

[ Theoretical ]  
**Astrophysics**  
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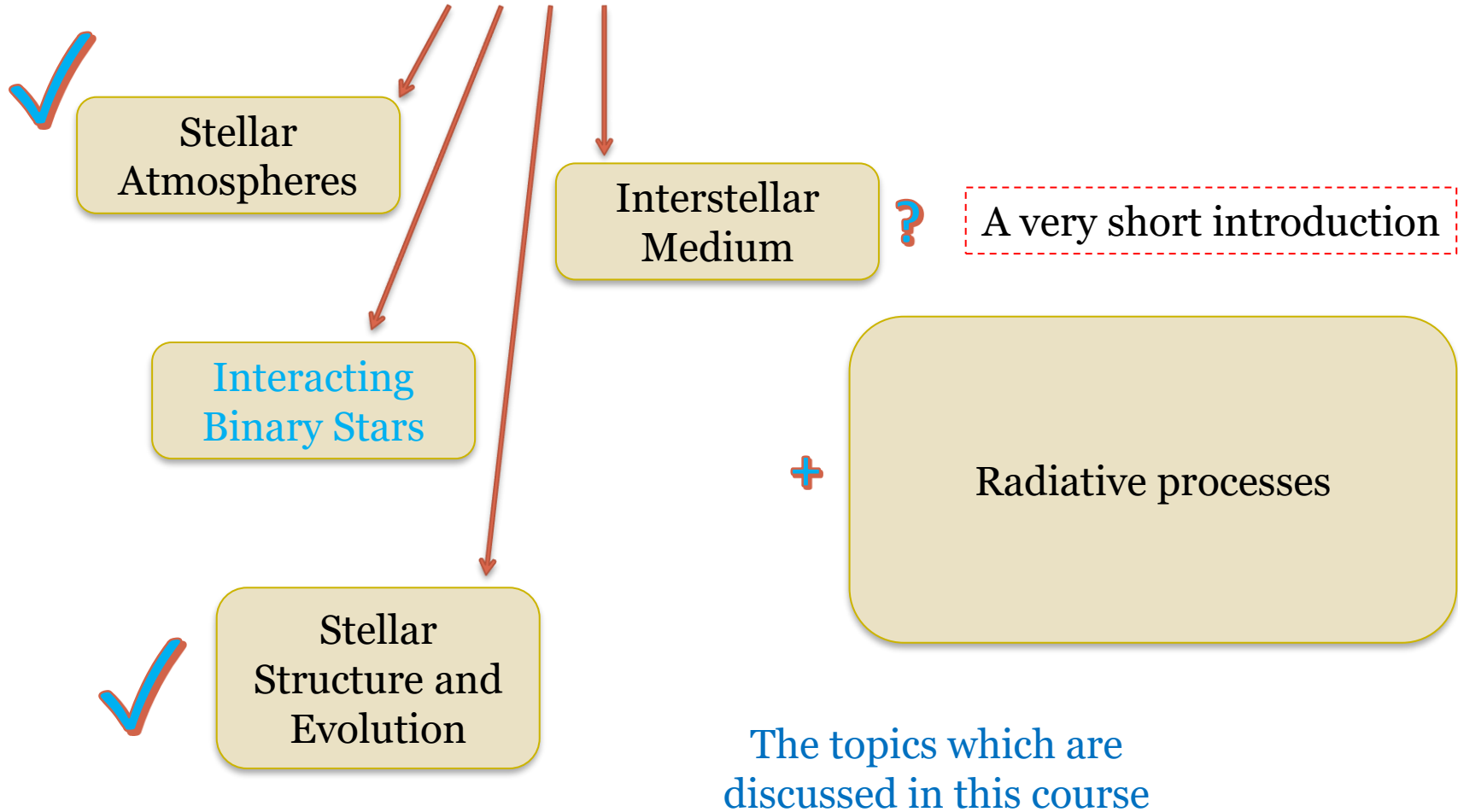
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**Part III**

# Theoretical Astrophysics



# A short introduction to the Interstellar Medium

3

DEFINITION OF THE INTERSTELLAR MEDIUM  
HISTORY  
THE ZOO OF OBJECTS IN THE ISM  
BASIC PROPERTIES AND PHASES OF THE ISM

# Definition of the interstellar medium

4

- The interstellar medium, or **ISM**, is the nearly-empty space inside our Galaxy in which all the objects we've studied thus far are embedded (note that there is also an **intergalactic** medium in the spaces between galaxies).
- The Milky Way (as well as other galaxies) is full of gas, dust, cosmic rays, and radiation. This forms a very complex medium, that often affects many processes involving stars and compact objects, as well as our observations of these objects.
  - The radiation of the ISM is faint, and the bulk of photons are emitted in the radio and infrared wavelength regime.
  - The ISM hosts only a few percent of the total mass of the galaxy.
  - The composition of the ISM is continuously modified by the stellar evolution.
  - The distribution and chemical composition of the ISM is also continuously modified by accretion of matter by the galaxy.
  - Density and temperature variations within the ISM trigger the star formation rate of a galaxy.

## History (1)

In 17th-19th centuries, it was assumed that the vast space between the stars is completely empty (vacuum).

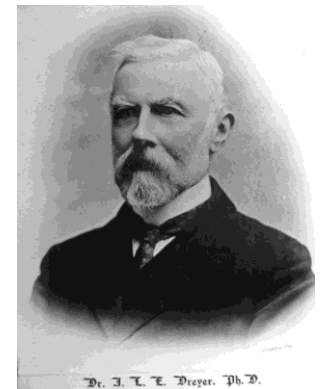
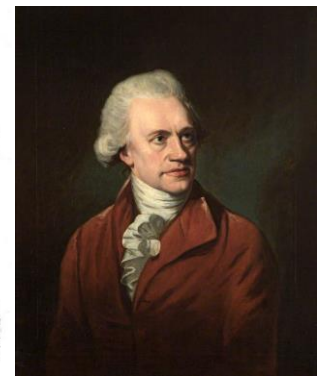
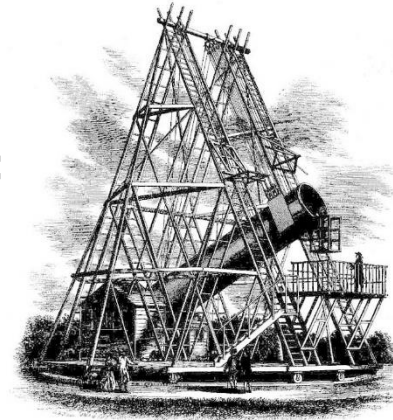
1610 - Discovery of the Orion Nebula (Nicolas Fabri de Peiresc) - “surprised to find a small illuminated cloud”.

1656 - First detailed description of the Orion Nebula (Christian Huygens).



# History (2)

- 1781 - Charles Messier compiled a catalogue of 103 objects:
  - The majority clusters of stars
  - galaxies (35)
  - nebulae (11)
- 1786-1802 – John Herschel discovered 2500 objects, later added 2200 objects:
  - Many of these were dark clouds:  
“here is truly a hole in Heaven”
- 1888 - New General Catalog (NGC) by J.L. Dreyer was published (13,000 objects)

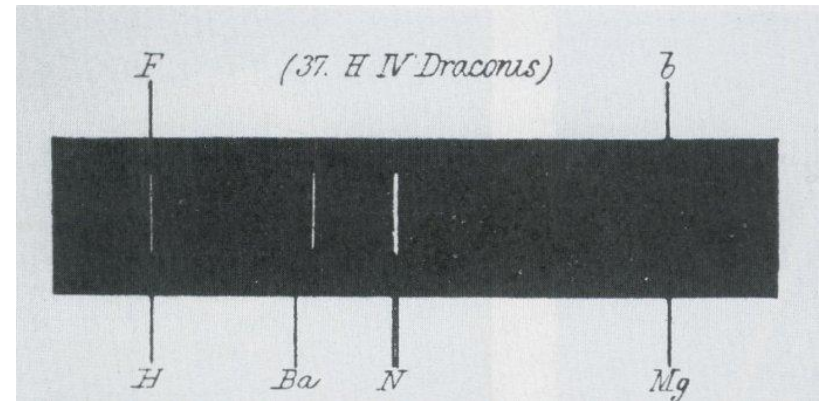


# 19th-century: Advent of spectroscopy

7

Helped distinguish between stellar and gas content:

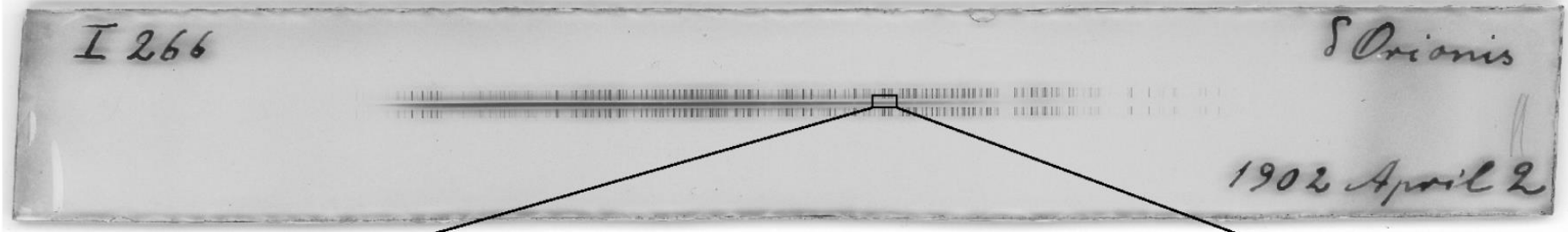
- 1864: Sir William Huggins observed the Cat's Eye Nebula and found **no continuous** spectrum like that seen in the Sun, but just a few strong **emission lines**.
- Two lines were first thought to be from nitrogen (hence the tag "N").
- However, it appeared later that these lines did **not** correspond to any known elements on Earth. These lines were attributed to an anonymous element called **Nebulium**.
- 1927: Ira Sprague Bowen showed that these lines belonged to **forbidden** transitions of oxygen [O III] at 5007 and 4959 Å.



# Interstellar absorption lines

8

- In the beginning of 20th century, in 1904, Johannes Hartmann using spectral analysis found 2 **static** lines of **Ca II H&K** in the spectra of the binary star  $\delta$  Orionis. Hartmann attributed these lines to interstellar medium (ISM). Doppler effect causes variations of stellar lines, but not of ISM absorption lines.

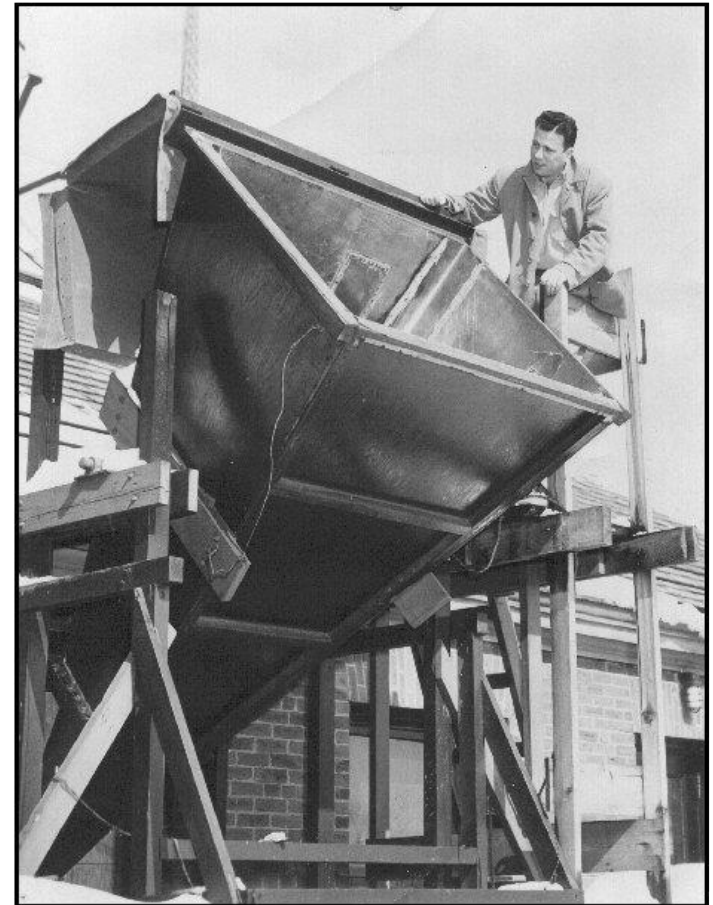
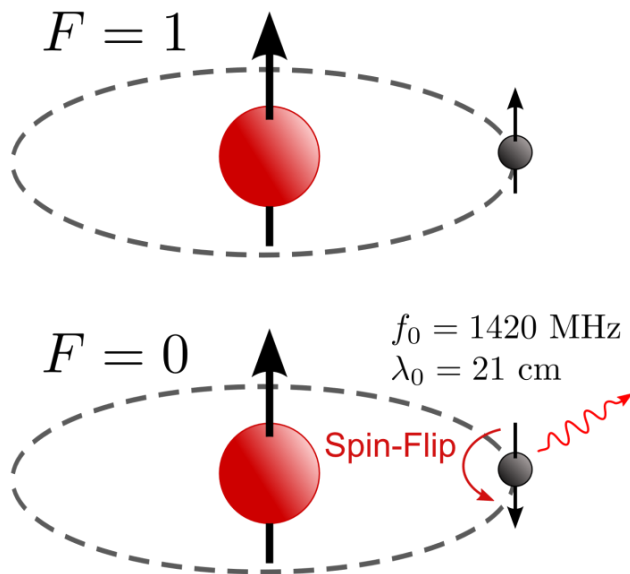


- Why Ca (also Na) was observed, but not H or He lines? Most atoms in the ISM are on their ground state. They can absorb stellar radiation in resonance lines (strongest). Ca and Na have these lines in the optical band, while H and He have them in the ultraviolet (UV) band ( $\text{Ly}\alpha$  1216 Å and 586Å, respectively). Earth atmosphere absorbs radiation with  $\lambda < 2900\text{\AA}$ . In stellar spectra, of course, one observes H and He lines of excited states.
- 1933: Plasket & Pearce found a correlation between the Ca II absorption line strength and the stellar distance.
- 1937: the first interstellar molecules CH, CH<sup>+</sup> and CN were discovered

# Radioastronomy and HI 21-cm line

9

- 1944 - Hendrik van de Hulst predicts the existence of the Neutral Hydrogen 21-cm hyperfine spin-flip transition.
- 1951: The 21 cm line was first detected by Harold “Doc” Ewen and Ed Purcell. Muller & Oort confirmed the detection.

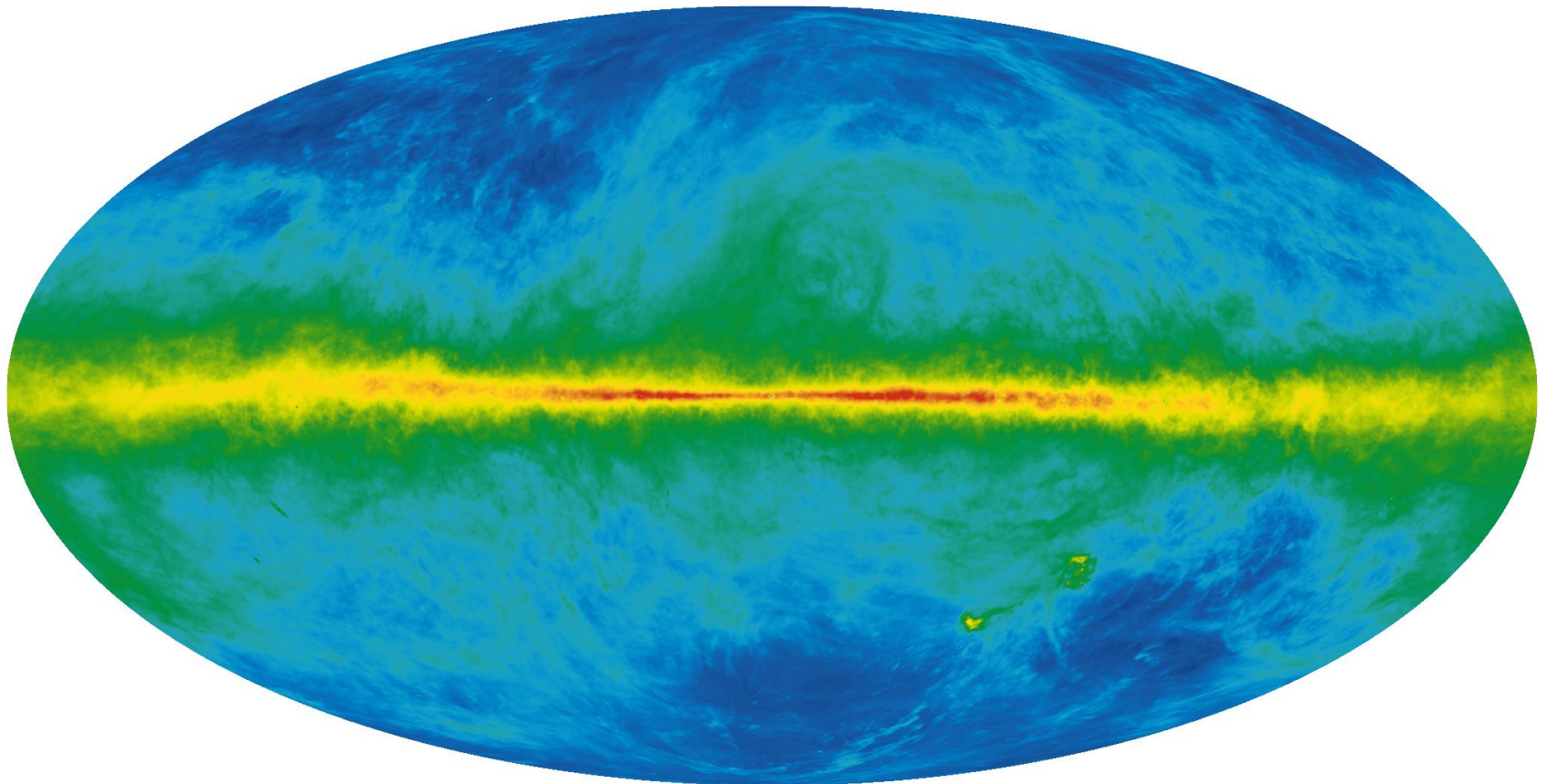


More detail later

# The neutral Hydrogen 21-cm line

10

Cold **H I** emitting at 21-cm makes up most of the mass of the ISM gas in the Milky Way. Its observation represented a revolution.



## Interstellar dust

In 1930s it was shown that space is not transparent in continuum also. Absorption is strongest in the galactic plane. Reddening of the stellar light is observed. **Dust!**

Dust: graphite (C), Si, ice (H<sub>2</sub>O). Size <1 μm.

Light becomes polarized as it goes through dust, since the elongated dust grains are aligned by interstellar magnetic field.



Spiral galaxy Messier 96



Sombrero Galaxy



# The zoo of objects in the ISM

12

- Dark clouds (molecular clouds) - not associated with bright stars, seen as black patches

## Horsehead Nebula

a dark nebula (also  
known as Barnard 33 in  
emission nebula IC 434)



# The zoo of objects in the ISM

14

- Dark clouds (molecular clouds) - not associated with bright stars, seen as black patches
- Reflection nebula - presence of hot stars (25000 K) the dust cloud may scatter the stellar radiation

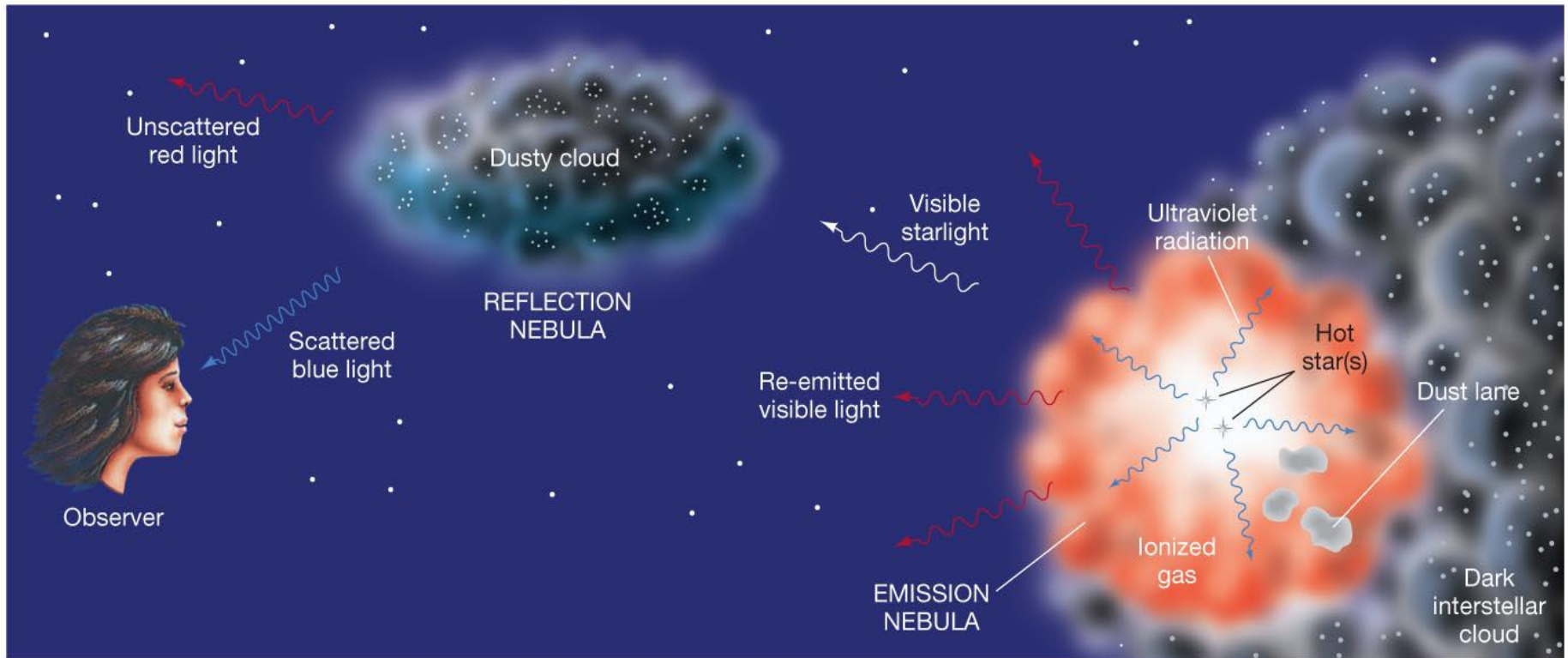
## The Pleiades, an open cluster

Reflection nebulae around the brightest stars were once thought to be left over material from their formation. Now they are considered likely to be an unrelated dust cloud in the interstellar medium through which the stars are currently passing.



# Why the reflection nebulae look like we see

16



# The zoo of objects in the ISM

17

- Dark clouds (molecular clouds) - not associated with bright stars, seen as black patches
- Reflection nebula - presence of hot stars (25000 K) the dust cloud may scatter the stellar radiation
- HII region (or emission nebula) - nebula associated with very hot stars ( $T_{\text{eff}} > 25000 \text{ K}$ ), gas surrounding is photoionized

## The HII region Sh 2-252

The field is  
approximately  $35' \times 30'$ .



# The zoo of objects in the ISM

19

- Dark clouds (molecular clouds) - not associated with bright stars, seen as black patches
- Reflection nebula - presence of hot stars (25000 K) the dust cloud may scatter the stellar radiation
- HII region (or emission nebula) - nebula associated with very hot stars ( $T_{\text{eff}} > 25000$  K), gas surrounding is photoionised
- Planetary nebula - former atmosphere of a solar mass star, now a hot white dwarf

## **Cat's Eye Nebula**

a planetary nebula.



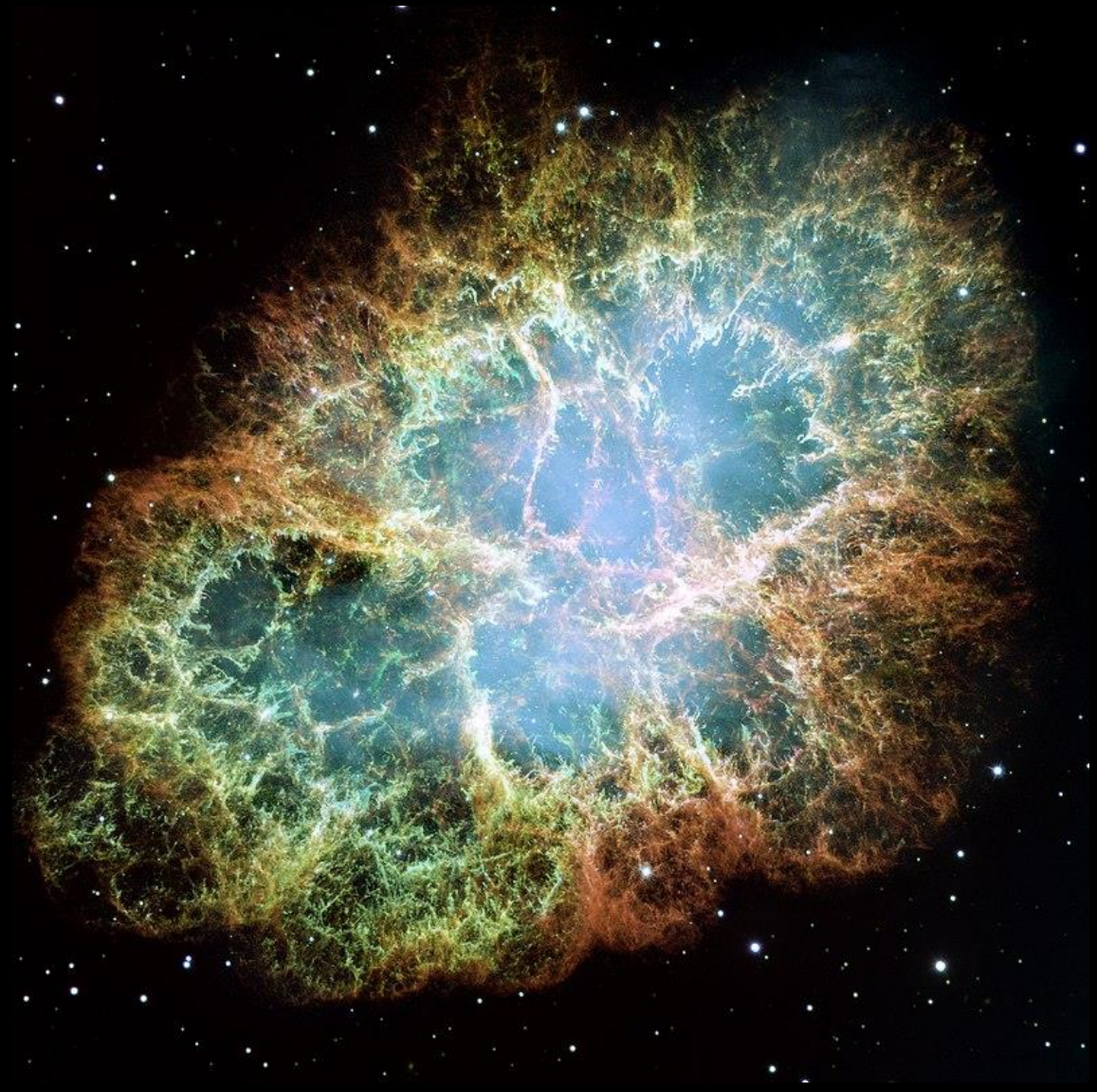
# The zoo of objects in the ISM

21

- Dark clouds (molecular clouds) - not associated with bright stars, seen as black patches
- Reflection nebula - presence of hot stars (25000 K) the dust cloud may scatter the stellar radiation
- HII region (or emission nebula) - nebula associated with very hot stars ( $T_{\text{eff}} > 25000$  K), gas surrounding is photoionised
- Planetary nebula - former atmosphere of a solar mass star, now a hot white dwarf
- **Supernova remnant - massive star's death, bright gaseous nebula**

## Crab Nebula

a supernova remnant  
and pulsar wind nebula.



# The zoo of objects in the ISM

23

- Dark clouds (molecular clouds) - not associated with bright stars, seen as black patches
- Reflection nebula - presence of hot stars (25000 K) the dust cloud may scatter the stellar radiation
- HII region (or emission nebula) - nebula associated with very hot stars ( $T_{\text{eff}} > 25000$  K), gas surrounding is photoionised
- Planetary nebula - former atmosphere of a solar mass star, now a hot white dwarf
- Supernova remnant - massive star's death, bright gaseous nebula

# Basic properties of the ISM

24

- Confined to the Galactic Plane (much flatter than a compact disc!)
- 1% of total visible mass of the Galaxy
- The interstellar medium consists mainly of hydrogen and helium
  - Chemical composition of the ISM comparable to the element's abundance of the Solar System
- Dust: ~1-10% of gas mass (grains: about 1% of the ISM mass)
  - grain size: 0.1 - 0.001 mm
  - ~ 1 grain/ $10^6 \text{ m}^3$
  - ~  $0.001 M_{\odot}/\text{pc}^3$
- Gas: ~1 atom/ $\text{cm}^3$ 
  - ~ $0.01 M_{\odot}/\text{pc}^3$
  - Gas atoms/dust grains by number =  $10^{12}$ .
- Temperature range  $4 \text{ K} < T < 10^6 \text{ K}$ 
  - The temperature is used as a measure for the physical conditions of the interstellar gas. The “phases” of the interstellar medium are characterized by the average gas temperature.
  - The state of Hydrogen determines the state of the ISM:
    - ✦ Molecular region  $\rightarrow \text{H}_2$
    - ✦ Neutral region  $\rightarrow \text{H I}$
    - ✦ Ionized region  $\rightarrow \text{H II}$  or  $\text{H}^+$
- Densities  $10^{-4} \text{ cm}^{-3} < n < 10^7 \text{ cm}^{-3}$
- Far from thermal equilibrium! Remember? **Not easy to calculate!**

# ISM phases

25

Component	Fractional Volume	Temperature (K)	Density (atoms/cm <sup>3</sup> )	State, Where
Molecular clouds	< 1%	20 - 50	$10^3 - 10^6$	hydrogen molecules (thin disk)
Cold Neutral Medium (CNM)	1-5%	50 - 100	$1 - 10^3$	neutral hydrogen atoms (thin disk)
Warm Neutral Medium (WNM)	10-20%	1000 - 5000	$10^{-1} - 10$	neutral hydrogen atoms (thin disk)
Warm Ionized Medium (WIM)	20-50%	$10^3 - 10^4$	0.01	ionized hydrogen (thick disk)
H II regions	~10%	$10^4$	$10^2 - 10^4$	ionized hydrogen
Coronal gas, Hot Ionized Medium (HIM)	30-70%	$10^6 - 10^7$	$10^{-4} - 10^{-2}$	highly ionized (halo)

# Is ISM a vacuum?

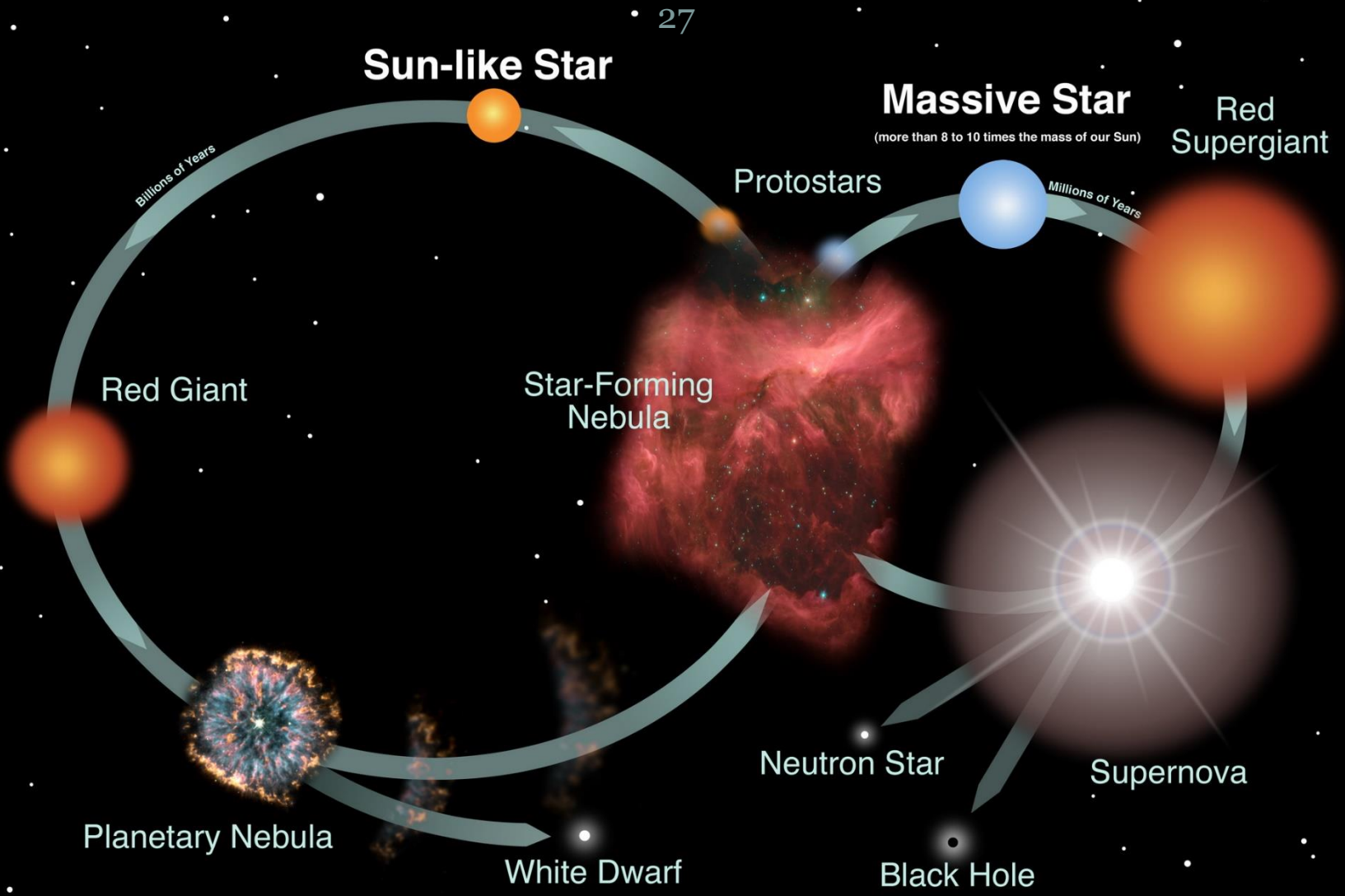
26

- Vacuum = mean free path  $l$  larger than the system size  $L$ :  $l > L$
- Laboratory lamp:  $n=10^{12} \text{ cm}^{-3}$ ,  $l = 1/(n \sigma)$ ,  $\sigma = 10^{-15} \text{ cm}^2$ ,  $l = 10 \text{ m}$ .  
For the lamp size  $L \approx 1 \text{ m}$ , it is **vacuum!**
- ISM:  $n=1 \text{ cm}^{-3}$ ,  $l = 10^{15} \text{ cm} = 3 \times 10^{-4} \text{ pc}$ ,  $L=200 \text{ pc}$ .  
 $l \ll L \Rightarrow$  **NOT a vacuum!**

One can apply gas-dynamics laws:

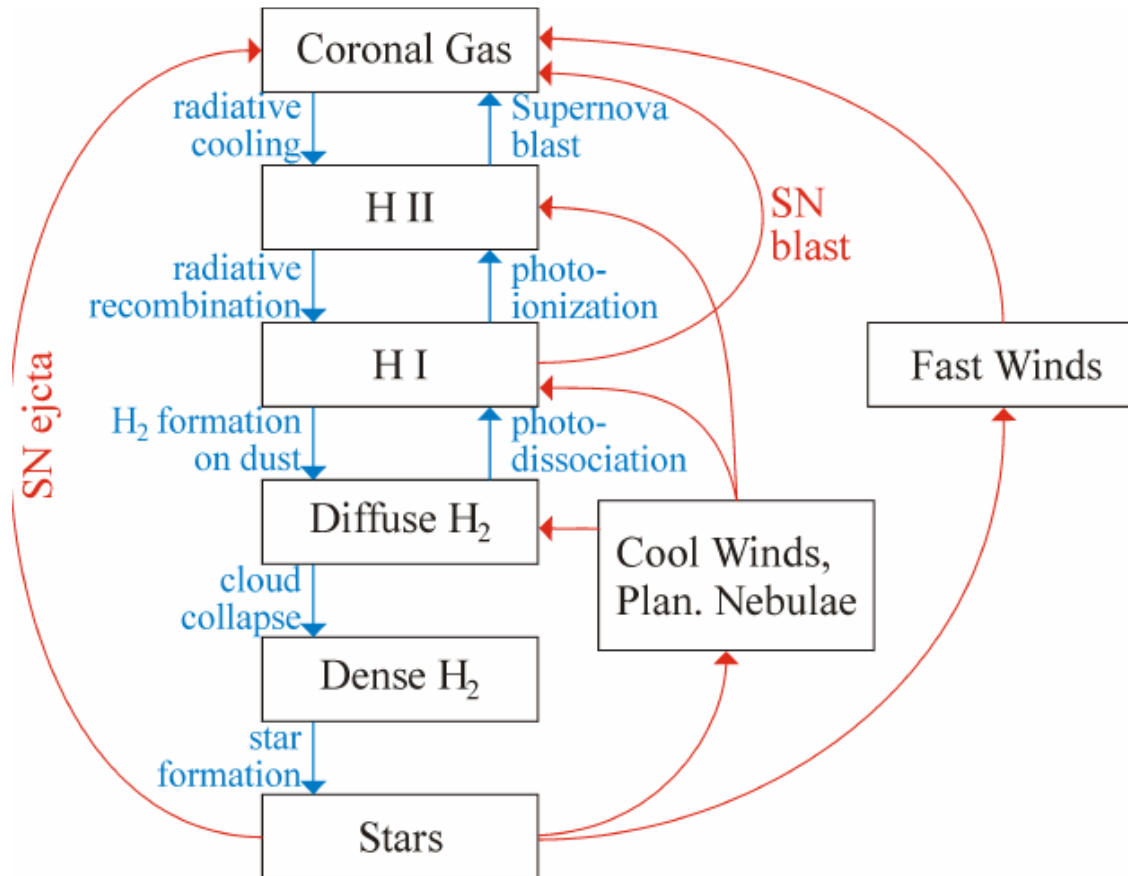
- The distribution of velocities remains generally Maxwellian and described by a kinetic temperature on scales greater than a mean free path.
- There are e.g. shock waves.

# Phase transitions



# Phase transitions

28



# Star formation?

29

- Earlier (**lecture 12**) we introduced the concept of the Jeans Mass, the mass required for gravitational collapse to occur:

$$M_J = \left( \frac{\mathfrak{R}}{\mu G} \right)^{3/2} \left( \frac{3}{4\pi} \right)^{1/2} T^{3/2} \rho^{-1/2}$$

- With more convenient for us units it can be written as

$$M_J = 2.3 M_{\odot} \left( \frac{T}{10 \text{ K}} \right)^{3/2} \left( \frac{n}{10^5 \text{ cm}^{-3}} \right)^{-1/2}$$

- Given the density and temperature of various stages of the ISM, we can calculate the Jeans mass in each phase. We can see why stars form in the molecular ( $\text{H}_2$ ) regions, in which  $M_J$  is of the order of a few tens  $M_{\odot}$ . **Calculate**  $M_J$  for other phases of the IMS and you will see that in  $\text{H}_2$  regions it is much less than even in neutral hydrogen gas (> a few hundred  $M_{\odot}$ ).
- Once the Jeans mass has been reached and gravitational collapse sets in, both  $n$  and  $T$  will increase in the cloud. These have competing effects: increasing  $n$  will tend to decrease the Jeans mass, while increasing  $T$  will increase the Jeans mass.
- If “density wins” and the net effect is a decrease in  $M_J$ , the large collapsing cloud will then be able to collapse on much smaller scales: the cloud fragments, and the result is multiple collapsing objects. If the collapsing object can no longer easily cool, then it has likely become a protostar. We discussed it in lecture 13.

# Summary

30

So, in the ISM we see:

- Dark clouds of dust
- Emission nebulae, star-forming regions
- Globules
- Protoplanetary disks
- Planetary nebulae
- Supernova remnants
- Radiation
- Cosmic rays
- Magnetic fields

In order to understand the physics of interstellar medium we need:

- Radiative transfer
- Atomic and molecular physics (microphysics):
  - cooling and heating
- Gasdynamics (macro-physics):
  - motions of ISM gases
  - shock fronts
  - ionization fronts
  - stellar winds
  - explosions

# Interstellar Absorption Lines

31

OPTICAL & UV ABSORPTION LINES  
DIFFUSE INTERSTELLAR BANDS  
RADIATIVE TRANSFER IN IS LINES  
COLUMN DENSITY  
CURVE OF GROWTH  
21 CM HYDROGEN LINE

# Interstellar Absorption Lines

32

## Optical & UV Absorption Lines:

- Familiar solar optical doublets, e.g., Fraunhofer Ca II K and Na I D provided early evidence for a pervasive ISM.
- These are “resonance” transitions with an electron going from the **ground** state to the next energy level (from an *s* to a *p* orbital).
- Similar transitions occur across UV and optical wavelengths. Some important examples are:

H I	1216 Å
C IV	1548, 1551 Å
Na I (Na D)	5890, 5896 Å
Mg II	2796, 2803 Å
K I	7665, 7645 Å
Ca II	3934, 3968 Å

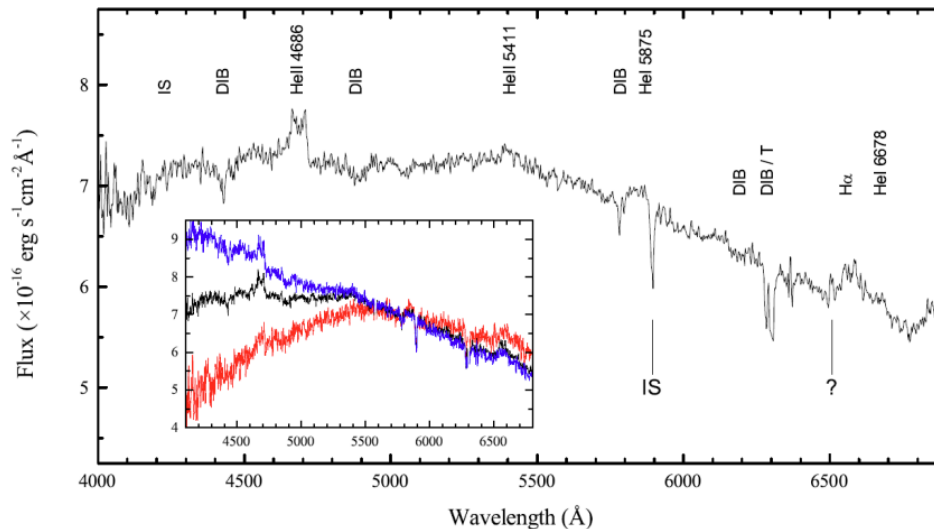
# Diffuse Interstellar Bands (DIBs)

33

DIBs are the most mysterious of the UV/Visible absorption components of the ISM. Since their discovery by Merrill in 1938, more than 200 DIBs have been identified in stellar spectra, with the strongest appearing at  $\lambda 4430 \text{ \AA}$ .

They have not been identified conclusively with any atomic or molecular species (neutral or ionized). They are characterized by being extremely broad (by the standards of interstellar absorption lines).

Some ideas are exotic molecular bands, transition from stuff on dust grain surfaces, exotica like ionized Fullerenes (3-D aromatic C molecules shaped like geodesic spheres), but none have produced consistent predictions of wavelengths.

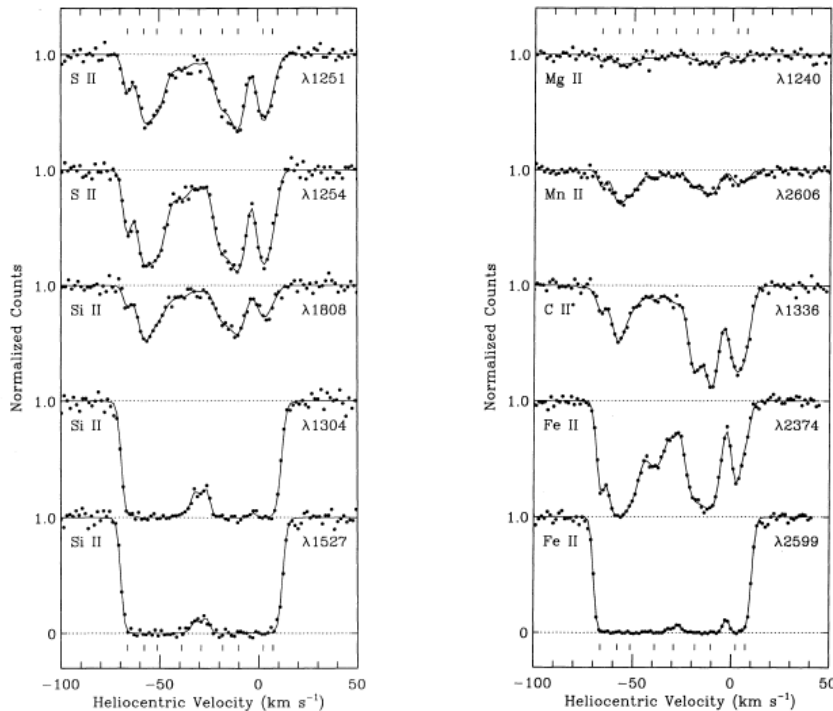


A spectrum of the distant Black Hole binary Swift J1753.5-0127, showing a number of DIBs [from Neustroev et al. 2014]

# Observations of IS Absorption Lines

34

At high spectral resolutions ( $R = \lambda / \Delta\lambda \gtrsim 10^4$ ) interstellar absorption lines resolve into narrow absorption lines that are Doppler shifted relative to each other.



Interstellar absorption lines towards the halo star HD93521. These spectra reveal velocities spanning  $\sim 90$  km/s and show multiple velocity components and effects of line saturation in different species. [From Spitzer & Fitzpatrick 1993]

Our ultimate aim is to deduce from measurements of these absorption lines the column densities and velocity dispersions of the absorbing atoms and ions.

# Radiative Transfer in IS lines

35

We have already learned many things in this course which can also be applied to studying the ISM:

- We defined the specific intensity  $I_\lambda$ , emission ( $j_\lambda$  and  $\epsilon_\lambda$ ) and absorption coefficients ( $\kappa_\lambda$  and  $\alpha_\lambda$ ), optical depth  $d\tau_\lambda = \kappa_\lambda \rho ds = \sigma_\lambda nds = \alpha_\lambda ds$ , the source function  $S_\lambda = j_\lambda/\kappa_\lambda$ .

- For the ISM, we can use the **parallel-ray** equation of radiative transfer:

$$\frac{dI_\lambda}{d\tau_\lambda} = -I_\lambda + S_\lambda$$

- We formally solved it (slides I-148-149), assuming that  $S_\lambda$  is constant along the path:

$$I_\lambda = S_\lambda(1 - e^{-\tau_\lambda}) + I_{\lambda 0}e^{-\tau_\lambda}$$

- The more accurate (constant  $S_\lambda$  along the path is a **rude assumption**), formal solution is

$$I_\lambda(\tau_\lambda) = \int_0^{\tau_\lambda} S_\lambda(t_\lambda) e^{-(\tau_\lambda - t_\lambda)} dt_\lambda + I_{\lambda 0} e^{-\tau_\lambda}$$

# Based on slide I.150

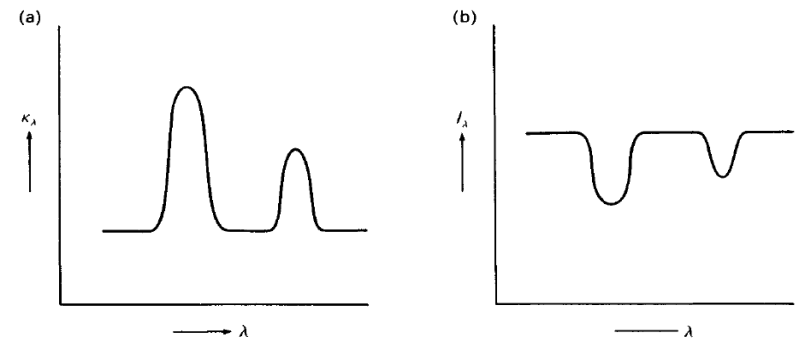
36

Imagine  $I_{\lambda 0} \neq 0$  (background source)

$$I_{\lambda} = S_{\lambda}(1 - e^{-\tau_{\lambda}}) + I_{\lambda 0}e^{-\tau_{\lambda}}$$

If  $I_{\lambda 0} > S_{\lambda}$ , so there is something subtracted from the original intensity which is proportional to the optical depth – we see absorption lines on the continuum intensity  $I_{\lambda}$ .

**EXAMPLE: stellar photospheres or interstellar absorption lines**



Opacity  $\kappa$  versus  $\lambda$   $\rightarrow$  Intensity versus  $\lambda$

An extreme case: **Optically thin case** ( $\tau_{\lambda} \ll 1$ ):

$$I_{\lambda} = I_{\lambda 0}(1 - \tau_{\lambda}) + \tau_{\lambda}S_{\lambda} = I_{\lambda 0} + \tau_{\lambda}(S_{\lambda} - I_{\lambda 0})$$

# Column Density

37

- Consider an arbitrary structure of gas – a “cloud” – with number density of atoms and ions which is a function of position within the cloud:  $n(\text{cm}^{-3}) = n(x, y, z)$ .
- A line-of-sight to a background source of light (e.g., a star) will probe a finite distance through the cloud for a length  $L$ .
- The column density  $N$  is defined as the integral of the number density along the line of sight:

$$N = \int_0^L n(l) dl$$

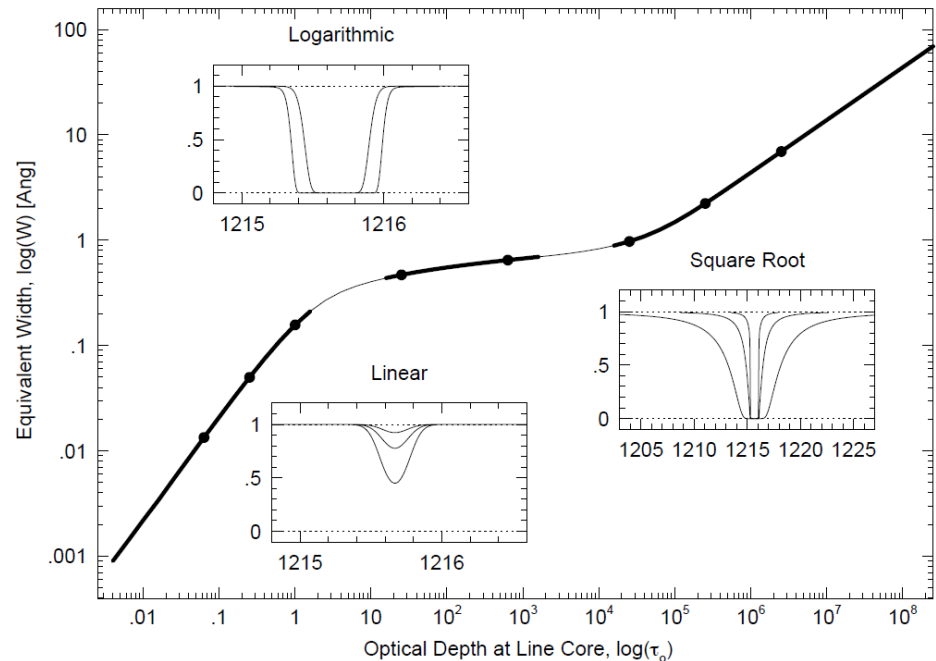
- The quantity  $N$  can be deduced from the strengths and profiles of IS absorption lines.
- If column densities can be measured for different ionization stages of the same ion, e.g. Si, Si<sup>+</sup>, Si<sup>++</sup>, Si<sup>3+</sup> from absorption lines of Si i, Si ii, Si iii, Si iv, the ionization conditions of the gas can be inferred.
- In some physical environments, for example clouds where H is predominantly neutral, the ionization structure of the gas can be simple, with most of the atoms/ions of a given element being concentrated in a dominant ionization stage.
- In this circumstances, if absorption lines from different elements are available, the chemical composition of the absorber can be investigated.

# The Equivalent Width Curve of Growth

38

A traditional method of analyzing absorption line data is via the **Curve of Growth** (Lecture II-23). The precise functional dependence of the equivalent width  $W$  on  $N$  is sensitive to the optical depth at the line core,  $\tau_0$  and the line profile. This behaviour defines three distinct portions of the Curve of Growth:

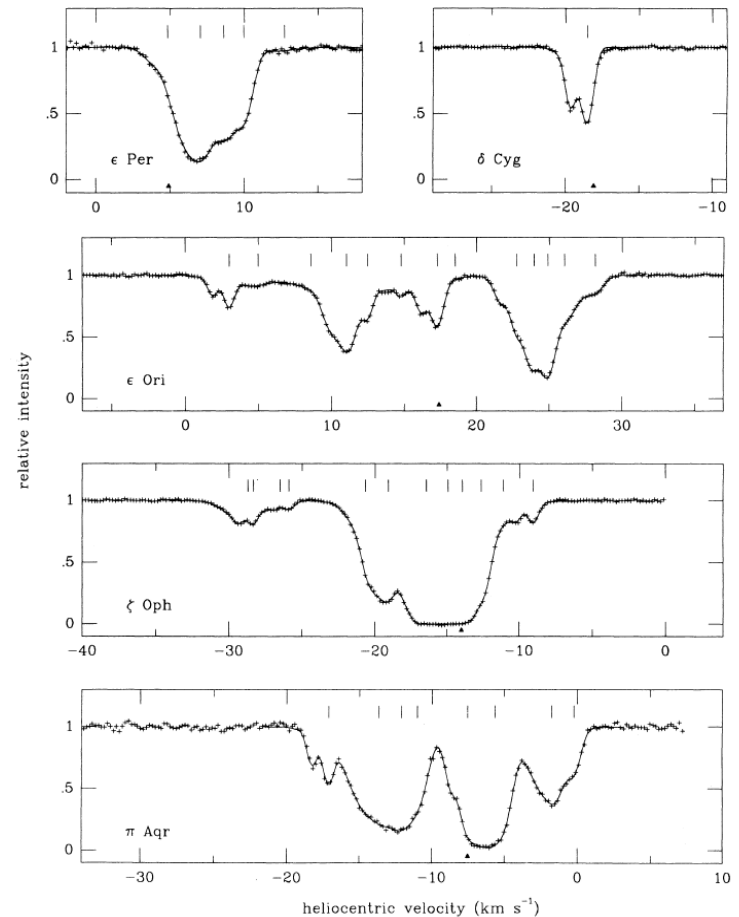
- The linear part, where  $\tau_0 < 1$  and  $W \propto N$ .  
The absorption line is optically thin and  $W$  is a sensitive measure of  $N$ , irrespectively of the values of the Doppler width  $\Delta\nu_D$  or the damping widths  $\gamma/2$ .
- The flat (logarithmic), or plateau, where  $10 \lesssim \tau_0 \lesssim 1000$  and  $W \propto (\ln N)^{1/2}$ .  
The absorption line is optically thick and  $W$  is not a good measure of  $N$ , but is sensitive to the Doppler width  $\Delta\nu_D$ .
- The damping, or square root part, where  $\tau_0 \gtrsim 10^4$  and  $W \propto N^{1/2}$ .  
In this regime, the optical depth in the damping wings provides an accurate estimate of  $N$ .



# IS absorption line profiles

39

- In practice, curve of growth methods are quite powerful, as they can relate the product  $N$  to a direct observable,  $W$ , that is relatively insensitive to the choice of spectral resolution.
- In principle, two different spectrometers working at very different resolutions and on different telescopes with different detectors should be able to measure the same equivalent widths to within the irreducible measurement uncertainties.
- Real interstellar absorption lines are often **highly structured** with a mixture of both saturated and unsaturated components because the line of sight to a particular star will often **intersect several interstellar clouds** with a wide range of column densities.

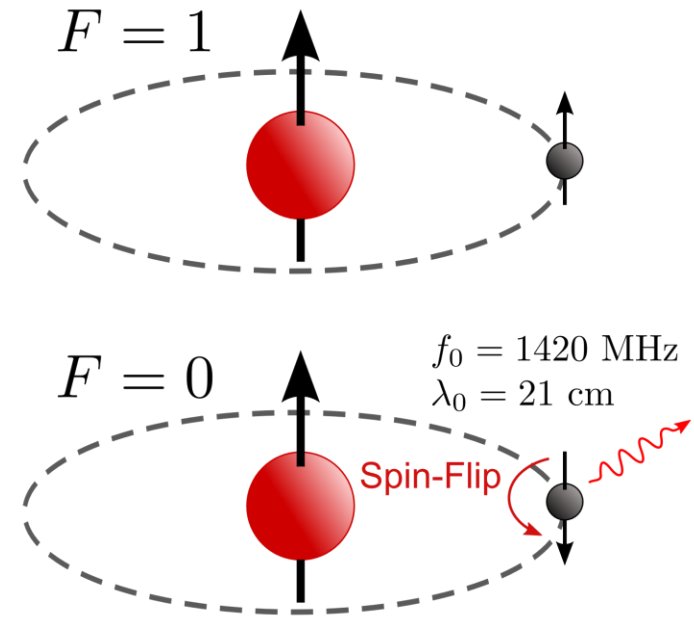


These Na D profiles show a mix of linear ( $\delta$  Cyg), flat ( $\epsilon$  Per &  $\epsilon$  Ori), and square-root ( $\zeta$  Oph) absorption lines.

# 21 cm hydrogen line (1)

40

- One can study ISM using absorption lines, but we need a **bright** background "lamp"! Which we don't have for all directions!!
- To map the distribution of interstellar atomic gas in any direction we wish to study emission lines.
- Most of our knowledge of the distribution of neutral atomic hydrogen in the ISM of the Milky Way and other galaxies come from observations of the **strong 21-cm line** (1420 MHz).
- This line arises from transitions between the hyperfine structure levels in the **ground state** of **Hydrogen**, and is seen in both emission and absorption.



21 cm hydrogen line corresponds to a flip of the electron spin relative to the spin of the proton:  
**Upper state:** electron and proton spins are parallel, statistical weight  $g_u=3$ .  
**Lower state:** electron and proton antiparallel,  $g_l=1$ .

# 21 cm hydrogen line (2)

41

**Upper state:** electron and proton spins are parallel, statistical weight  $g_u=3$ .

**Lower state:** electron and proton are antiparallel,  $g_l=1$ .

$A_{lu}$  = transition probability =  $2.9 \times 10^{-15} \text{ sec}^{-1} \rightarrow$

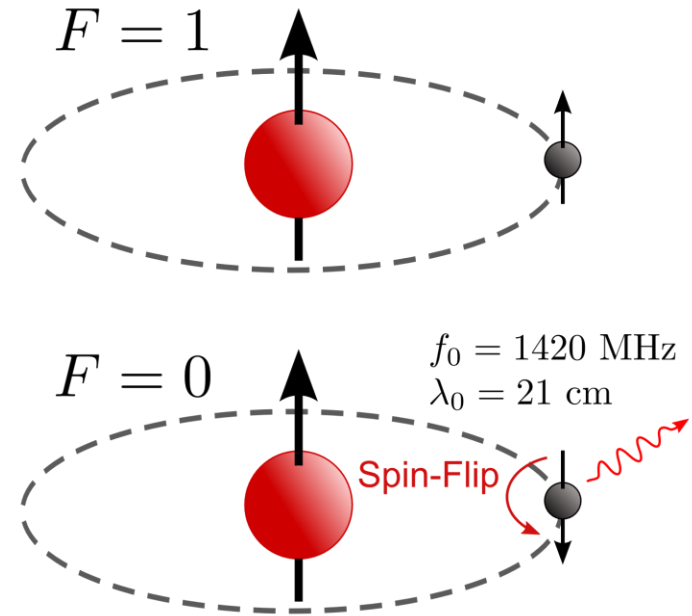
Lifetime of upper level = 11 million years!

Thus, for  $n_H \approx 1 \text{ cm}^{-3}$  collisions dominate - levels are populated according to the Boltzmann equation:

$$\frac{N_u}{N_l} = \frac{g_u}{g_l} e^{-(E_u-E_l)/kT} \approx \frac{g_u}{g_l} \approx 3$$

Since the energy difference between levels is **very small!!**

Thus, the populations of the levels are essentially independent of temperature in the ISM.



The transition energy in units of Kelvin is **0.07 K**, which is much lower than in most astrophysically interesting conditions.

# ISM: Ionized regions

42

SOURCE OF IONIZATION  
IONIZATION AND RECOMBINATION  
STRÖMGREN SPHERES

# The non-equilibrium ISM

43

The ISM is generally not in thermodynamic equilibrium due to its low density and consequent low collision rates and low optical depths. Collisions between particles may result in **radiative decay** rather than redistributing internal energies. Photons can escape the system, or enter from external sources. Whereas the distribution of velocities **remains generally Maxwellian**, and described by a **kinetic** temperature on scales greater than a mean free path, the distribution of energy levels may be significantly **different** from the Boltzmann distribution. This can be formulated either as departure coefficients,

$$b_i = \frac{n_{i,actual}}{n_{i,LTE}}$$

or, more commonly, by defining an excitation temperature,  $T_{ex}$ , such that

$$\frac{n_u}{n_l} = \frac{g_u}{g_l} e^{-(E_u - E_l)/kT_{ex}}$$

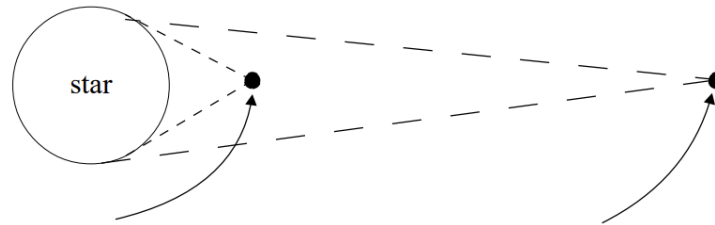
Note that although  $T_{ex}$  has units of K, it is not a physical temperature and **may not be** equal to the **kinetic** temperature. It is a function of the energy level and parameterizes how far the distribution of states is from Boltzmann.

# Radiation field

44

Interstellar radiation field is characterized by a huge discrepancy between the frequency-integrated radiation density and spectral composition.

Compare photons close to and far from an extended source like a star:



Photons here are crowded in physical space (high density) but spread out in solid angle (direction)

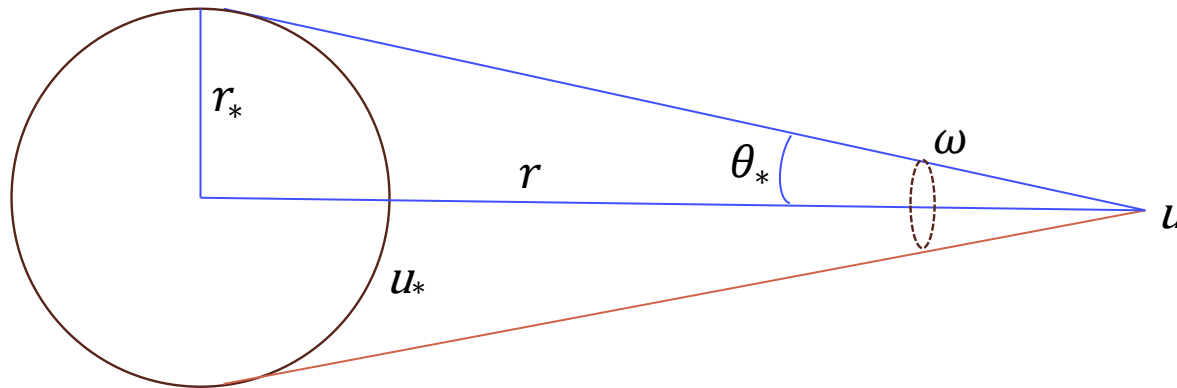
Photons here are diffuse in physical space (low density) but highly collimated in angle (high density).

It is sometimes convenient to approximate the density of the background starlight as a “diluted blackbody” of “dilution factor”  $W$  and “color temperature”  $T_c$ .

# Dilution factor (1)

45

Compare photons close to and far from an extended source like a star:



The dilution factor  $W$  is defined to be the ratio of the actual energy density  $u$  to the energy density of (undiluted) blackbody radiation of temperature  $T_c$ .

$\omega$  is the solid angle, then  $u = \frac{\omega}{4\pi} u_* = u_* W$ , where  $W$  is the **dilution factor**.

$$W = \frac{\omega}{4\pi} = \frac{1}{4\pi} \int_0^{2\pi} d\phi \int_0^{\theta_*} \sin \theta d\theta = \frac{1}{2} (1 - \cos \theta_*) = \frac{1}{2} \left( 1 - \sqrt{1 - \frac{r_*^2}{r^2}} \right) \approx \frac{1}{2} \frac{r_*^2}{r^2}$$

# Dilution factor (2)

46

The dilution factor  $W \approx \frac{1}{2} \frac{r_*^2}{r^2}$

Planetary Nebula?  $r_* \approx 10^9$  cm,  $r \approx$

Slide I-157: the energy contained in volume

Assume Blackbody, then  $u_{*,\lambda} = \frac{4\pi}{c} B_\lambda(T_*)$

The integrated radiation density  $u_* =$

Then  $T_d^4 = WT_*^4$ , or

$T_d$  is the temperature corresponded to radiation of temperature  $T_*$ . For  $W \sim 1$

But a spectrum corresponds to  $T_c = T_*$

Indeed, **Rosseland's theorem** dictates that

**in low-density regions, short-wavelength radiation is transformed into long-wavelength radiation.**

This immediately explains a PNe observable, that the nebula emits much more energy in the optical than does the central star. This is because UV radiation is being processed into optical photons.

## Mean intensity and Energy density

157

$$J_\lambda = \frac{1}{4\pi} \oint I_\lambda d\omega$$

- The **mean intensity**  $J_\lambda$  is related to the energy density  $u_\lambda$ :
- Energy radiated through area element  $d\sigma$  during time  $dt$ :

$$dE_\lambda = I_\lambda d\lambda d\sigma d\omega dt$$

$$l = c dt \rightarrow dV = l d\sigma = c dt d\sigma$$

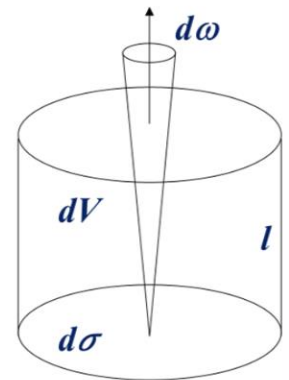
- Hence, the energy contained in volume element  $dV$  per wavelength interval is:

$$u_\lambda dV d\lambda = \oint I_\lambda d\omega d\lambda d\sigma dt = 4\pi J_\lambda \frac{dV}{c} d\lambda$$

$$u_\lambda = \frac{4\pi}{c} J_\lambda \left[ \frac{\text{erg}}{\text{cm}^3 \text{\AA}} \right]$$

$$u = \int_0^\infty u_\lambda d\lambda = \frac{4\pi}{c} \int_0^\infty J_\lambda d\lambda \left[ \frac{\text{erg}}{\text{cm}^3} \right]$$

Total radiation emerge in volume element



# Dilution factor (2)

47

The dilution factor  $W \approx \frac{1}{2} \frac{r_*^2}{r^2}$

Planetary Nebula?  $r_* \approx 10^9$  cm,  $r \approx 10^{16}$  cm  $\rightarrow W \sim 10^{-14}$

**Slide I-157:** the energy contained in volume element per wavelength interval is  $u_{*,\lambda} = \frac{4\pi}{c} J_\lambda \left[ \frac{\text{erg}}{\text{cm}^3 \text{\AA}} \right]$

Assume Blackbody, then  $u_{*,\lambda} = \frac{4\pi}{c} B_\lambda(T_*)$ , where  $T_* = T_{\text{eff}}$  of a star.

The integrated radiation density  $u_* = aT_*^4$  for the star and  $u = \int_0^\infty u_\lambda d\lambda = aT_d^4$  far from the star.

Then  $T_d^4 = WT_*^4$ , or

$$T_d = W^{1/4} T_*$$

$T_d$  is the temperature corresponded to the integrated energy density of **diluted** blackbody radiation of temperature  $T_*$ . For  $W \sim 10^{-14}$ ,  $T_d = \text{a few K}$ .

But a spectrum corresponds to  $T_c = T_*$ ! The ISM must work as a transformer, decreasing  $T_c$ .

Indeed, **Rosseland's theorem** dictates that

**in low-density regions, short-wavelength radiation is transformed into long-wavelength radiation.**

This immediately explains a PNe observable, that the nebula emits much more energy in the optical than does the central star. This is because UV radiation is being processed into optical photons.

# The non-equilibrium ISM (2)

48

The interstellar radiation field is very dilute but populated by discrete energetic sources (e.g. OB stars) and can contain multiple spectral features from the gas. It is therefore generally very different from the Planck function which we can parameterize in terms of a brightness temperature.

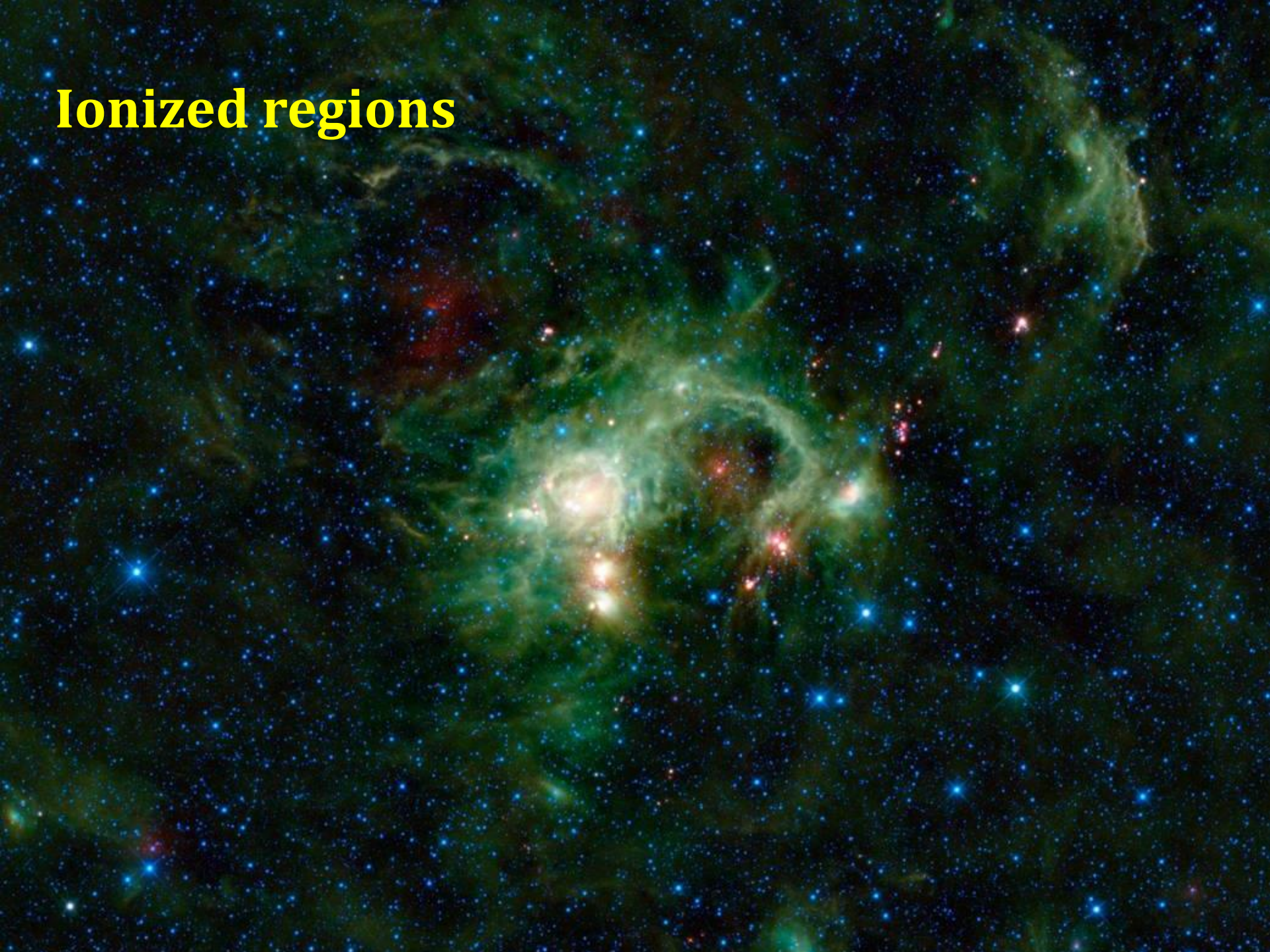
Despite these complications, we can still look for a statistical equilibrium solution to the distribution of energy levels,

$$\frac{dn_i}{dt} = \sum_{j \neq i}^N n_j P_{ji} - n_i \sum_{j \neq i}^N P_{ij} = \sum_{j \neq i}^N n_j (R_{ji} + C_{ji}) - n_i \sum_{j \neq i}^N (R_{ij} + C_{ij}) = 0$$

where  $P_{ji}$  is the (radiative  $R_{ji}$  plus collisional  $C_{ji}$ ) rate from level  $j$  to  $i$ . This matching of forward and reverse rates is also known as the principle of detailed balance.

Remember? See [lecture 24](#).

# Ionized regions



# Source of ionization?

50

HII regions are zones of ionized atomic hydrogen. They are often associated with nebulae, and require gas that contains a **continuous** source of ionizing radiation.

$$\text{Ionization potential of hydrogen } h\nu_0 = 13.6 \text{ eV} \\ (\nu_0 = 3.29 \times 10^{15} \text{ Hz}, \lambda_0 = 912 \text{ \AA})$$

Thus, for ionization we need  $h\nu > 13.6 \text{ eV}$

(it is  $\gg kT$  in neutral ISM  $\Rightarrow$  collisions **unimportant**)

This could be a massive star or a white dwarf, but either way it must be **very hot**.

**Are** OB stars hot enough?

$$T \approx 3\text{-}5 \times 10^4 \text{ K} \Rightarrow E = kT \approx 3\text{-}5 \text{ eV} < 13.6 \text{ eV} \Rightarrow \text{cannot ionize H...?}$$

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \Rightarrow \text{can ionize H!}$$

Check it with Wien's displacement law

**Yes**, the OB stars are hot enough.

# HII regions

51

Without that **hot** central source, the protons and electrons in the ISM **will quickly recombine** – even with the ionizing source, it can only ionize a region of some given volume before recombinations will be happening as quickly as ionizations.

The result will be a bubble of ionized gas, termed a **Strömgren Sphere**.

Such HII regions are easily observable via the strong emission lines resulting from recombination. Thus, to add further to the nomenclature, they are also sometimes known as **emission-line nebulae** (remember the spectrum of the Cat's Eye Nebula?)

Our goal now is to understand the size of the ionized bubble and its detailed ionization structure. In this effort, we define the ionization fraction

$$f \equiv \frac{n_{H^+}(r)}{n_H(r)}$$

For a fully neutral, atomic ISM  $f = 0$ , while full ionization implies  $f = 1$ .

As hinted at in the preceding argument, to maintain a constant  $f$  we will want to make use of ionization equilibrium, where

$$\# \text{ of recombinations / sec} = \# \text{ of ionizations / sec}$$

# Ionization and Recombination

52

- Consider pure hydrogen gas
- Reaction:  $H + h\nu \leftrightarrow p + e$
- Ionization produces an electron of energy  $E_{\text{kin},e} = h\nu - \chi_{\text{ion}}$   
where  $\chi_{\text{ion}}$  is the ionization potential (one can assume all atoms to be in the ground state)
- Recombination to level  $i$  gives a photon

$$h\nu = E_{\text{kin},e} + \chi_{\text{ion}} / i^2 \approx \chi_1 (1/i^2 + 0.07 T_e / 10^4 \text{ K})$$

Transition energy between levels  $u$  and  $l$ :

$$\chi_{ul} = C \left( \frac{1}{u^2} - \frac{1}{l^2} \right)$$

where  $C = \chi_{\text{ion}} = -13.6 \text{ eV}$

where  $\chi_{\text{ion}} \approx kT_e$  is kinetic energy of electron,  $i$  – is the main quantum number, plus a cascade of line (Balmer+Lyman etc.) photons.

Photons recombining to the 1st level can still **ionize** hydrogen.

- Equilibrium: # of recombinations = # of ionizations

# Energy redistribution and temperature

53

- We have **e**, **p**, and neutral **H**.
- Why should we assume that all of them are described by the same temperature  
 $T_e = T_p = T_H$ ?
- Why should they have Maxwellian distribution?
- The distribution of injected electrons depends on incident photon spectrum.
- Electrons thermalize rapidly by collisions.
- Electrons and protons exchange energy much slower (since the masses are very different).
- Protons and neutrals exchange energy even slower  $\tau_{ee} \ll \tau_{ep} \ll \tau_{pH}$  but still  $\ll \tau_{\text{dyn}}$   
 $\Rightarrow$
- Electrons lose energy in inelastic collisions (by radiation). Energy is taken from the gas energy as a whole. We associate gas temperature with  $T_e$ .

# HII regions: recombination (case A)

54

- Recombination rate proportional to  $n_e n_p$ . Depends on  $T_e$ , since the recombination probability and the flux of electrons both depend on  $T_e$
- $e^-$  can be captured in any level
- Recombination rate to level  $i$  is

$$N_{\text{rec},I} = n_e n_p \alpha_{i,\text{rec}}(T_e) \text{ cm}^{-3} \text{ s}^{-1}$$

where  $\alpha_{i,\text{rec}}$  is the recombination coefficient.

- Total “case A” recombination coefficient

$$\alpha_A = \sum_{n=2}^{\infty} \sum_{l=0}^{n-1} \alpha_{nl}(T) \approx 4.2 \times 10^{-13} (T_e/10^4)^{3/4} \text{ cm}^3 \text{ s}^{-1}$$

- Thus,

$$N_{\text{rec}} = n^2 \alpha_A(T_e) \text{ cm}^{-3} \text{ s}^{-1}$$

**However**

# HII regions: recombination (case B)

55

- Photon produced by recombination to level 1 still **can ionize** gas. These photons produce “**diffuse**” radiation field. Assuming that these photons are absorbed nearby (“on-the-spot” approximation), one can just neglect these recombinations in the total recombination rate:

$$N_{\text{rec}} = \sum_{i=2} N_{\text{rec},i} = n_e^2 \alpha_B (T_e) \text{ cm}^{-3} \text{ s}^{-1}$$

where “case B” recombination coefficient

$$\alpha_B = \alpha' = \alpha_A - \alpha_1$$

$$\alpha_B (T_e) \approx 2.6 \times 10^{-13} (T_e / 10^4 \text{ K})^{-3/4} \text{ cm}^3 \text{ s}^{-1}$$

# Steady State in HII Regions

56



**Case A recombination:** gas optically thin to radiation produced after recombination

**Case B recombination:** gas optically thick to radiation just above 13.6 eV;  
photons produced in recombination are absorbed for  
photoionization of another atom

# Photoionization/ Recombination

57

- Transitions down are fast :  $A \sim 10^8 \text{ s}^{-1}$   
All atoms are on ground state (absorption in Balmer lines is negligible).
- Total ionization rate at distance  $r$  from the star is:

$$N_{\text{ion}} = n_{\text{H}} \Gamma_i \approx n_{\text{H}} \sigma_0 N_* / (4\pi r^2) \text{ cm}^{-3} \text{ s}^{-1}$$

where  $\Gamma_i$  is the number of ionizations per atom per second from level  $l$   
 $N_*$  is the number of stellar ionizing photons (with  $h\nu > 13.6 \text{ eV}$ ) per second,  
and  $\sigma_0 = 6.3 \times 10^{-18} \text{ cm}^2$  is the ionization cross-section at  $\nu = \nu_i$ ,

$$\Gamma_i = \int_{\nu_i}^{\infty} \frac{F_\nu}{h\nu} \sigma_i(\nu) d\nu, \quad \text{where } F_\nu = L_\nu / 4\pi r^2$$

$\sigma_i$  is the ionization cross-section:  $\sigma_i = 6.3 \times 10^{-18} (\nu_i / \nu)^{3.5} \text{ cm}^2$

# HII regions: ionization

58

- Let's introduce  $n_e = x n$   
 $n = n_p + n_H = n_e + n_H$

- Then  $n_H = (1-x) n$

$$N_{\text{rec}} = N_{\text{ion}} \Rightarrow x^2 n^2 \alpha'_{\text{rec}}(T_e) = (1-x) n \sigma_0 N_* / (4\pi r^2)$$

- For O6.5 star:  $N_* = 10^{49} \text{ s}^{-1}$ ,  $r = 1 \text{ pc}$ ,  $n = 10^2 \text{ cm}^{-3}$ ,  $T_e = 10^4 \text{ K}$ ,  
we get  $(1-x) = 3 \times 10^{-5}$ , i.e.  $x \sim 1$  and hydrogen is almost fully ionized.

# Strömgren Spheres

59

- Two components to local ionizing flux near a star:
  - direct ionizing flux
  - diffuse flux from recombinations to ground state
- Calculate radius of ionized sphere in uniform density:  
balance total number of ionizations (=number of ionizing photons the star produces) to the total number of recombinations to levels above ground  $\alpha^{(2)}=\alpha'=\alpha_B$

$$\frac{4\pi}{3} r_S^3 x^2 n^2 \alpha_B = N_* \quad \Rightarrow \quad r_S = \left( \frac{3N_*}{4\pi n^2 \alpha_B} \right)^{1/3}$$

- Strömgren radius for an O star:  $r_S = 70 \text{ pc } n^{-2/3}$ ;  $n = 10^2 \text{ cm}^{-3}$ ,  $r_S \sim 3 \text{ pc}$

→ OB stars have an enormous impact on the ISM

# Edges of HII regions

60

- $n_{\text{H}} \uparrow$ , absorption  $\uparrow$ , ionizing photon flux  $\downarrow$ ,  $n_{\text{H}} \uparrow$
- The optical depth  $\tau = \int n \sigma dl$  for ionization is huge.
- At Lyman edge,  $\sigma = 6.3 \times 10^{-18} \text{ cm}^2$ , so if  $n = 1 \text{ cm}^{-3}$ , the mean free path  $\Delta r$  (where  $\tau=1$ ) is  
$$\Delta r = \tau / n \sigma = 1.5 \times 10^{17} \text{ cm} / n = 0.05 \text{ pc} / n$$

$$\Delta r / r_{\text{S}} = 10^{-3} n^{-1/3} \ll 1$$

- Observed H II regions limited:
  - ionization bounded: all photons are used up for ionization, interstellar cloud has a larger extent than the nebula
  - density bounded: all atoms ionized, there are still photons left
- Ionization bounded H II regions have **sharp** edges

# Edges of HII regions

