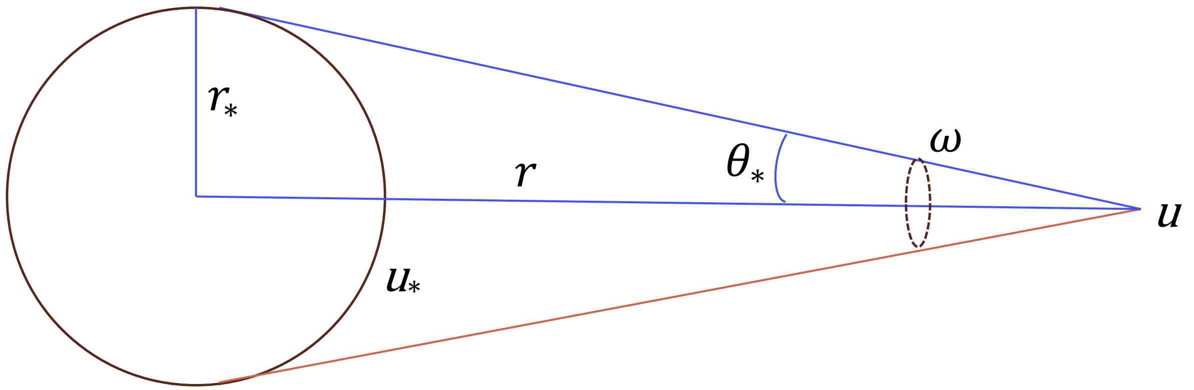
Photons here are crowded in physical space (high density) but spread out in solid angle (direction)

Photons here are diffuse in physical space (low density) but highly collimated in angle (high density).

It is sometimes convenient to approximate the density of the background starlight as a “diluted blackbody” of “dilution factor” W and “color temperature” T_c.

Dilution factor (1)

Compare photons close to and far from an extended source like a star:



The dilution factor W is defined to be the ratio of the actual energy density u to the energy density of (undiluted) blackbody radiation of temperature T_c.

ω is the solid angle, then $u = \frac{\omega}{4\pi} u_* = u_* W$, where W is the **dilution factor**.

$$W = \frac{\omega}{4\pi} = \frac{1}{4\pi} \int_0^{2\pi} d\varphi \int_0^{\theta_*} \sin\theta \, d\theta = \frac{1}{2}(1 - \cos\theta_*) = \frac{1}{2}\left(1 - \sqrt{1 - \frac{r_*^2}{r^2}}\right) \approx \frac{1}{2}\frac{r_*^2}{r^2}$$

http://dx.doi.org/10.1016/j.nima.2015.12.060

$r \gg r_*$

Dilution factor (2)

The dilution factor $W \approx \frac{1}{2}\frac{r_*^2}{r^2}$

Planetary Nebula? $r_* \approx 10^9$ cm, $r \approx$

Slide I-157: the energy contained in vo

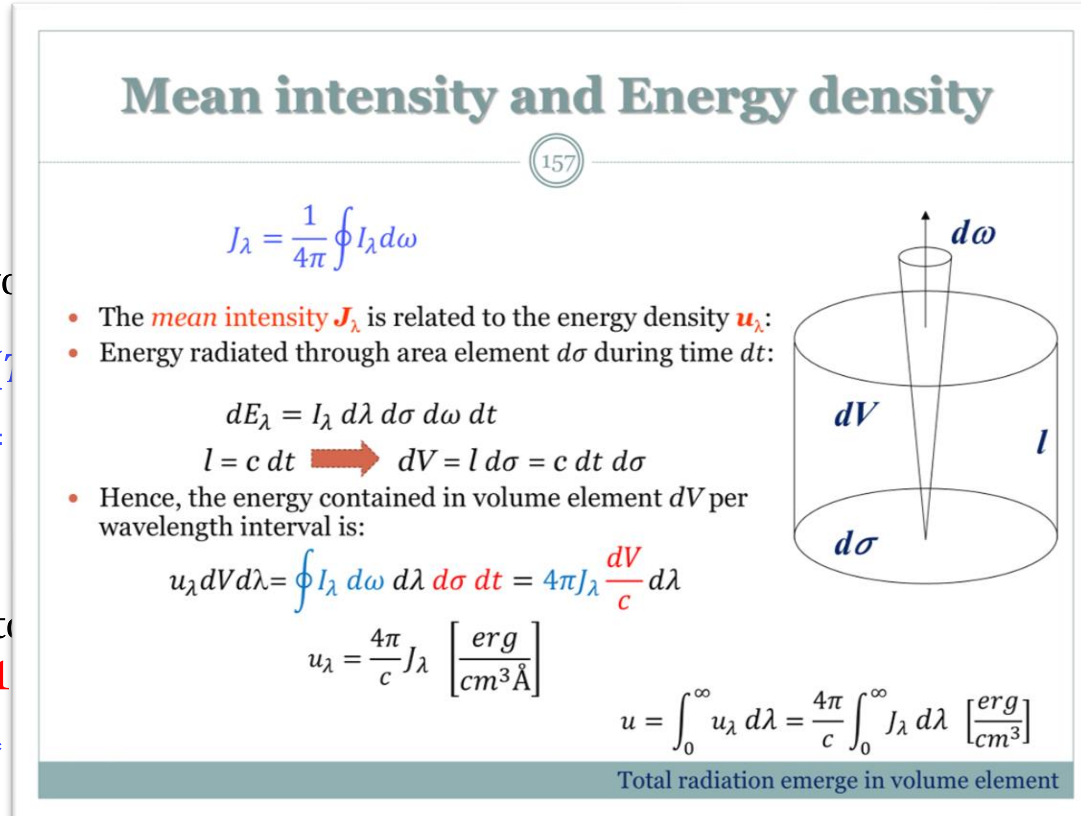
Assume Blackbody, then $u_{*,\lambda} = \frac{4\pi}{c} B_\lambda($

The integrated radiation density $u_* =$

Then $T_d^4 = W T_*^4$, or

T_d is the temperature corresponded t
radiation of temperature T_*. For W~1

But a spectrum corresponds to T_C=T_*



Indeed, Rosseland's theorem dictates that
in low-density regions, short-wavelength radiation is transformed into long-wavelength radiation.
This immediately explains a PNe observable, that the nebula emits much more energy in the optical than does the central star. This is because UV radiation is being processed into optical photons.

Dilution factor (2)

The dilution factor $W \approx \frac{1}{2}\frac{r_*^2}{r^2}$

Planetary Nebula? $r_* \approx 10^9$ cm, $r \approx 10^{16}$ cm → W~10^{-14}

Slide I-157: the energy contained in volume element per wavelength interval is $u_{*,\lambda} = \frac{4\pi}{c} J_\lambda \left[\frac{erg}{cm^3 Å}\right]$

Assume Blackbody, then $u_{*,\lambda} = \frac{4\pi}{c} B_\lambda(T_*)$, where $T_* = T_{\mathrm{eff}}$ of a star.

The integrated radiation density $u_* = aT_*^4$ for the star and $u = \int_0^\infty u_\lambda \, d\lambda = aT_d^4$ far from the star.

Then $T_d^4 = WT_*^4$, or

$$T_d = W^{1/4} \, T_*$$

T_{d} is the temperature corresponded to the integrated energy density of diluted blackbody radiation of temperature T_*. For W~10^{-14}, T_{d}=a few K.

But a spectrum corresponds to T_{C}=T_*! The ISM must work as a transformer, decreasing T_{C}.

Indeed, Rosseland's theorem dictates that

in low-density regions, short-wavelength radiation is transformed into long-wavelength radiation.

This immediately explains a PNe observable, that the nebula emits much more energy in the optical than does the central star. This is because UV radiation is being processed into optical photons.

The non-equilibrium ISM (2)

The interstellar radiation field is very dilute but populated by discrete energetic sources (e.g. OB stars) and can contain multiple spectral features from the gas.
It is therefore generally very different from the Planck function which we can parameterize in terms of a brightness temperature.

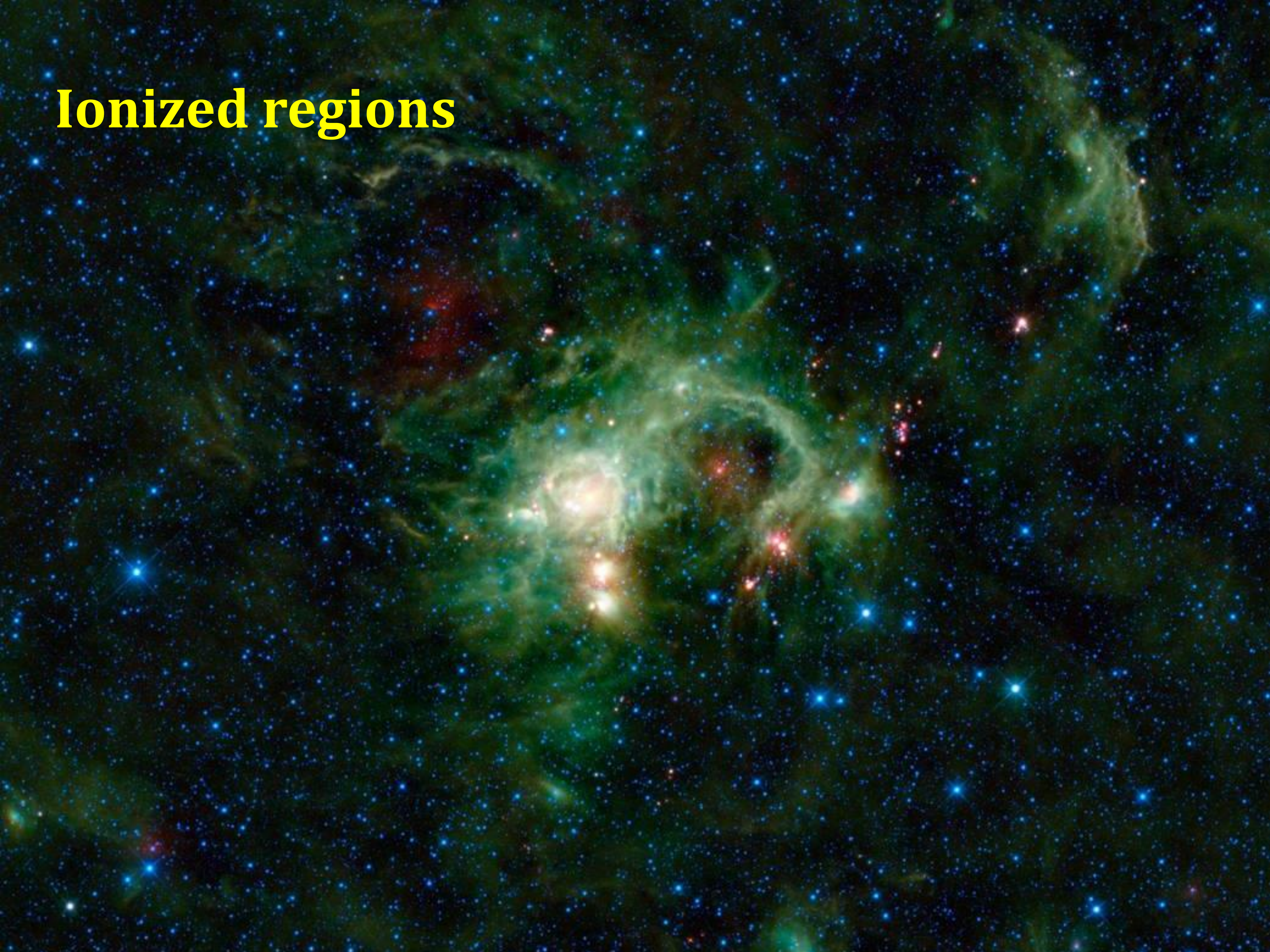
Despite these complications, we can still look for a statistical equilibrium solution to the distribution of energy levels,

$$\frac{dn_i}{dt} = \sum_{j\neq i}^{N} n_j P_{ji} - n_i \sum_{j\neq i}^{N} P_{ij} = \sum_{j\neq i}^{N} n_j (R_{ji} + C_{ji}) - n_i \sum_{j\neq i}^{N} (R_{ij} + C_{ij}) = 0$$

where P_{ji} is the (radiative R_{ji} plus collisional C_{ji}) rate from level j to i. This matching of forward and reverse rates is also known as the principle of detailed balance.

Remember? See lecture 24.

Ionized regions



Source of ionization?

H II regions are zones of ionized atomic hydrogen. They are often associated with nebulae, and require gas that contains a continuous source of ionizing radiation.

Ionization potential of hydrogen $h\nu_0 = 13.6$ eV

($\nu_0 = 3.29 \times 10^{15}$ Hz, $\lambda_0 = 912$Å)

Thus, for ionization we need $h\nu > 13.6$ eV
(it is >> kT in neutral ISM => collisions unimportant)

This could be a massive star or a white dwarf, but either way it must be very hot.

Are OB stars hot enough?

$T \approx 3\text{-}5 \times 10^4$ K => $E = kT \approx 3\text{-}5$ eV < 13.6 eV => cannot ionize H…?

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1}$$ => can ionize H!

Check it with Wien's displacement law

Yes, the OB stars are hot enough.

H II regions

Without that **hot** central source, the protons and electrons in the ISM will quickly recombine – even with the ionizing source, it can only ionize a region of some given volume before recombinations will be happening as quickly as ionizations.
The result will be a bubble of ionized gas, termed a **Strömgren Sphere**.
Such H II regions are easily observable via the strong emission lines resulting from recombination. Thus, to add further to the nomenclature, they are also sometimes known as emission-line nebulae (remember the spectrum of the Cat's Eye Nebula?)

Our goal now is to understand the size of the ionized bubble and its detailed ionization structure. In this effort, we define the ionization fraction

$$f \equiv \frac{n_{H^+}(r)}{n_H(r)}$$

For a fully neutral, atomic ISM $f = 0$, while full ionization implies $f = 1$.
As hinted at in the preceding argument, to maintain a constant f we will want to make use of ionization equilibrium, where

# of recombinations / sec = # of ionizations / sec

Ionization and Recombination

- Consider pure hydrogen gas
- Reaction: H+ hν ↔ p + e
- Ionization produces an electron of energy $E_{\text{kin,e}} = h\nu - \chi_{\text{ion}}$

  where χ_{ion} is the ionization potential (one can assume all atoms to be in the ground state)
- Recombination to level i gives a photon

$$h\nu = E_{\text{kin,e}} + \chi_{\text{ion}} / i^2 \approx \chi_1 \, (1/i^2 + 0.07 \; T_e / 10^4 \text{ K})$$

Transition energy between levels u and l:

$$\chi_{ul} = C\left(\frac{1}{u^2} - \frac{1}{l^2}\right)$$

where $C=\chi_{\text{ion}}$=-13.6 eV

where $\chi_{\text{ion}} \approx kT_e$ is kinetic energy of electron, i – is the main quantum number, plus a cascade of line (Balmer+Lyman etc.) photons.

Photons recombining to the 1st level can still ionize hydrogen.

- Equilibrium: # of recombinations = # of ionizations

Energy redistribution and temperature

- We have e, p, and neutral H.
- Why should we assume that all of them are described by the same temperature $T_e = T_p = T_H$?
- Why should they have Maxwellian distribution?
- The distribution of injected electrons depends on incident photon spectrum.
- Electrons thermalize rapidly by collisions.
- Electrons and protons exchange energy much slower (since the masses are very different).
- Protons and neutrals exchange energy even slower $\tau_{ee} \ll \tau_{ep} \ll \tau_{pH}$ but still $\ll \tau_{dyn}$ $\Rightarrow$
- Electrons lose energy in inelastic collisions (by radiation). Energy is taken from the gas energy as a whole. We associate gas temperature with T_e.

HII regions: recombination (case A)

- Recombination rate proportional to $n_e n_p$. Depends on T_e, since the recombination probability and the flux of electrons both depend on T_e
- e^- can be captured in any level
- Recombination rate to level i is

$$N_{\mathrm{rec},I} = n_e n_p \, \alpha_{i,\mathrm{rec}} (T_e) \ \mathrm{cm^{-3}\ s^{-1}}$$

where $\alpha_{i,\mathrm{rec}}$ is the recombination coefficient.

- Total "case A" recombination coefficient

$$\alpha_A = \sum_{n=2}^{\infty} \sum_{l=0}^{n-1} \alpha_{nl}(T) \approx 4.2 \times 10^{-13} \, (T_e/10^4)^{3/4} \ \mathrm{cm^3 s^{-1}}$$

- Thus,

$$N_{\mathrm{rec}} = n^2 \alpha_A (T_e) \ \mathrm{cm^{-3}\ s^{-1}}$$

However

HII regions: recombination (case B)

- Photon produced by recombination to level 1 still can ionize gas. These photons produce "**diffuse**" radiation field. Assuming that these photons are absorbed nearby ("on-the-spot" approximation), one can just neglect these recombinations in the total recombination rate:

$$N_{rec} = \sum_{i=2} N_{rec,i} = n_e^2\, \alpha_B\, (T_e)\ \mathrm{cm^{-3}\ s^{-1}}$$

where "case B" recombination coefficient

$$\alpha_B = \alpha' = \alpha_A - \alpha_1$$

$$\alpha_B\, (T_e) \approx 2.6 \times 10^{-13}\, (T_e\, /\, 10^4\ \mathrm{K})^{-3/4}\ \mathrm{cm^3\ s^{-1}}$$

Steady State in HII Regions

$H^+ + e^- \rightarrow H + h\nu$
$H + h\nu \rightarrow H^+ + e^-$

Case A recombination: gas optically thin to radiation produced after recombination

Case B recombination: gas optically thick to radiation just above 13.6 eV;
photons produced in recombination are absorbed for photoionization of another atom

Photoionization/ Recombination

- Transitions down are fast : $A \sim 10^{8}$ s^{-1}
  All atoms are on ground state (absorption in Balmer lines is negligible).

- Total ionization rate at distance r from the star is:

$$N_{\rm ion} = n_{\rm H}\,\Gamma_i \approx n_{\rm H}\,\sigma_0\,N_*/\,(4\pi r^2)\ \text{cm}^{-3}\ \text{s}^{-1}$$

where Γ_i is the number of ionizations per atom per second from level l
N_* is the number of stellar ionizing photons (with $h\nu$ >13.6 eV) per second,
and σ_0=6.3 × 10^{-18} cm^2 is the ionization cross-section at $\nu=\nu_i$,

$$\Gamma_i = \int_{\nu_i}^{\infty} \frac{F_\nu}{h\nu}\sigma_i(\nu)d\nu, \qquad \text{where} \quad F_\nu = L_\nu/4\pi r^2$$

σ_i is the ionization cross-section: $\sigma_i = 6.3 \times 10^{-18}\,(\nu_i/\nu)^{3.5}$ cm^2

HII regions: ionization

- Let's introduce $n_e = x\, n$

  $n = n_p + n_H = n_e + n_H$

- Then $n_H = (1-x)\, n$

$$N_{rec} = N_{ion} \quad \Rightarrow \quad x^2\, n^2\, \alpha'_{rec}\,(T_e) = (1-x)\, n\, \sigma_0\, N_* / \,(4\pi r^2)$$

- For O6.5 star: $N_* = 10^{49}$ s^{-1}, r = 1 pc, $n = 10^2$ cm^{-3}, $T_e = 10^4$ K,
  we get $(1-x) = 3 \times 10^{-5}$, i.e. $x \sim 1$ and hydrogen is almost fully ionized.

Strömgren Spheres

- Two components to local ionizing flux near a star:
  - direct ionizing flux
  - diffuse flux from recombinations to ground state

- Calculate radius of ionized sphere in uniform density:
  balance total number of ionizations (=number of ionizing photons the star produces) to the total number of recombinations to levels above ground $\alpha^{(2)}=\alpha'=\alpha_B$

$$\frac{4\pi}{3} r_S^3 x^2 n^2 \alpha_B = N_* \quad \Longrightarrow \quad \boldsymbol{r_S = \left(\frac{3N_*}{4\pi n^2 \alpha_B}\right)^{1/3}}$$

- Strömgren radius for an O star: r_S=70 pc $n^{-2/3}$; n=10^2 cm^{-3}, $r_S \sim 3$ pc

→ OB stars have an enormous impact on the ISM

Edges of HII regions

- $n_H \uparrow$, absorption ↑, ionizing photon flux ↓, $n_H \uparrow$
- The optical depth $\tau = \int n\, \sigma\, dl$ for ionization is huge.
- At Lyman edge, $\sigma = 6.3 \times 10^{-18}$ cm^2, so if $n = 1$ cm^{-3}, the mean free path Δr (where τ=1) is

  $\Delta r = \tau / n\, \sigma = 1.5 \times 10^{17}$ cm / n = 0.05 pc / n

$$\Delta r / r_S = 10^{-3}\, n^{-1/3} \ll 1$$

- Observed H II regions limited:
  - ionization bounded: all photons are used up for ionization, interstellar cloud has a larger extent than the nebula
  - density bounded: all atoms ionized, there are still photons left
- Ionization bounded H II regions have sharp edges

Edges of HII regions