# Other sources of opacity

He ABSORPTION
METALLIC ABSORPTION
SCATTERING
EFFECT OF NONGREYNESS OF THE TEMPERATURE STRUCTURE

# Many physical processes contribute to opacity



- **Bound-Bound Transitions** – absorption or emission of radiation from electrons moving between bound energy levels.
- **Bound-Free Transitions** – the energy of the higher level electron state lies in the continuum or is unbound.
- **Free-Free Transitions** – change the motion of an electron from one free state to another.
- **Electron Scattering** – deflection of a photon from its original path by a particle, without changing its wavelength.
  - **Rayleigh scattering** – photons scatter off **bound** electrons (varies as $\lambda^{-4}$).
  - **Thomson scattering** –photons scatter off **free** electrons (independent of wavelength).
- **Photodissociation** may occur for molecules.

# What can various particles do?

- Free electrons – Thomson scattering
- Atoms and Ions –
  - Bound-bound transitions
  - Bound-free transitions
  - Free-free transitions
- Molecules –
  - BB, BF, FF transitions
  - Photodissociation
- Most continuous opacity is due to hydrogen in one form or another

# He opacity?



Helium is the next most abundant element after H, so
is it important for the continuous absorption in the Sun or other stars?

Ionization of He to $He^+$ requires an energy of 24.6eV ($\lambda$<504Å are needed). Indeed, even the first excited level lies 19.8eV above the ground state, which can contribute only below 600 Å where there is very little radiation coming from the Sun. From the Boltzmann formula ($g_1$=1, $g_2$=3):

$$\log(N_{He}(2s^3S) / N_{He}(1s^1S)) = 0.48\text{-}19.8\times(5040/5777) = -16.8$$

So, only $10^{-17}$ of the He atoms can contribute to the absorption, and since He is 10% as abundant as H, only one in $10^{-18}$ atoms are He atoms in the 1st excited state.

Consequently, He opacity plays a **negligible** role for the Sun. The bound-free absorption from $He^-$ is generally negligible, whilst free-free $He^-$ (with a form similar to free-free $H^-$) can be significant at long wavelengths in cool stars.
**Photoionization (bound-free processes) from He only plays a significant role for the hottest, O-type, stars.**

# Metal (Iron) opacity



- If He only plays a role for very hot stars, do any metals contribute to the continuous opacity in cools stars?

- **Iron** (Fe/H=$10^{-4}$) is generally the dominant metal continuous opacity source in stellar atmospheres.

- In the Sun, let's consider absorption by atomic Fe in the ultraviolet (2000 Å) for which an excitation energy of ~1.7 eV is required. The fraction of excited Fe atoms is $4 \times 10^{-2}$ relative to the ground-state (from Boltzmann formula), whilst the fraction of ionized to neutral Fe is approximately 6 (from Saha equation).
- Accounting for the abundance of Fe, we obtain the fraction of atomic Fe atoms absorbing at 2000 Å relative to the total number of H atoms to be $4 \times 10^{-2} \times 10^{-4} \times 1/6 = 6 \times 10^{-7}$,

- We previously obtained $2 \times 10^{-8}$ for $H^-$, so metallic lines in the UV are much more important for the absorption than the $H^-$ ion, or the neutral H atom. Even more important is the absorption by the metal atoms in the ground level, which is <1570 Å for Fe, <1520 Å for Si.

# Molecular opacity



- $CN^-$, $C_2^-$, $H_20^-$ , $CH_3$, TiO are important sources of opacity in late (K-type) & very late (M-type) stars.

- Molecular Hydrogen ($H_2$) is more common than atomic H in stars cooler than mid-M (brown dwarfs!)

- $H_2$ does not absorb in the visible spectrum, so only plays a role in the IR.

- $H_2^+$ does absorb in the visual but is less than 10% of $H^-$. $H_2^+$ is a significant absorber in the UV for such very cool stars.

# Scattering



In the classical picture of an atom, we can consider the electron as being bound to the atom. Any force trying to remove it will be counteracted by an opposing force. If a force were to pull on the electron and then let go, it would oscillate with eigenfrequencies $\omega = 2\pi\nu$.

The scattering cross-section for a *classical oscillator* can be written as

$$\sigma_s = \frac{8\pi}{3}\frac{e^4}{m_e^2 c^4}\left[\frac{\nu^4}{(\nu^2-\nu_0^2)^2+\gamma^2\omega^2}\right] \qquad \omega = 2\pi\nu$$

where $\nu_0$ is the eigenfrequency of an atom and $\gamma$ is the damping constant.

# Thomson & Rayleigh Scattering



Two cases are of interest:

1. **Thompson (electron) scattering** ($\nu_o$=0, $\gamma$=0) (photons scatters off a free electron, no change in $\lambda$, just direction):

classical electron radius

$$\sigma_T = \frac{8\pi}{3}\frac{e^4}{m_e^2 c^4} = \frac{8\pi}{3} r_e^2 = 6.65 \times 10^{-25}\ \mathrm{cm}^2/\mathrm{electron}$$

2. **Rayleigh scattering** by atoms/molecules ($\nu \ll \nu_o$, $\gamma \ll \nu_o$)

$$\sigma_R(\nu) \propto \sigma_T \nu^4 = \sigma_T \lambda^{-4}$$

# Electron scattering vs. f-f transition



- Electron scattering (Thomson scattering) – the path of the photon is altered, but not the energy.

- Free-Free transition – the electron emits or absorbs a photon. A free-free transition **can only occur in the presence of an associated nucleus**. An electron in free space cannot gain the energy of a photon.

# Thompson Scattering



Since an electron is tiny it makes a poor target for an incident photon so the cross-section for Thomson scattering is very small ($\sigma_T$=6.65x10$^{-25}$ cm$^2$ ) and has the same value for photons of all wavelengths: **As such electron scattering is the only grey opacity source.**

Although e$^-$ are very abundant in the Solar photosphere, the small cross-section makes it unimportant.

Electron scattering is most effective as a source of opacity at high temperatures. In atmospheres of OB stars where most of the gas is completely ionized, other sources of opacity involving bound electrons are excluded. In this regime, $\alpha_T$ **dominates** the continuum opacity.

# Rayleigh Scattering



- Rayleigh scattering by H atoms in Solar-type is more relevant than $e^-$ scattering since atoms are much more common (recall **N(H)≫N(H⁺)**).
- In M stars, $H_2$ becomes the dominant form for hydrogen, with strong electronic transitions in the UV, so Rayleigh scattering by molecular $H_2$ can be important.
- The cross-section for Rayleigh scattering is much smaller than $\sigma_T$, and is proportional to $\lambda^{-4}$ so increases steeply towards the blue. (In the same way the sky appears blue, due to a steep increase in the scattering cross-section of sunlight scattered by molecules in our atmosphere).
- The cross-section is sufficiently small relative to metallic absorption coefficients that Rayleigh scattering only plays a dominant role in extended envelopes of supergiants.

# Total extinction coefficient κ



- The total extinction coefficient is given by:

$$\kappa_\nu = (1 - e^{-h\nu/kT}) \sum_j x_j(\kappa_j^{bb} + \kappa_j^{bf} + \kappa_j^{ff}) + \kappa^s$$

where the sum is over all elements $j$ of number fraction $x_j$.

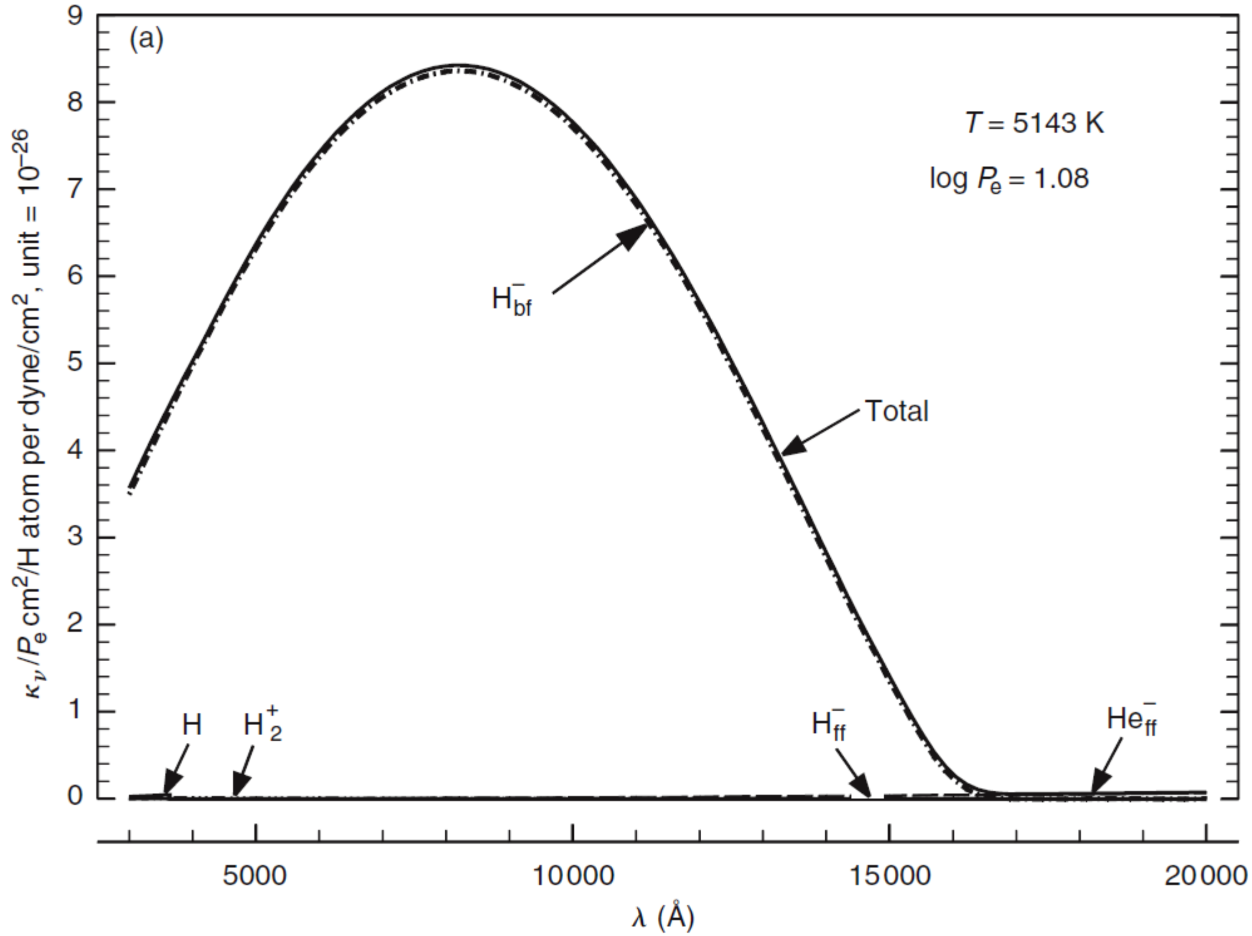
Here the ($e^{-\frac{h\nu}{kT}}$) term accounts for stimulated emission (incident photon stimulates electron to de-excite and emit photon with identical energy, as in a laser). We shall discuss it later.

- What is the total extinction coefficient for different types of star?

# G-type (optical depth unity)



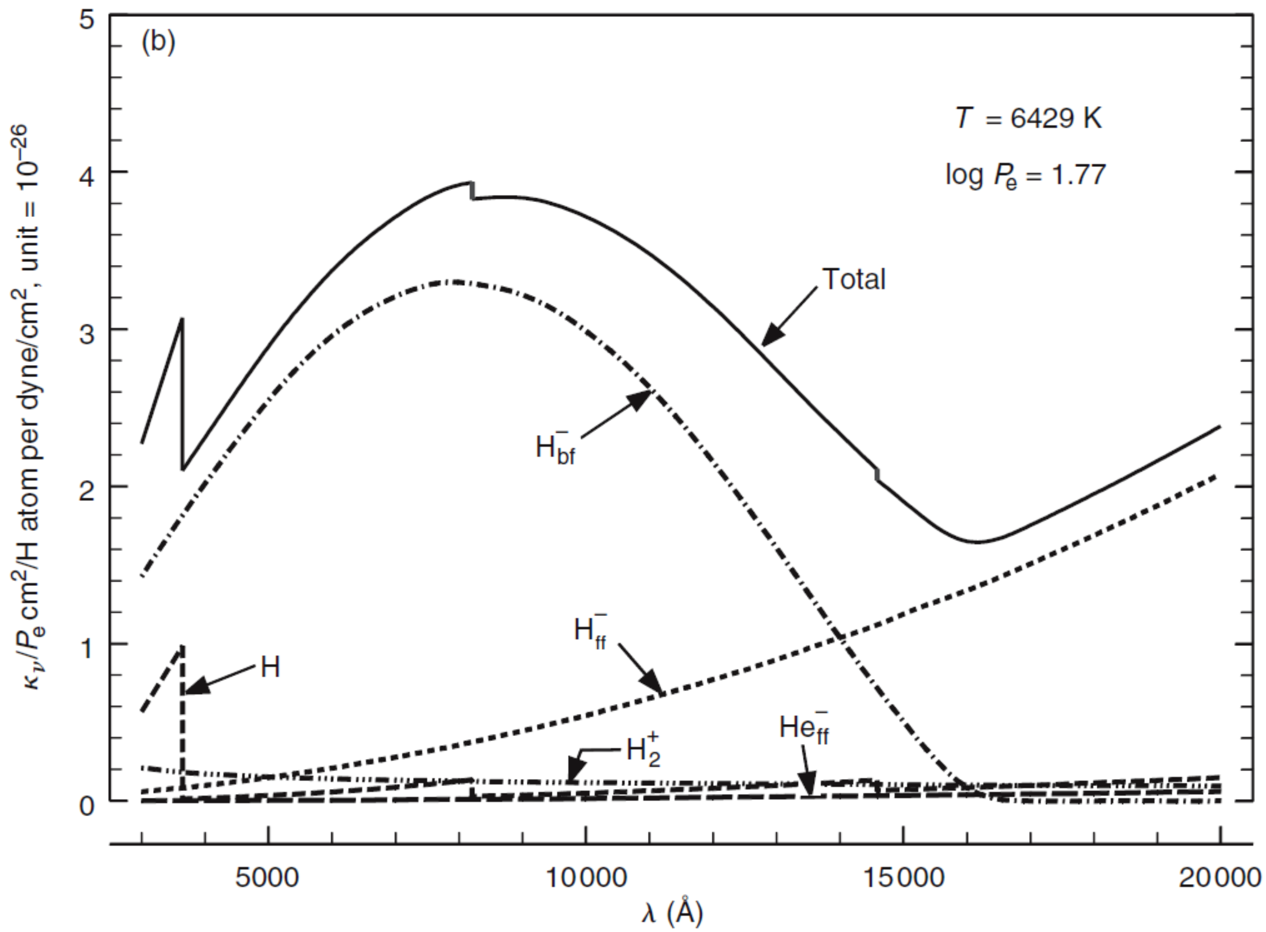
For G stars, the $H^-$ ion (bound-free) dominates for optical.



# F-type (optical depth unity)



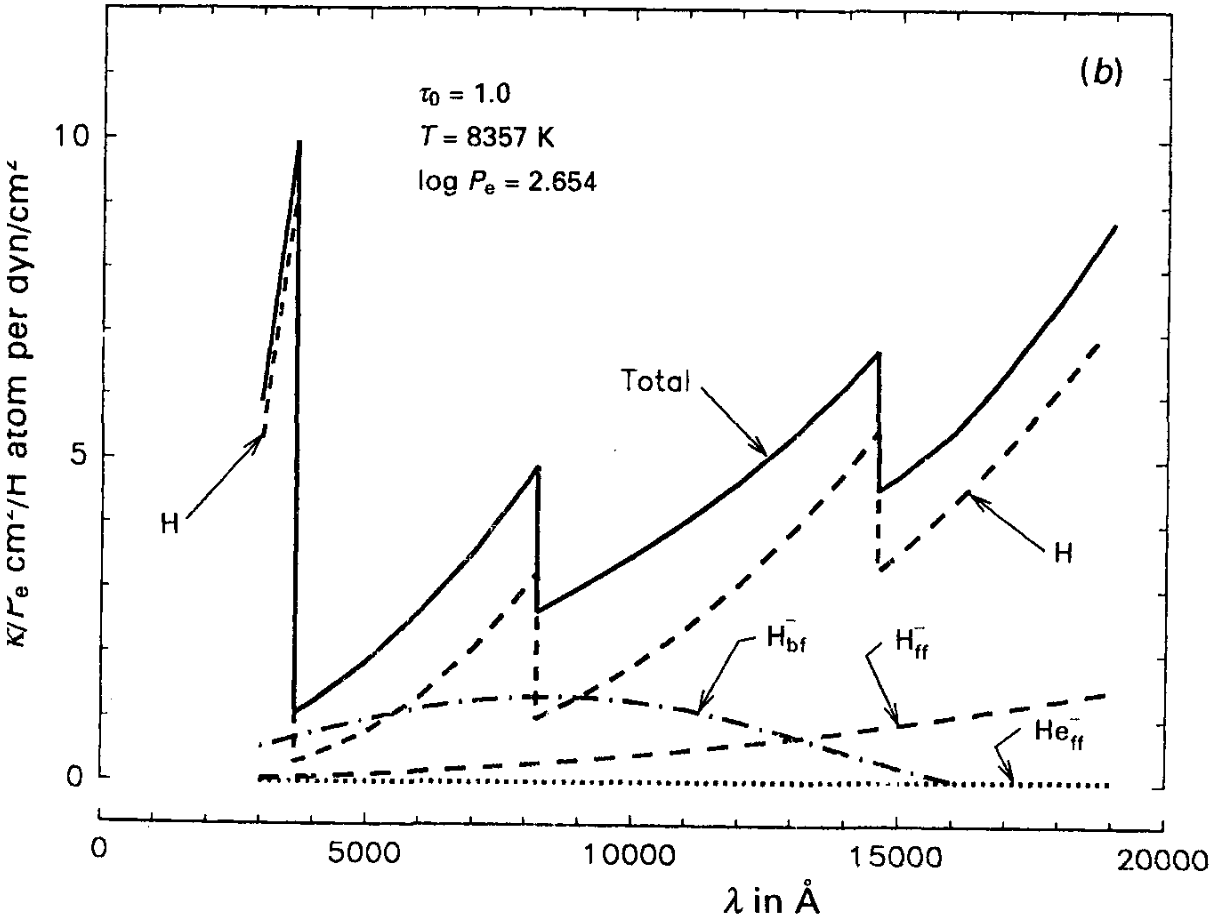
For F stars, the absorption is dominated by the two components of the $H^-$ ion (bound-free) and (free-free), with a contribution from the Balmer continua below 3647Å.



# A-type (optical depth unity)



For a late A star, absorption from the $H^-$ ion is dropping back compared to the cooler cases, while neutral hydrogen has grown with increasing temperature.
H (bound-free) Balmer, Paschen and Brackett continua start to dominate.

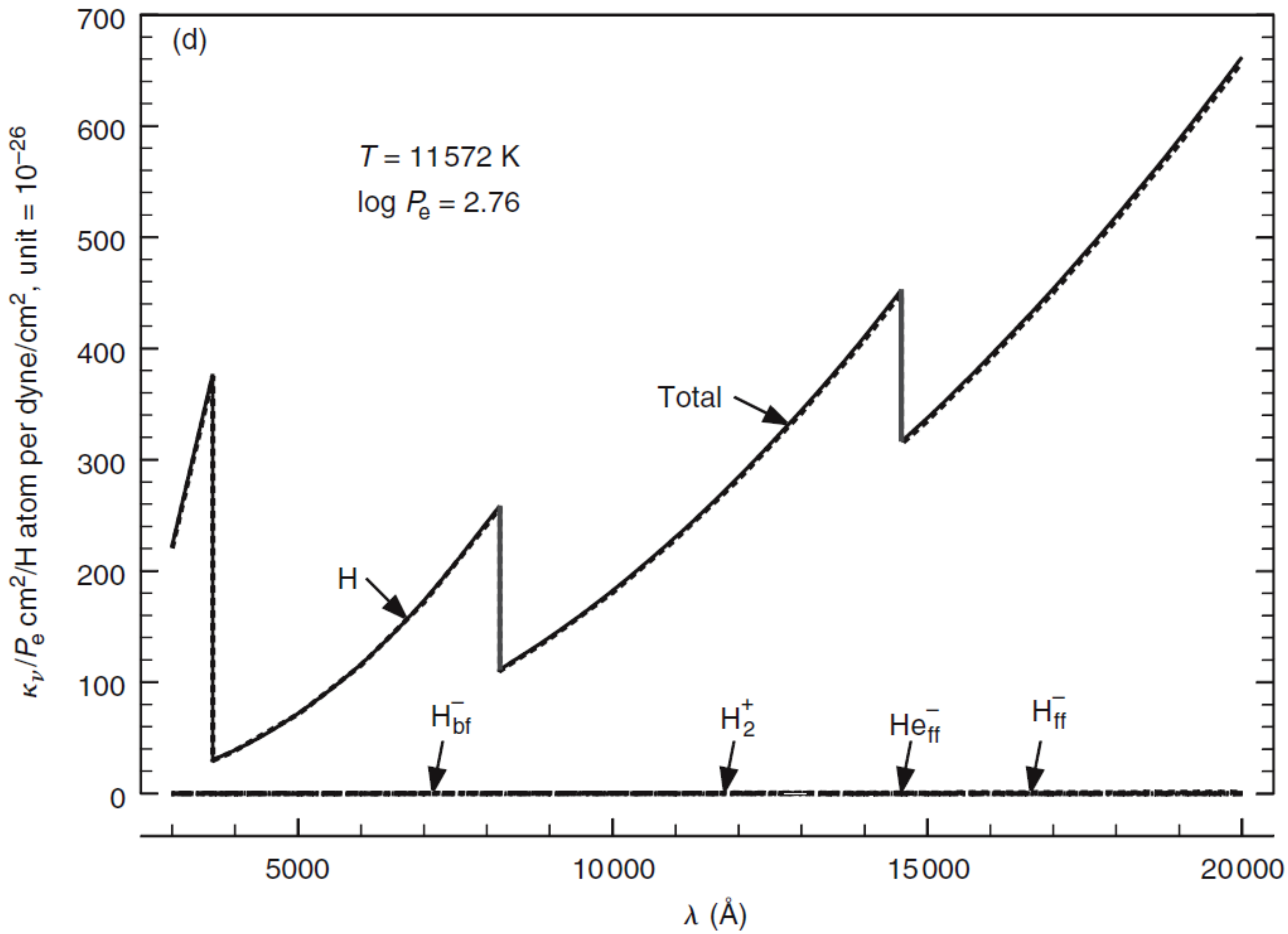


# B-type (optical depth unity)



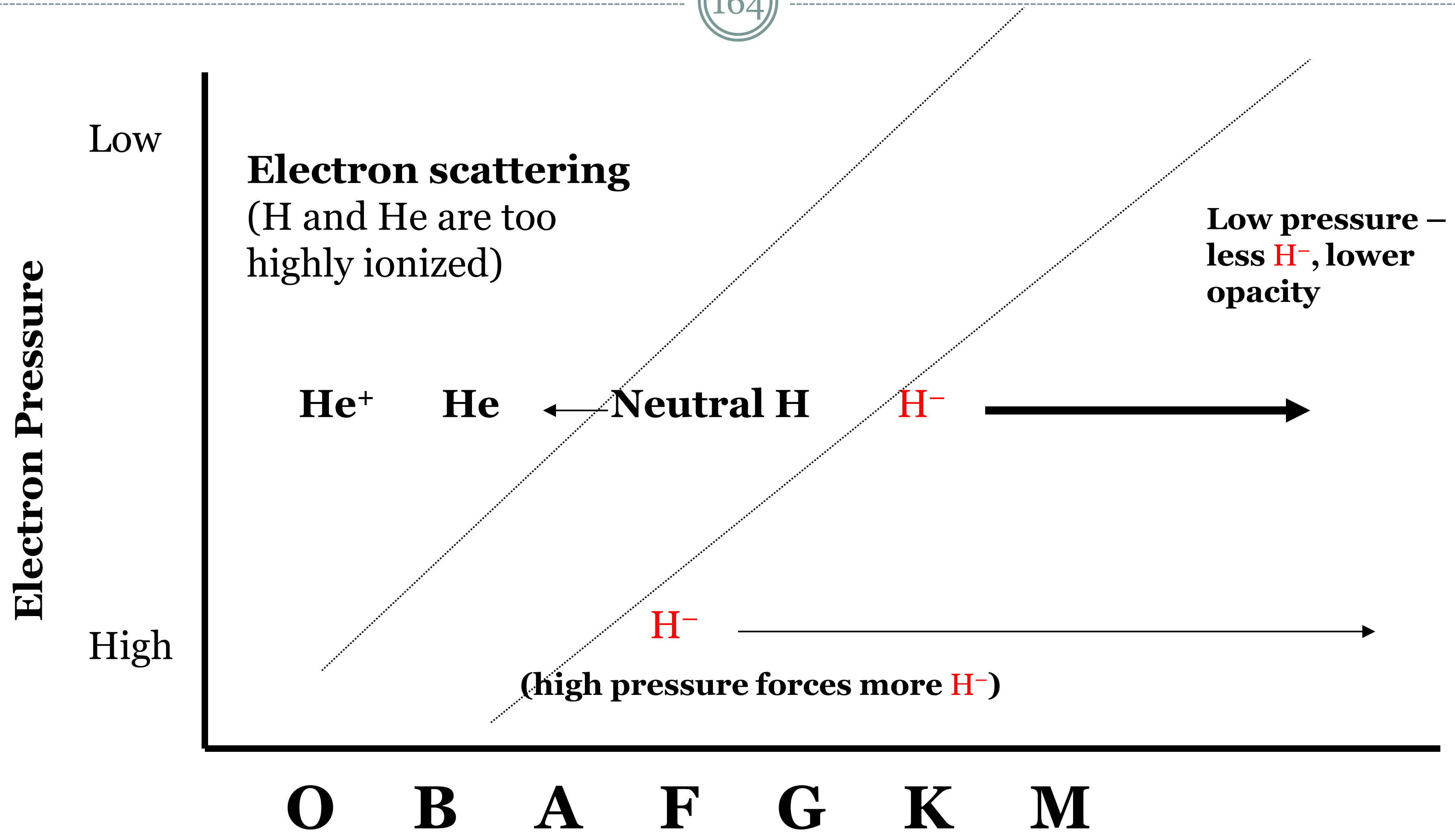
At late B, the H (bound-free) Balmer, Paschen & Brackett continua completely dominate.

For O stars electron scattering is the primary opacity source.



# Dominant Opacity vs. Spectra Type





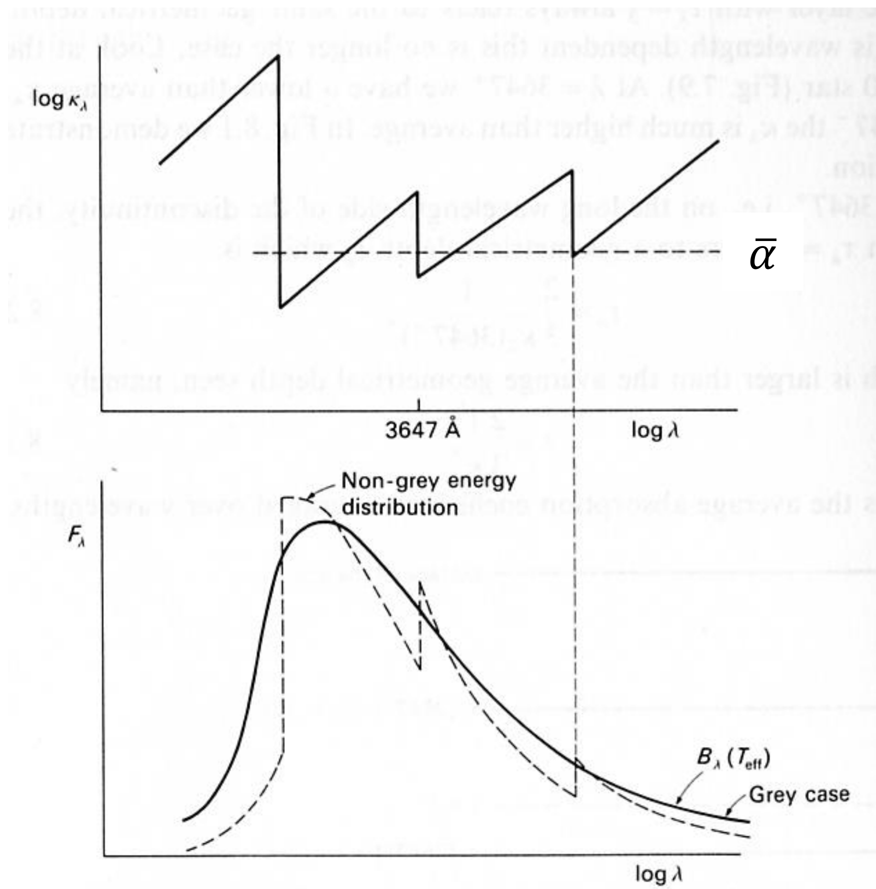
# Continuum Energy Distribution



What is the effect of the $\lambda$ dependence of $\alpha_\lambda$ on the emergent spectrum?

Consider the Balmer discontinuity at 3647Å. Immediately above the discontinuity (3647$^+$), the opacity $\alpha_\lambda$ is **lower** than average (shown as $\bar{\alpha}$), so we probe **deeper** than average into the atmosphere, where $S_\lambda$ (and $F_\lambda$) is **higher** than the grey case, so $F_\lambda$ exceeds the Planck function.

For 3647$^-$, the opacity is **higher** than average, so we probe **less deep** into the atmosphere (where $T$ is smaller), and so receive a lower $F_\lambda$.



# Balmer jump. Why is important?



- In hot stars, T>9000K, $H^-$ negligible, only H contributes to opacity.

$$\frac{\alpha^+}{\alpha^-} = \frac{\sigma^+(\mathrm{H})}{\sigma^-(\mathrm{H})} \frac{N_H(n=3)}{N_H(n=2)}$$

Function of $T$ only

"observed" known From Boltzmann law(T)

Thus, we can obtain the temperature.

In cooler stars (Solar-type)

$$\frac{\alpha^+}{\alpha^-} = \frac{\sigma(H^-)N(H^-) + \sigma^+(H)N_H(n=3)}{\sigma(H^-)N(H^-) + \sigma^-(H)N_H(n=2)}$$

small

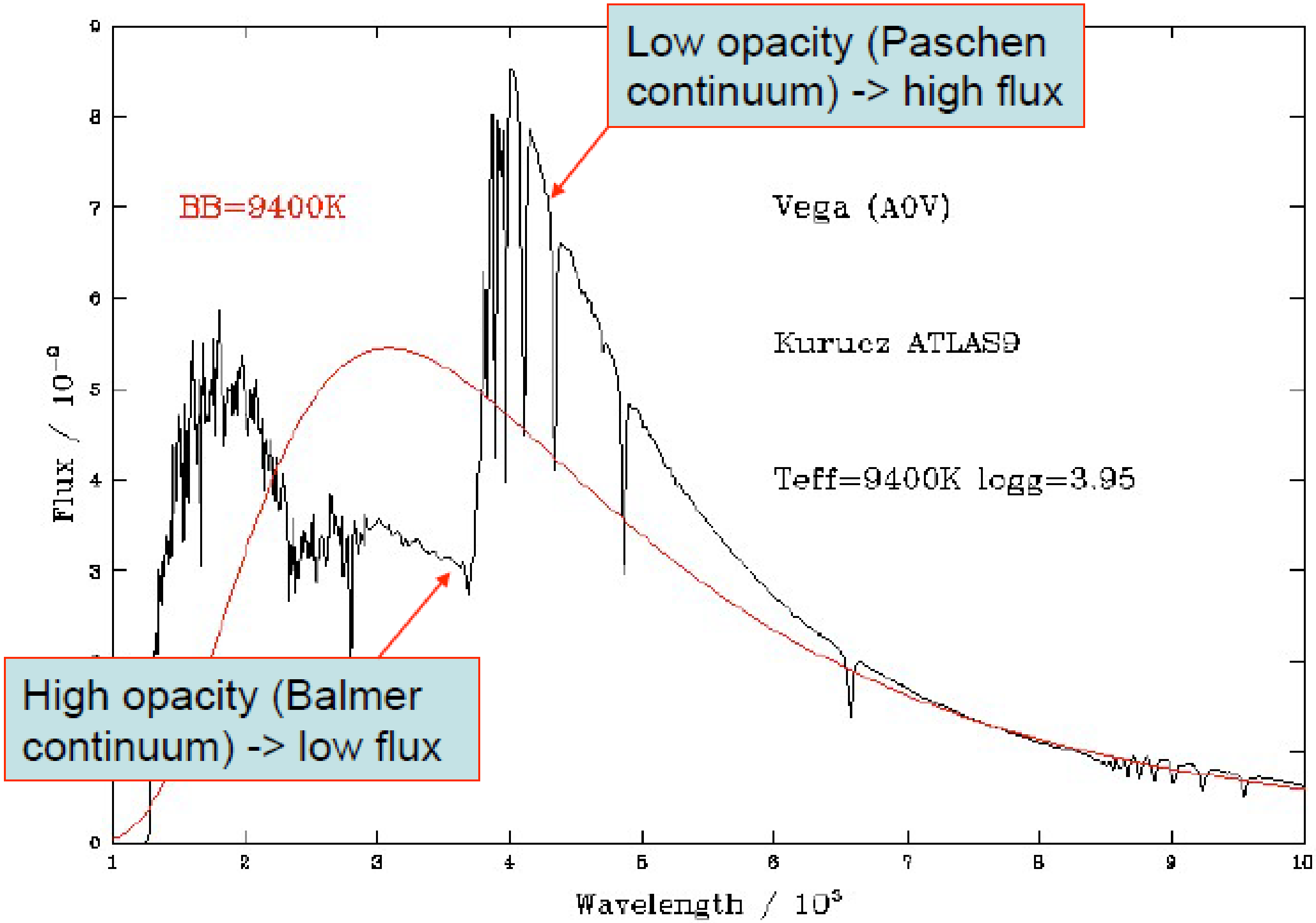
$$N(H^-) = N_H(n=1)n_e f(T) \quad \text{Saha eq} \Rightarrow \frac{\alpha^+}{\alpha^-}(n_e, T)$$

One of $n_e$ or $T$ can be determined

$$\text{if } n_e \uparrow \text{ then } \frac{\alpha^+}{\alpha^-} \rightarrow 1$$

# Balmer jump in Vega





# Summary



- Bound-bound transitions contribute to the line absorption. Bound-free and free-free transitions (plus scattering) contribute to the continuous absorption, mostly by H & He.
- Atomic H absorption coefficient highly *T* sensitive. For late-type stars in the optical and IR, bound-free and free-free transitions of the $H^-$ ion dominate the continuous opacity, since the population of atomic H in n=3 (Paschen series) is so low.
- For early-type stars, atomic H dominates, producing strong jumps in the opacity at the Lyman, Balmer & Paschen edges.
- Negative H ion confirmed as dominant Solar optical & IR opacity source from limb darkening.
- He b-f opacity relevant only for very hot stars. Metal (Fe) opacity contributes to opacity in Solar-type stars in ultraviolet.
- Thompson (electron) scattering is grey & dominates continuum opacity in hot stars. Rayleigh scattering most important for late-type supergiants in UV
- Observed form of e.g. Balmer jump in A stars can be understood from the discontinuity in continuous H b-f opacity.
- Nongreyness changes the temperature structure.

# Spectral lines

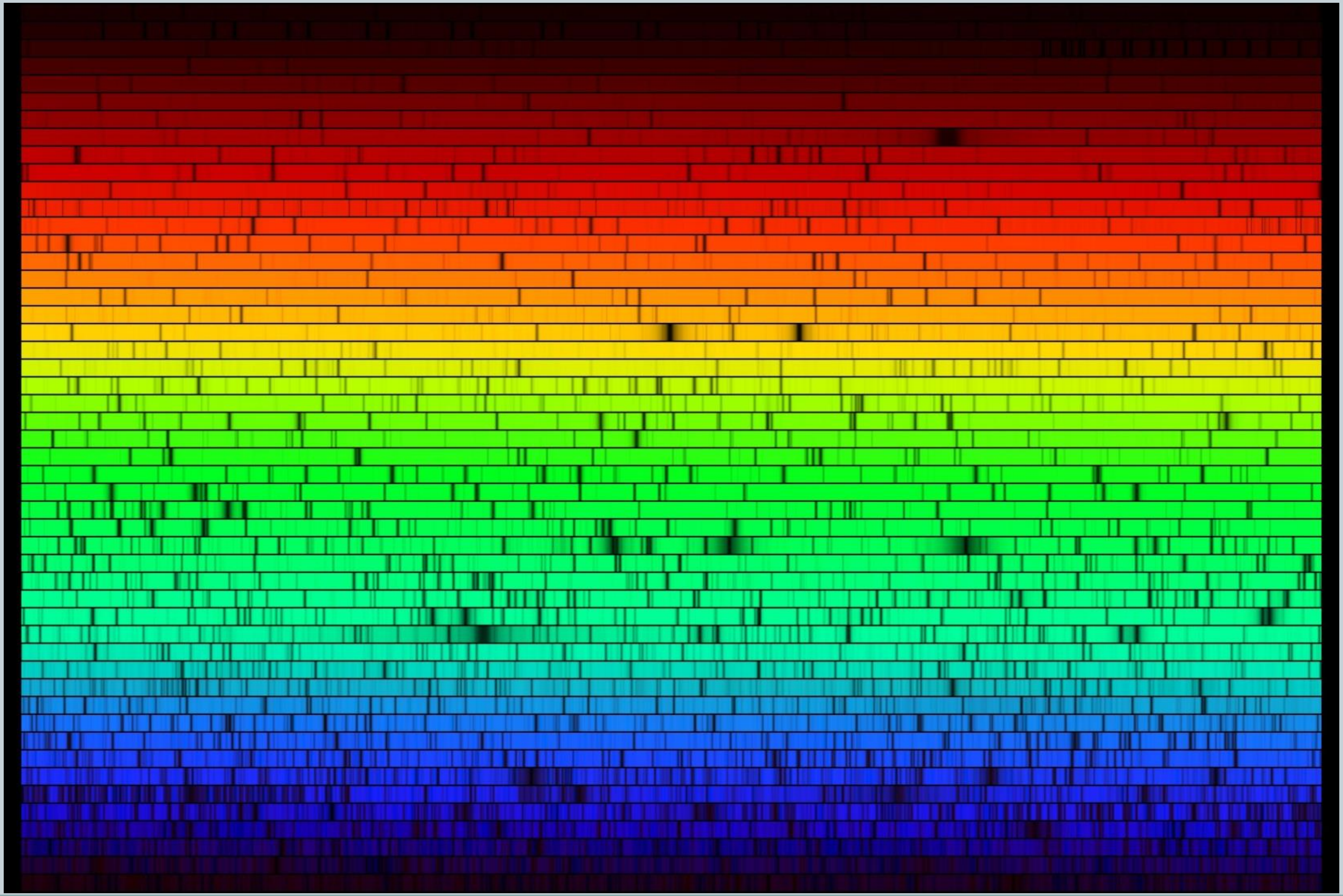


EQUIVALENT WIDTH
FWHM
FWZI

# Spectral Lines



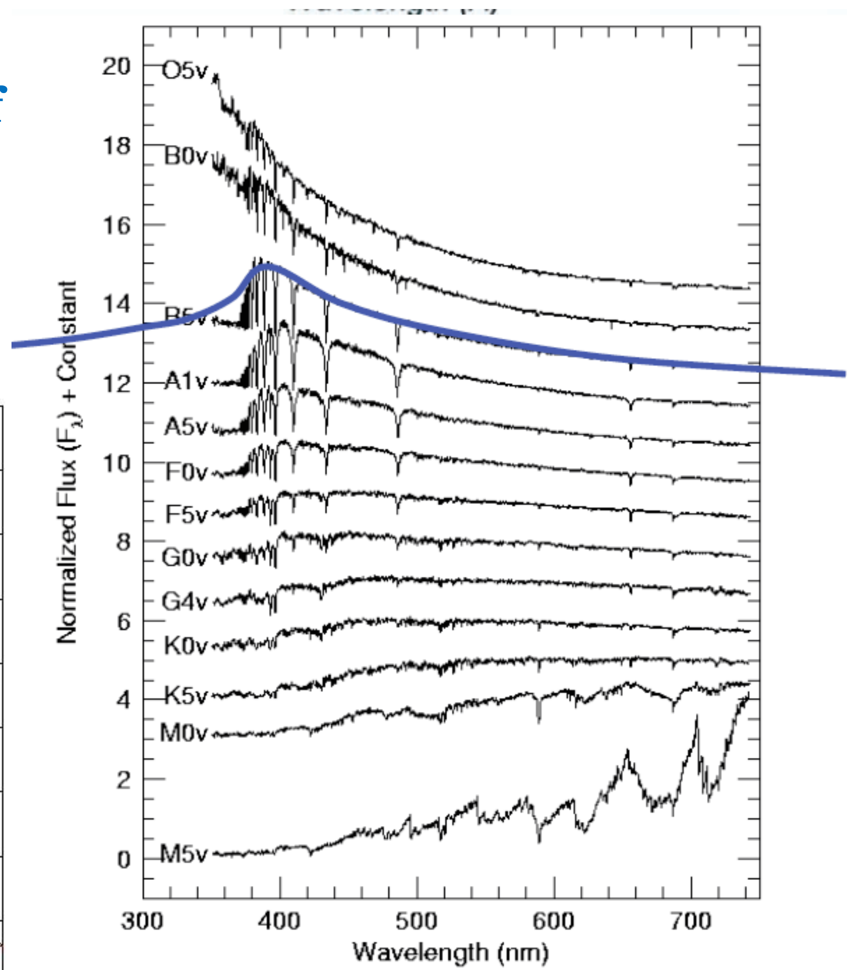
(e.g. 2D echelle image of optical Solar spectrum)



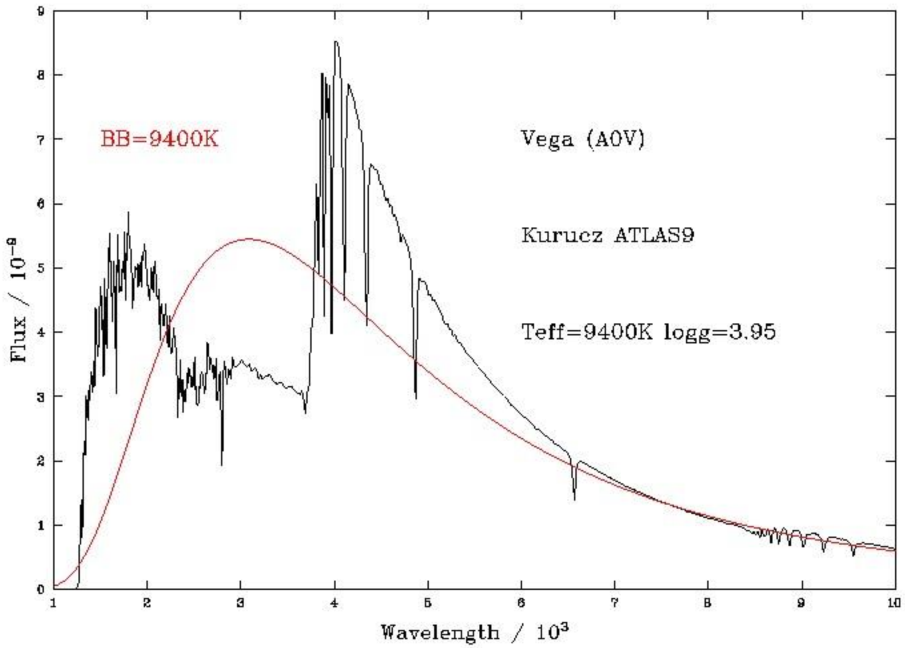
# Continuous Energy Distribution



Dwarf Stars



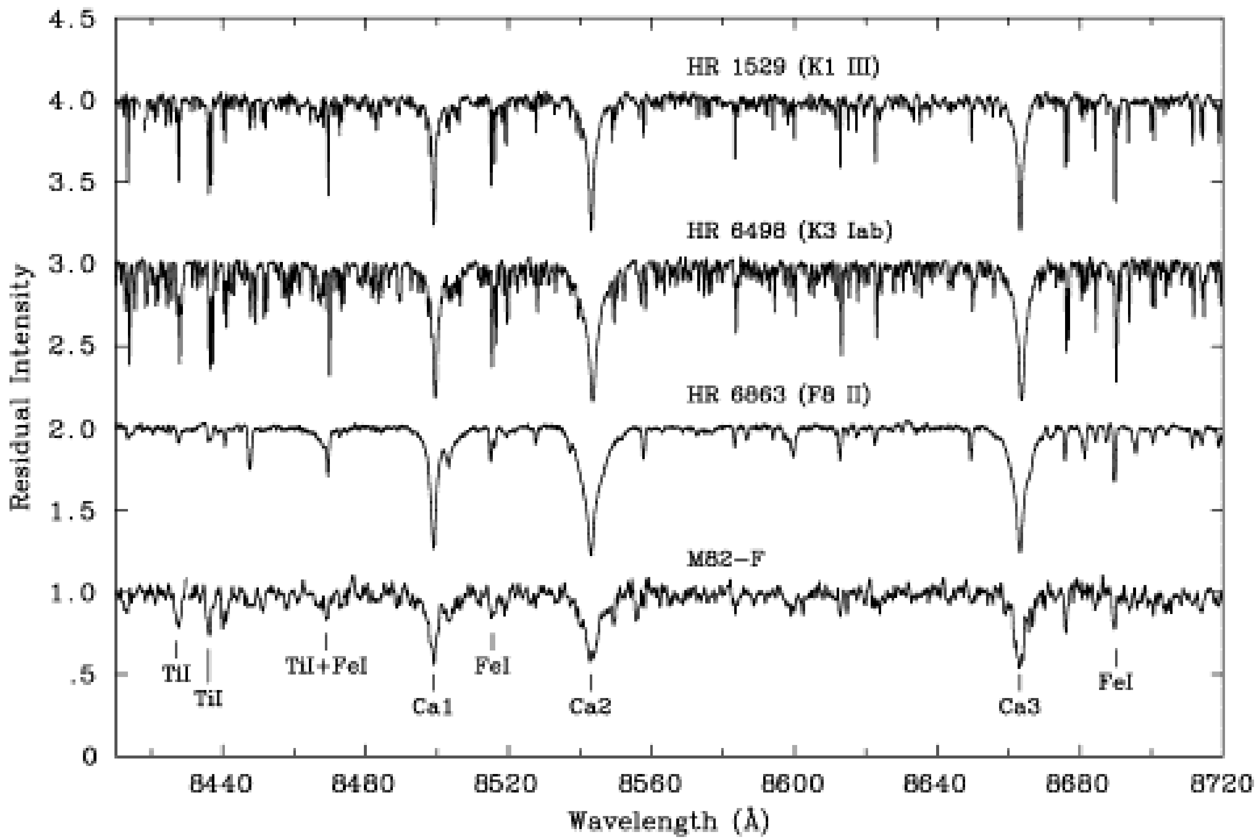

Vega



# Spectra of stars, clusters, galaxies…



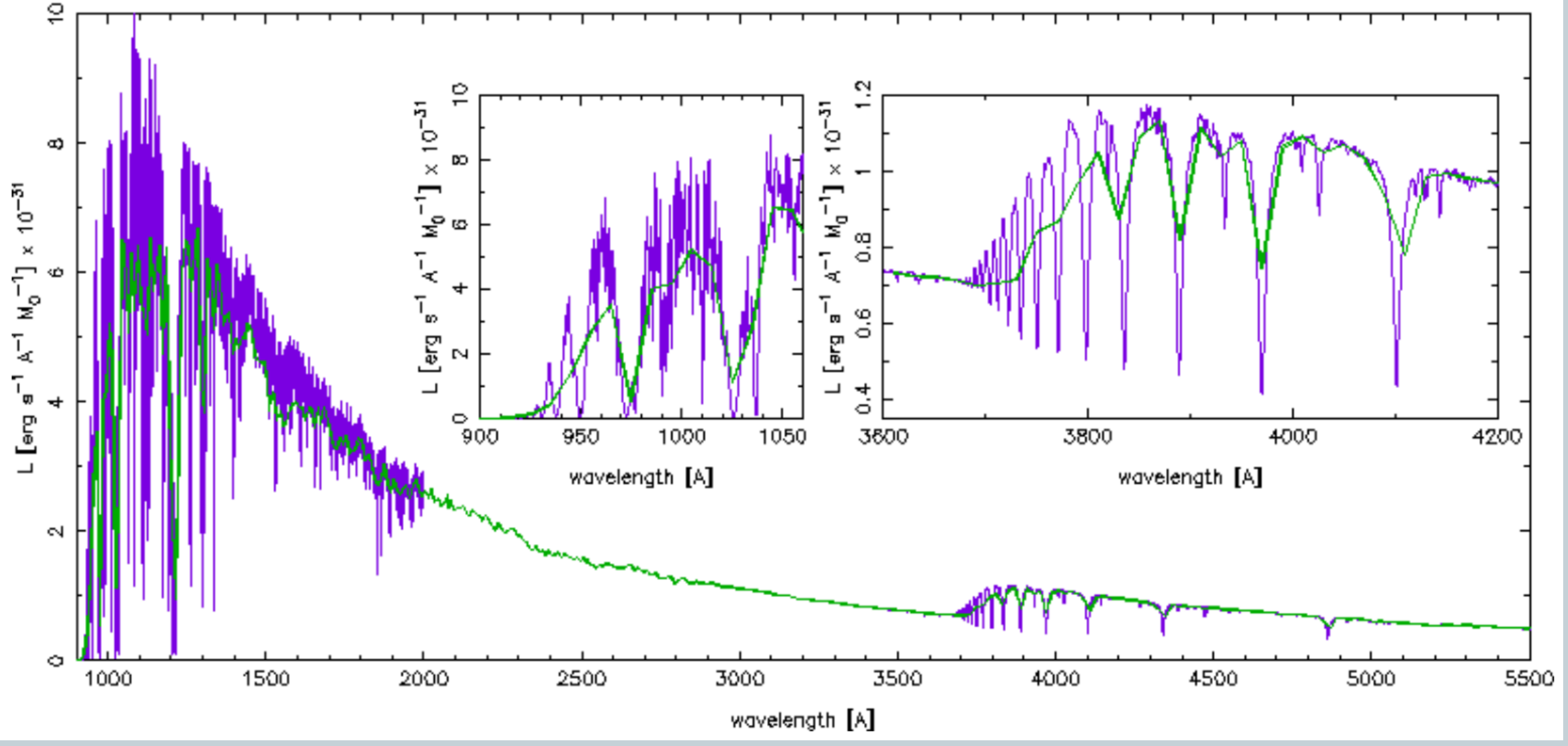
Spectral lines and continuum energy distributions provide temperatures and metallicity of individual stars, plus ages of clusters & galaxies (since the highest mass stars are visually the brightest).



# Spectral Lines



## Impact of Spectral Resolution



# Line depth

We now turn from the continuous energy distribution to the line spectrum.

Relative intensity $r_\lambda$ (*not very common term, usually applied to emission lines*):

$$r_\lambda = \frac{F_\lambda}{F_c}$$

The line depth $R_\lambda$:

$$R_\lambda = \frac{F_c - F_\lambda}{F_c} = 1 - \frac{F_\lambda}{F_c}$$

The largest $R_{\lambda,0}$ — the central line depth

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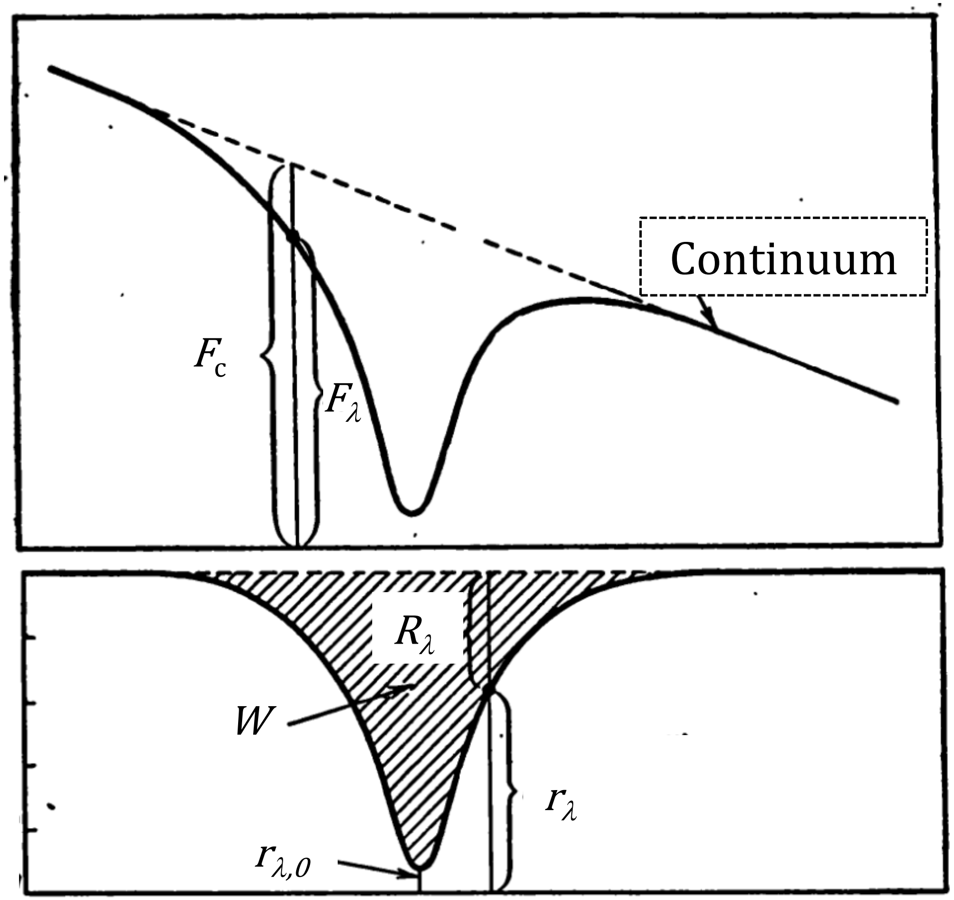


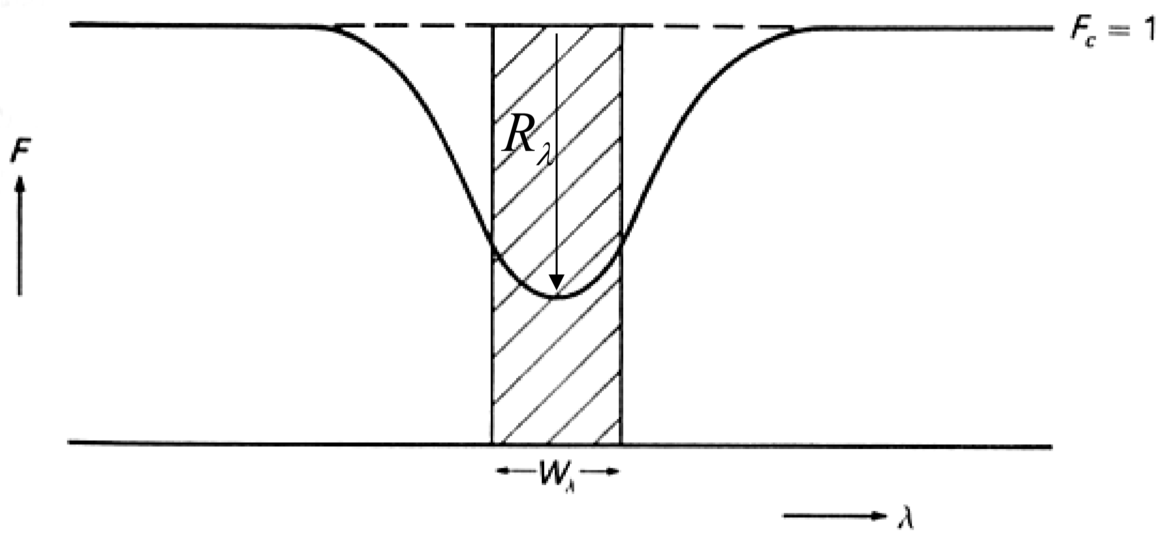
# Equivalent Width



- The total area in a spectral line divided by the continuum flux $F_c$ is called the line **equivalent width**, i.e. an integral over a line **depth $R_\lambda$**

$$W_\lambda = \int \frac{F_c - F_\lambda}{F_c} d\lambda = \int R_\lambda d\lambda$$

- The division by the continuum flux means that this is a measurement of the flux in units of the continuum – the equivalent width is identical to a rectangular line of width $W_\lambda$.
- EW of absorption lines is positive, emission lines have negative EWs, and are measured in Ångströms (at optical wavelengths).



# Energy Flux of (emission) line



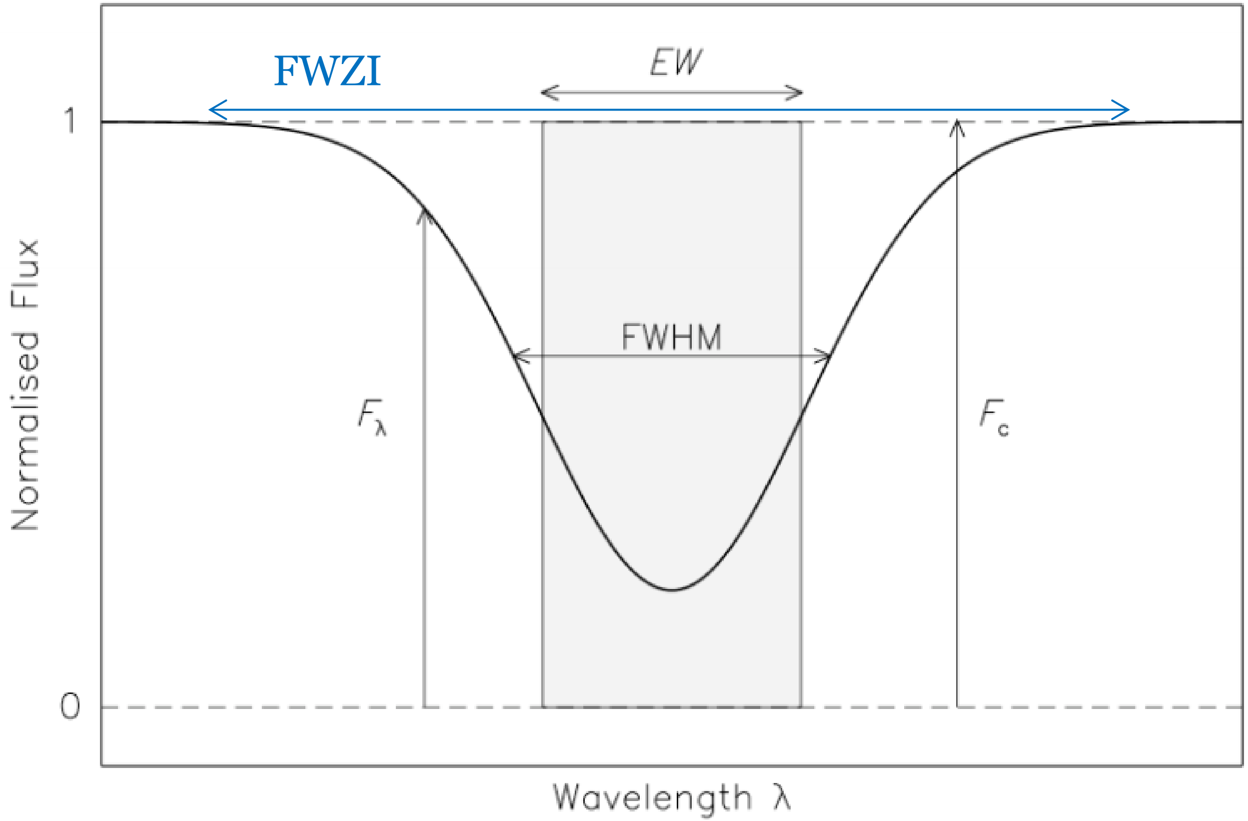
- The total area in a spectral line from which the continuum flux $F_c$ subtracted is called the **energy flux of the line**

$$F = \int (F_\lambda - F_c)\, d\lambda$$

# FWHM and FWZI



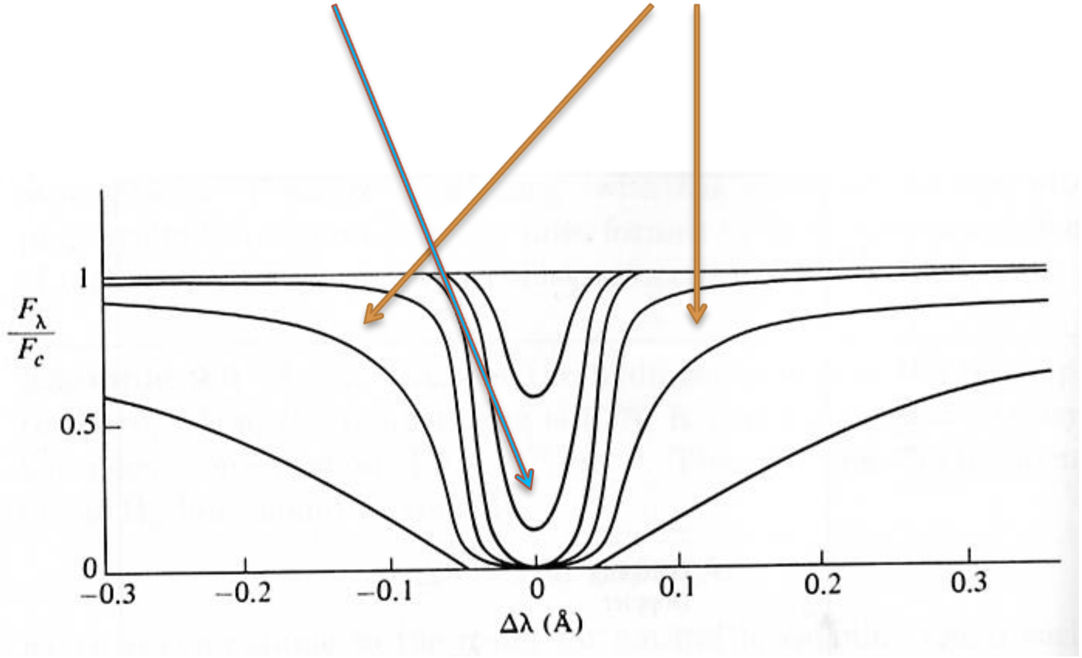
- Other measures of the line width are the **Full Width at Half Maximum (FWHM)**, the distance between the half line depth from blue to red, i.e. $(\Delta\lambda)_{1/2}$ , and the **Full Width at Zero Intensity (FWZI)**,



# Line core and the wings



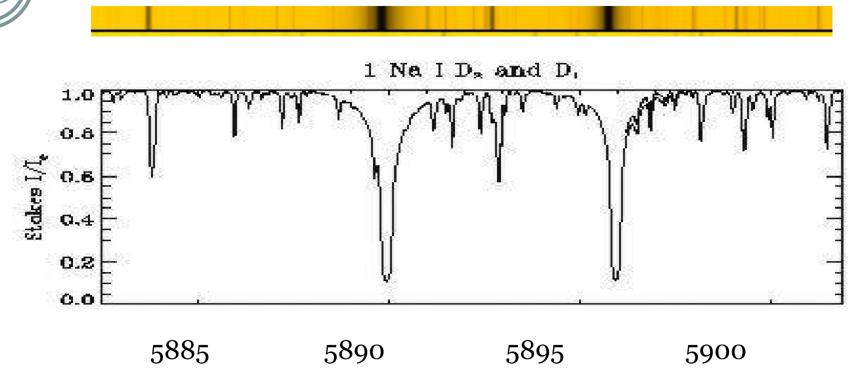
- We denote **optically** (**thin**) **thick** lines as those in which the line core is (**not**) **saturated**, i.e. reaching zero intensity. In reality, zero intensity is only reached for lines in non-LTE.
- The region close to the centre of the spectral line is referred to as the line **core**, whilst the **wings** sweep up the local continuum.



# Example: Solar spectrum



| λ | Element | $W$(Å) | Name |
|---|---|---|---|
| 4920.51 | Fe I | 0.43 | |
| 4957.61 | Fe I | 0.45 | |
| 5167.33 | Mg I | 0.65 | $b_4$ |
| 5172.70 | Mg I | 1.26 | $b_2$ |
| 5183.62 | Mg I | 1.58 | $b_1$ |
| 5232.95 | Fe I | 0.35 | |
| 5269.55 | Fe I | 0.41 | |
| 5324.19 | Fe I | 0.32 | |
| 5238.05 | Fe I | 0.38 | |
| 5528.42 | Mg I | 0.29 | |
| 5889.97 | Na I | 0.63 | $D_2$ |
| 5895.94 | Na I | 0.56 | $D_1$ |
| 6122.23 | Ca I | 0.22 | |
| 6162.18 | Ca I | 0.22 | |
| 6562.81 | $H_\alpha$ | 4.02 | C |
| 6867.19 | $O_2$ | tell | B |
| 7593.70 | $O_2$ | tell | A |
| 8194.84 | Na I | 0.30 | |
| 8498.06 | Ca II | 1.46 | |
| 8542.14 | Ca II | 3.67 | |
| 8662.17 | Ca II | 2.60 | |
| 8688.64 | Fe I | 0.27 | |
| 8736.04 | Mg I | 0.29 | |




Strong spectral lines in the Solar spectrum typically have equivalent widths $W_\lambda \approx 1$Å, such as the Na I D lines in the yellow. In other stars, line equivalent widths can reach tens or even hundreds of Angstroms. EWs are by definition measured relative to the continuum strength, unlike line fluxes.

# Formation of absorption lines



- We obtained earlier that the emergent flux from the stellar surface is π times the Source function at an optical depth of 2/3:

$$F_\lambda(0) = \pi S_\lambda(\tau_\lambda = 2/3) = \pi B_\lambda(T(\tau_\lambda = 2/3))$$

↑ **LTE**

- In spectral lines, the opacity is much larger, thus we see much higher layers at these wavelengths. These layers have a lower temperature and so $B_\lambda$ is smaller, leading to a smaller $F_\lambda$ in the line than $F_c$, the continuum flux in the neighbourhood of the line.

- In the following few lectures, we will study theory of line formation.

# Spectral line formation



**EINSTEIN COEFFICIENTS**

LINE PROFILES: NATURAL BROADENING

BROADENING OF SPECTRAL LINES

NATURAL LINE BROADENING

THERMAL (DOPPLER) BROADENING

CONVOLUTION OF DIFFERENT BROADENING PROCESSES

PRESSURE BROADENING

INGIS-TELLER RELATION

ROTATIONAL AND INSTRUMENTAL BROADENING

# Bound-Bound transitions



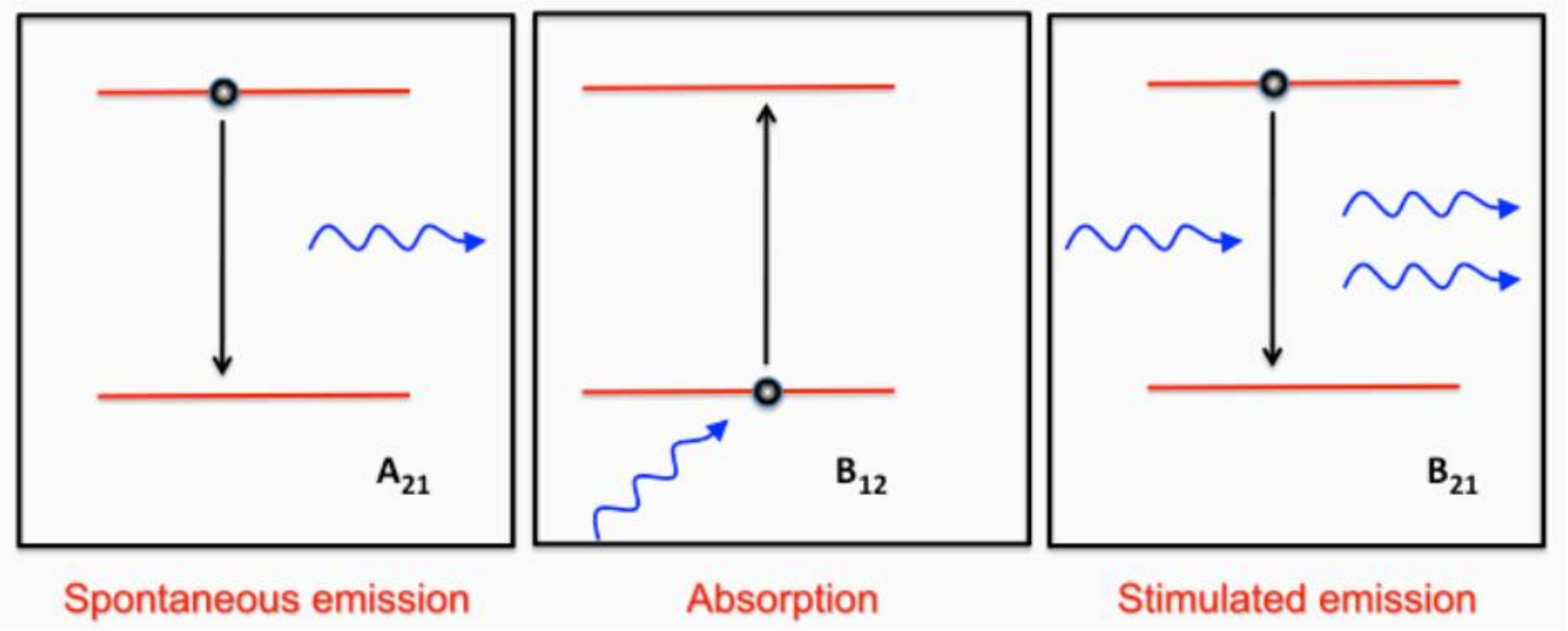
There are 3 basic kinds of line processes associated with bound-bound transitions of atoms or ions:

1. **Direct Absorption**, in which the absorbed photon induces a bound electron to go into a higher energy level.
2. **Spontaneous Emission**, in which an electron in a higher energy level spontaneously decays to lower level, emitting the energy difference as a photon.
3. **Stimulated Emission**, in which an incoming photon induces an electron in a higher energy level to decay to a lower level, emitting in effect a second photon that is nearly identical in energy (and even phase) to the original photon.

The probability that the atom will emit (or absorb) its quantum of energy is described by **Einstein probability coefficients**, written as $B_{ij}$ , $A_{ji}$, and $B_{ji}$.

# Einstein coefficients





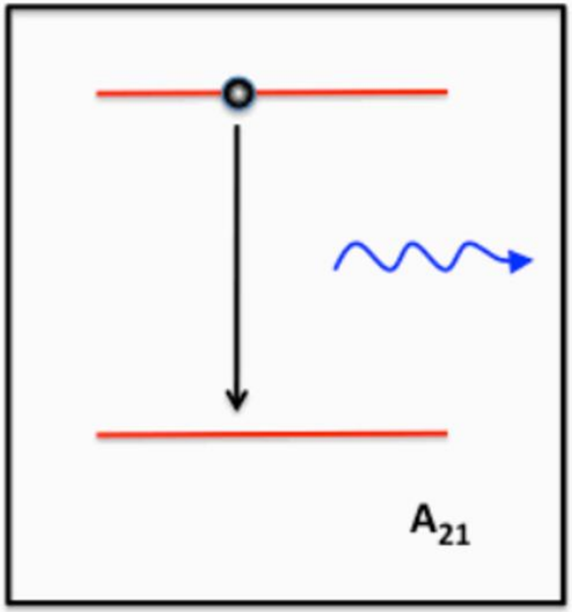

Einstein coefficients concern the probability that a particle spontaneously emits a photon, the probability to absorb a photon, and the probability to emit a photon under the influence of another incoming photon. Einstein's coefficients are valid for all radiation fields.

# Spontaneous emission



Consider an upper level $u$ and a lower level $l$ separated by an energy $h\nu$.

- The probability that the atom will spontaneously emit its quantum of energy within a time $dt$ and in a solid angle $d\omega$ is $A_{ul}\, dt\, d\omega$.
- The proportionality constant, $A_{ul}$, is the Einstein probability coefficient for spontaneous emission [$s^{-1}$].
- Occurs independently of the radiation field.
- Emits **isotropically**.




Spontaneous emission

For H$\alpha$, $A_{32}$=4.4×10$^7$ s$^{-1}$. If at time $t_0$=0 there are $N_u(0)$ atoms in level $u$, then at time $t$ the population is $N_u(t)=N_u(0)\exp(-A_{ul}\, t)$.

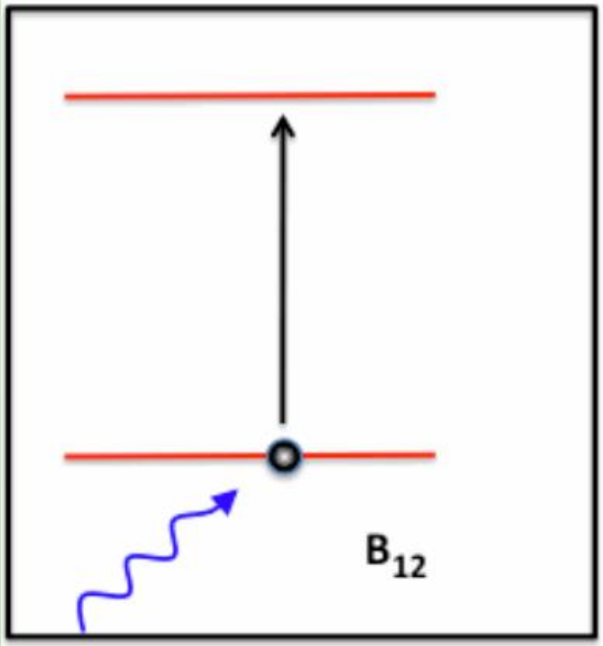
Lifetime = $1/A_{ul}$

# Absorption



Consider an upper level $u$ and a lower level $l$, separated by an energy $h\nu$.

- Photons with energies *close* to $h\nu$ cause transitions from levels $l$ to $u$.
- The probability per unit time for this process will evidently be proportional to the mean intensity $J_\nu$ at the frequency $\nu$.
- $B_{lu} J$ : transition probability of absorption per unit time.
- The proportionality constant $B_{lu}$ is one of the Einstein $B$-coefficients.



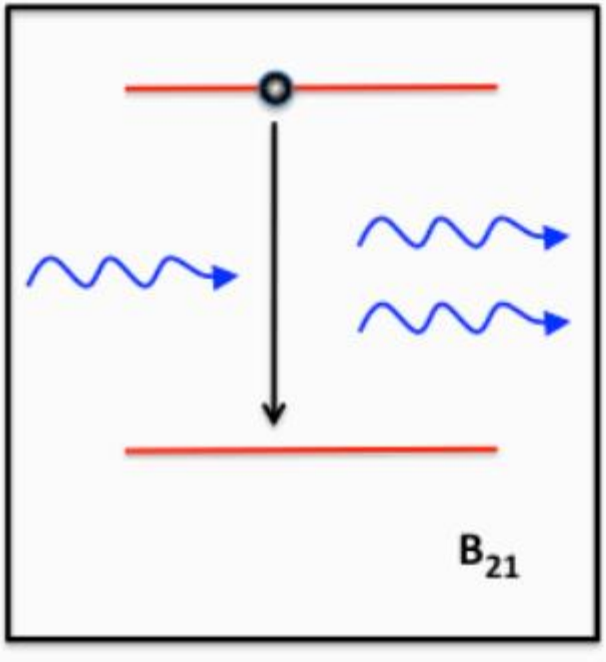

Absorption

# Stimulated emission



Planck's law does **not** follow from considering only spontaneous emission and absorption. Must also include stimulated emission, which like absorption is proportional to the mean intensity *J*.

- The system goes from an upper level *u* to a lower level *l* stimulated by the presence of a radiation field ($h\nu$ corresponding to the energy difference between levels *u* and *l* ).
- The energy of the emitted photon is the **same** as of the incoming photon (also direction and phase are the same).
- $B_{ul} J$ : transition probability of stimulated emission per unit time.
- The proportionality constant $B_{ul}$ is a second Einstein *B*-coefficient.
- The process of stimulated emission is sometimes referred to as a process of *negative absorption*.
- Stimulated emission occurs into the **same** state (frequency, direction, polarization) as the photon that stimulated the emission.




Stimulated emission

# Relation between Einstein coefficients



Einstein's Coefficients are not independent. To find a relation between them, let's assume strict Thermodynamic Equilibrium (TE), and, for simplicity, adopt a 2-level approximation.

In TE, each process is in equilibrium with its inverse, i.e., within one line there is no netto destruction or creation of photons (detailed balance)

$$n_1 B_{12} J_\nu = n_2 A_{21} + n_2 B_{21} J_\nu$$

Transitions 1→2 equal to 2→1
$n_1$, $n_2$: number density of $e^-$ in levels 1,2

$$J_\nu = \frac{A_{21}/B_{21}}{\left(\frac{n_1}{n_2}\right)\left(\frac{B_{12}}{B_{21}}\right) - 1}$$

Thermodynamic equilibrium:
Boltzmann, $J = B_\nu(T)$

$$\frac{n_1}{n_2} = \frac{g_1}{g_2} e^{h\nu_{21}/kT}$$

# Relation between Einstein coefficients



TE: blackbody, $J = B_\nu(T)$

$$B_\nu(T) = \frac{A_{21}/B_{21}}{\left(\frac{g_1 B_{12}}{g_2 B_{21}}\right) e^{h\nu_{21}/kT} - 1}$$

Comparison with Planck blackbody radiation:

$$B_\nu(T) = \frac{A_{21}}{B_{21}} \left( \frac{g_1 B_{12}}{g_2 B_{21}} e^{\frac{h\nu_{21}}{kT}} - 1 \right)^{-1} = \frac{2h\nu_{21}^3}{c^2} \left( e^{\frac{h\nu_{21}}{kT}} - 1 \right)^{-1}$$

$$\frac{A_{21}}{B_{21}} = \frac{2h\nu_{21}^3}{c^2} \rightarrow A_{21} = B_{21} \frac{2h\nu_{21}^3}{c^2}$$

$$\frac{g_1 B_{12}}{g_2 B_{21}} = 1 \rightarrow g_1 B_{12} = g_2 B_{21}$$

# Einstein coefficients



Thus, if one of the Einstein Coefficients is known then two other can be calculated.

**Important:** The Einstein's coefficients are **atomic constants**.
Although the above relations were derived under the conditions of TE, these relations hold in any non-TE state.

Total amount of absorbed photons per unit time at a given frequency is

$$n_1 B_{12} J_\nu - n_2 B_{21} J_\nu = n_1 B_{12} J_\nu \left(1 - \frac{n_2 B_{21}}{n_1 B_{12}}\right) = n_1 B_{12} J_\nu \underbrace{\left(1 - \frac{g_1 n_2}{g_2 n_1}\right)}$$

Thus, to take into account negative absorption (stimulated emission), one must multiply the number of absorbed photons by

$$\left(1 - e^{-h\nu_{12}/kT}\right)$$

(we already did it before)

# Comparison of induced and spontaneous emission

Home work:

- When (at what temperatures, wavelengths) is spontaneous or induced emission stronger?

  Assume LTE (blackbody)

# Lifetime of atom in excited state



In the absence of collisions and of any other transitions than the ***ul*** one, the mean lifetime of particles in state ***u*** is **Lifetime = 1/$A_{ul}$**

If at time $t_0$=0 there are $N_u(0)$ atoms in level $u$, then at time $t$ the population is

$$N_u(t) = N_u(0)e^{-A_{ul}t}.$$

Typical value of $A_{ij}$ is $10^7$- $10^8$ $s^{-1}$ (for Hα, $A_{32}=4.4\times10^7$ $s^{-1}$), so lifetime is ~$10^{-8}$ s.

**However, not all transitions are allowed, some are strictly ~~forbidden~~!**

In practice, strictly forbidden means **very low probability of occurrence** ➔ **Metastable states** at which a lifetime is much longer than of the ordinary excited states but shorter than of the ground state.

**Lifetimes at metastable states can reach several hours and even longer!**

Forbidden line transitions are noted by placing square brackets around the atomic species in question, e.g. [O III] or [S II]. A semi-forbidden line, designated with a single square bracket, such as C III], occurs where the transition probability is about a thousand times higher than for a forbidden line.

# Einstein A-coefficients for Hydrogen



| k \ i | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 2 | $4{,}67 \cdot 10^{8}$ | — | — | — | — | — | — |
| 3 | $5{,}54 \cdot 10^{7}$ | $4{,}39 \cdot 10^{7}$ | — | — | — | — | — |
| 4 | $1{,}27 \cdot 10^{7}$ | $8{,}37 \cdot 10^{6}$ | $8{,}94 \cdot 10^{6}$ | — | — | — | — |
| 5 | $4{,}10 \cdot 10^{6}$ | $2{,}52 \cdot 10^{6}$ | $2{,}19 \cdot 10^{6}$ | $2{,}68 \cdot 10^{6}$ | — | — | — |
| 6 | $1{,}64 \cdot 10^{6}$ | $9{,}68 \cdot 10^{5}$ | $7{,}74 \cdot 10^{5}$ | $7{,}67 \cdot 10^{5}$ | $1{,}02 \cdot 10^{6}$ | — | — |
| 7 | $7{,}53 \cdot 10^{5}$ | $4{,}37 \cdot 10^{5}$ | $3{,}34 \cdot 10^{5}$ | $3{,}03 \cdot 10^{5}$ | $3{,}24 \cdot 10^{5}$ | $4{,}50 \cdot 10^{5}$ | — |
| 8 | $3{,}85 \cdot 10^{5}$ | $2{,}20 \cdot 10^{5}$ | $1{,}64 \cdot 10^{5}$ | $1{,}42 \cdot 10^{5}$ | $1{,}38 \cdot 10^{5}$ | $1{,}55 \cdot 10^{5}$ | $2{,}26 \cdot 10^{5}$ |