Chemical composition (Population I)

- Stellar atmosphere = mixture, composed of many chemical elements, present as atoms, ions, or molecules
- Abundances, e.g., given as mass fractions β_k
- **Solar abundances**

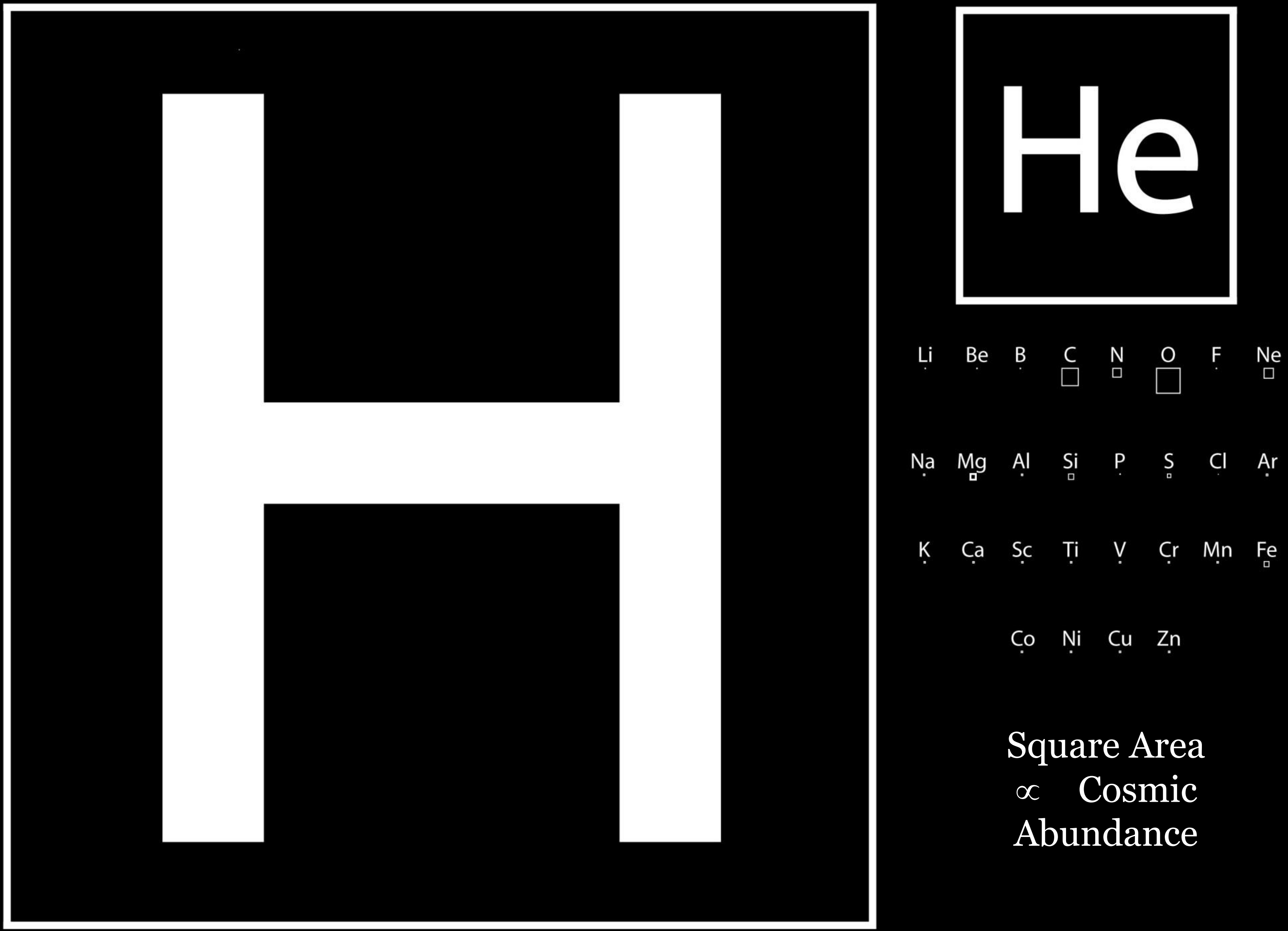
$$\left.\begin{array}{rl} & \beta_H = 0.71 \\ & \beta_{He} = 0.28 \\ \text{“Metals” } (Z): & \left[\begin{array}{l} \beta_C = 0.004 \\ \beta_N = 0.001 \\ \beta_O = 0.009 \\ \vdots \\ \beta_{Fe} = 0.001 \end{array}\right. \\ & \vdots \end{array}\right\}$$

$\longrightarrow$ **X**

$\longrightarrow$ **Y**

Universal abundance for Population I stars

X+Y+Z=1



Square Area $\propto$ Cosmic Abundance

Chemical composition (Population II)

- Population II stars

$$\beta_H = \beta_H^{\odot}$$

$$\beta_{He} = \beta_{He}^{\odot}$$

$$\beta_Z = 0.1 \cdots 0.00001\ \beta_Z^{\odot}$$

- Chemically peculiar stars, e.g., helium stars

$$\beta_H \le 0.002 << \beta_H^{\odot}$$

$$\beta_{He} = 0.964 >> \beta_{He}^{\odot}$$

$$\beta_C = 0.029 >> \beta_C^{\odot}$$

$$\beta_N = 0.003 \approx \beta_N^{\odot}$$

$$\beta_O = 0.002 < \beta_O^{\odot}$$

- Chemically peculiar stars, e.g., PG1159 stars

$$\beta_H \le 0.05 << \beta_H^{\odot}$$

$$\beta_{He} = 0.25 >> \beta_{He}^{\odot}$$

$$\beta_C = 0.55 >> \beta_C^{\odot}$$

$$\beta_N < 0.02$$

$$\beta_O = 0.15 >> \beta_O^{\odot}$$

Other definitions

- Particle number density N_k = number of atoms/ions of element k per unit volume. Relation to mass density:

$$\beta_k \rho = A_k m_H N_k$$

with A_k = mean mass of element k in atomic mass units (AMU)
m_H = mass of hydrogen atom

- Particle number fraction $\frac{N_k}{\sum N_{k'}}$
- Logarithmic $\varepsilon_k = \log(N_k/N_H) + 12.00$

- Iron(Fe)-to-Hydrogen(H) ratio, for the Sun: $\log\left(\frac{N_{Fe}}{N_H}\right) \cong -4.3$

For other stars: $[\mathrm{Fe/H}] = \log\frac{(\mathrm{Fe/H})_*}{(\mathrm{Fe/H})_\odot} = \log(\mathrm{Fe/H})_\odot$- $\log(\mathrm{Fe/H})_*$

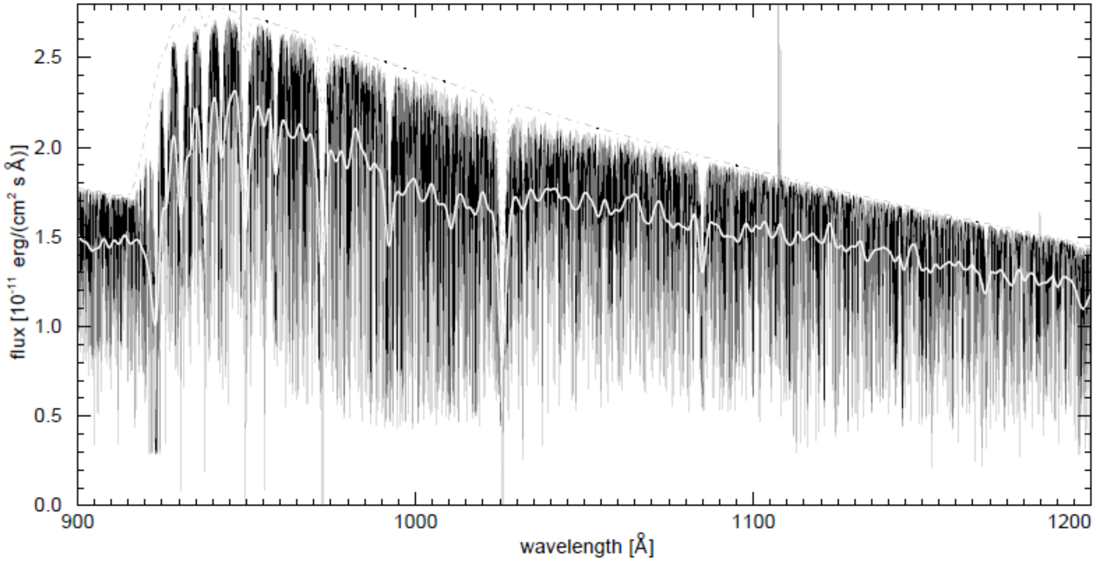
$$[\mathrm{Fe/H}]_\odot \equiv 0$$

Line absorption

- A bound-bound transition absorbs or emits at $h\nu = hc/\lambda = \chi_u - \chi_l$ where χ is the excitation of the upper and lower levels above the ground state. Such transitions contribute to the **line** absorption. We will discuss spectral lines later.

- The cumulative effect of many lines can behave much as continuous opacity in the upper photosphere. Problems associated with line opacity are due to the large numbers of lines involved.

- Data for millions of atomic line transitions have been calculated by Kurucz and more recently by the OP (Opacity Project).

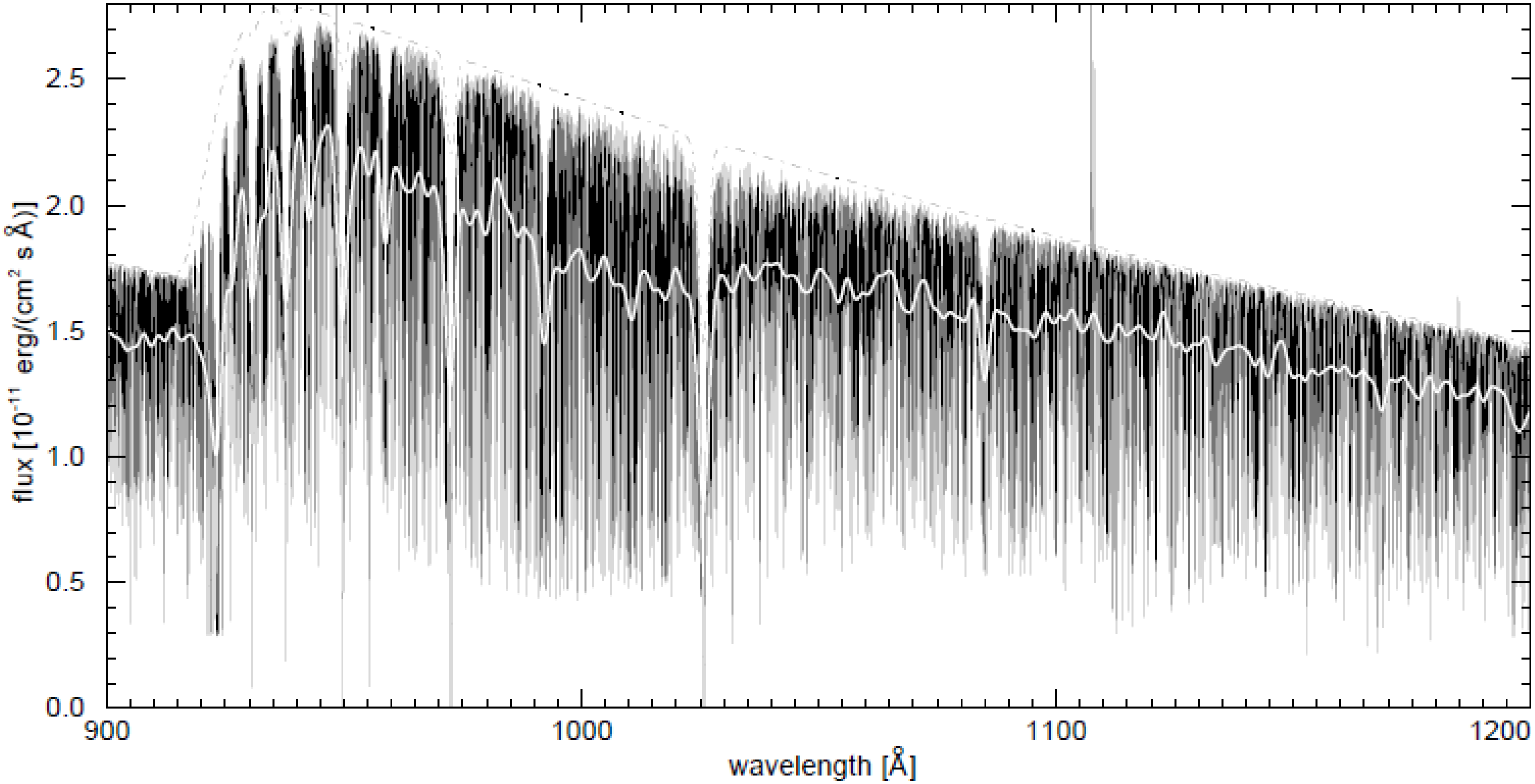


Here is the effect of many lines (Fe and Ni) on the emergent UV continuum of the subdwarf O star Feige 67.

(From Deetjen 2000)

Observations

Here is the effect of many lines (Fe II and Ni) on the emergent UV continuum of the subdwarf O star Feige 67.



(From Deetjen 2000)

Continuous absorption

For **continuous** sources of absorption, there must be a continuum of energy levels, i.e. at least one end of the transition involving a free state of the electron (at an energy above $\chi_{\rm ion}$). Two possibilities…

1. A transition from a bound state (level n) to a free state with velocity v. The energy of the absorbed bound-free photon is given by

   $$h\nu = hc/\lambda = (\chi_{\rm ion} - \chi_{\rm n}) + mv^2/2$$

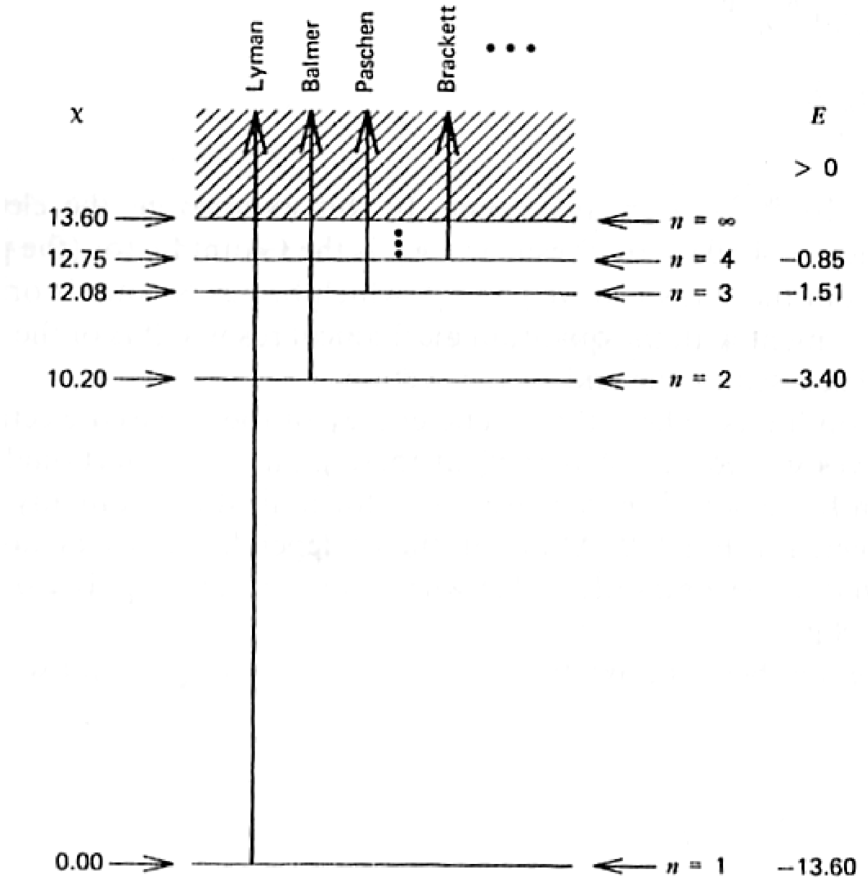
   Each bound-free transition corresponds to an ionization process (since the electron is free afterwards). The emission of a photon by a free-bound transition corresponds to a recombination process.
2. Finally, one can get a continuum of transitions if the electron goes from one free-state (with velocity v_1) to another free-state (with velocity v_2) . The energy of the free-free transition is

   $$h\nu = \frac{hc}{\lambda} = \frac{mv_2^2}{2} - \frac{mv_1^2}{2}$$

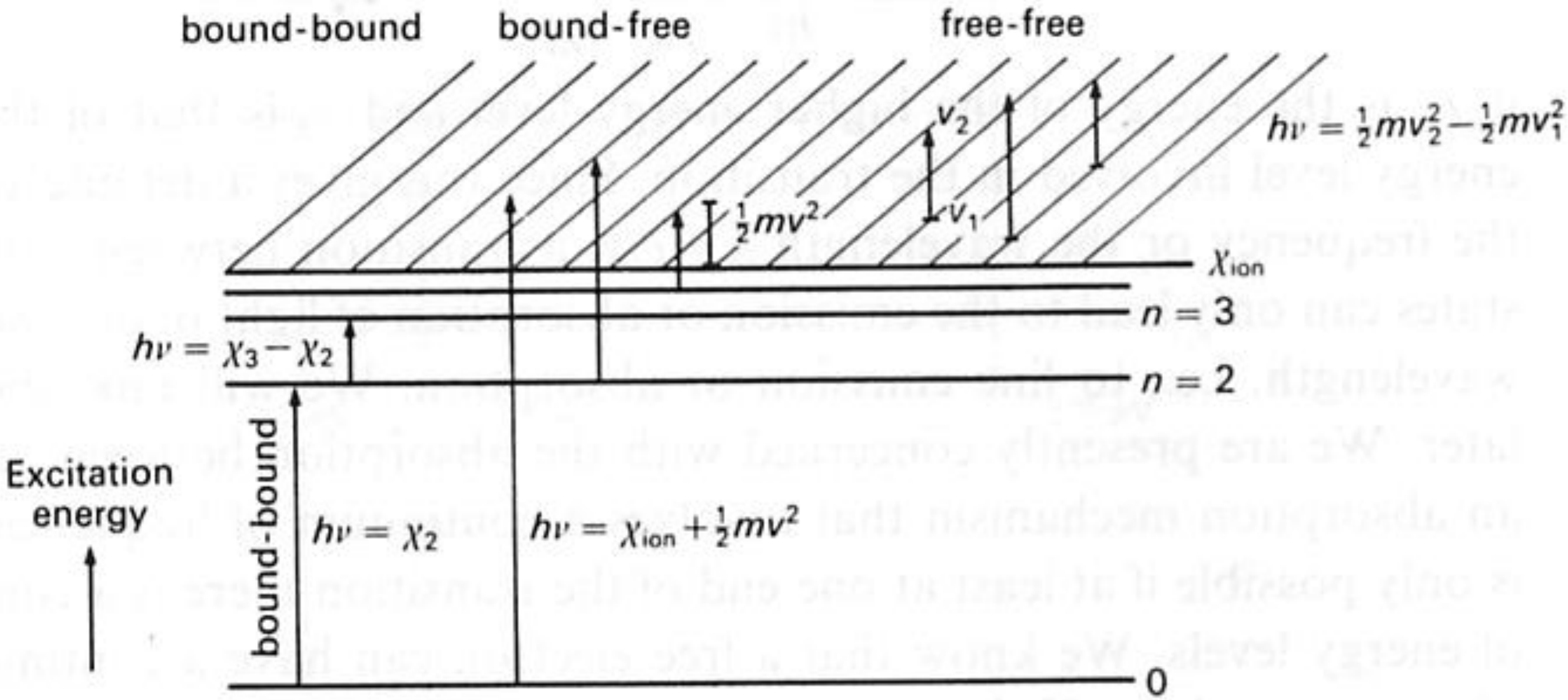
Lyman, Balmer, Paschen continua

- For hydrogen, transitions occurring between n=1 and another bound state n=2, 3, 4, etc. are known as the Lyman series (observed in the UV), between n=2 and higher members are the Balmer series (seen in the optical), with higher series observed in the IR: Paschen (n=3), Brackett (n=4), Pfund (n=5), etc.

- The Lyman continuum refers to a bound-free transition between n=1 and the H^+ continuum. Accordingly, the Balmer continuum between n=2 and the H^+ continuum, Paschen (n=3), Brackett (n=4), etc.



bb, bf, ff-processes



1 eV=11604 K =1.602 10^{-12} erg

χ_{ion}=13.6 eV, χ_{ion}/k=157820 K

k=8.6174x10^{-5} eV/K

Continuous absorption

Which states contribute at a given wavelength?

- Photons need an energy great enough to overcome the ionization energy i.e. $h\nu > \chi_{ion} - \chi_n$ or $\lambda < hc/(\chi_{ion} - \chi_n)$. At long wavelengths only energy levels with very large χ_n can contribute to α, so most continuous opacity is from mainly free-free transitions.
- The contribution of level n will start at $\lambda_n = hc/(\chi_{ion} - \chi_n)$ and continue for shorter λ. There is a **discontinuity** at λ_n because of a sudden change in the number of absorbing atoms, e.g.

  Lyman jump (912Å) due to the contribution of **n=1**.
  Balmer jump (3647Å) due to the contribution of **n=2**.

- To derive α_λ, the total absorption at wavelength λ, we have to multiply σ_n by the number of atoms in this state and sum up all states n that contribute at this wavelength.
  For this we need to use the Boltzmann formula.

$$\boldsymbol{\alpha_\lambda = \sigma_\lambda n}$$

Bound-free absorption coefficient

Kramers approximation for continuous cross-section for level *n* for H-like nucleus of charge Z:

Gaunt factor ≈ 1

$$\sigma_{bf}(\mathrm{H}) = \frac{32\pi^2}{3\sqrt{3}} \frac{e^6}{c^3\, h^3} R \frac{\lambda^3}{n^5} Z^4 G_{bf} = a_0 \frac{\lambda^3}{n^5} G_{bf} \; [\mathrm{cm}^2 \text{ per neutral H atom}]$$

Rydberg constant

$R=2\pi^2 me^4/h^3c$

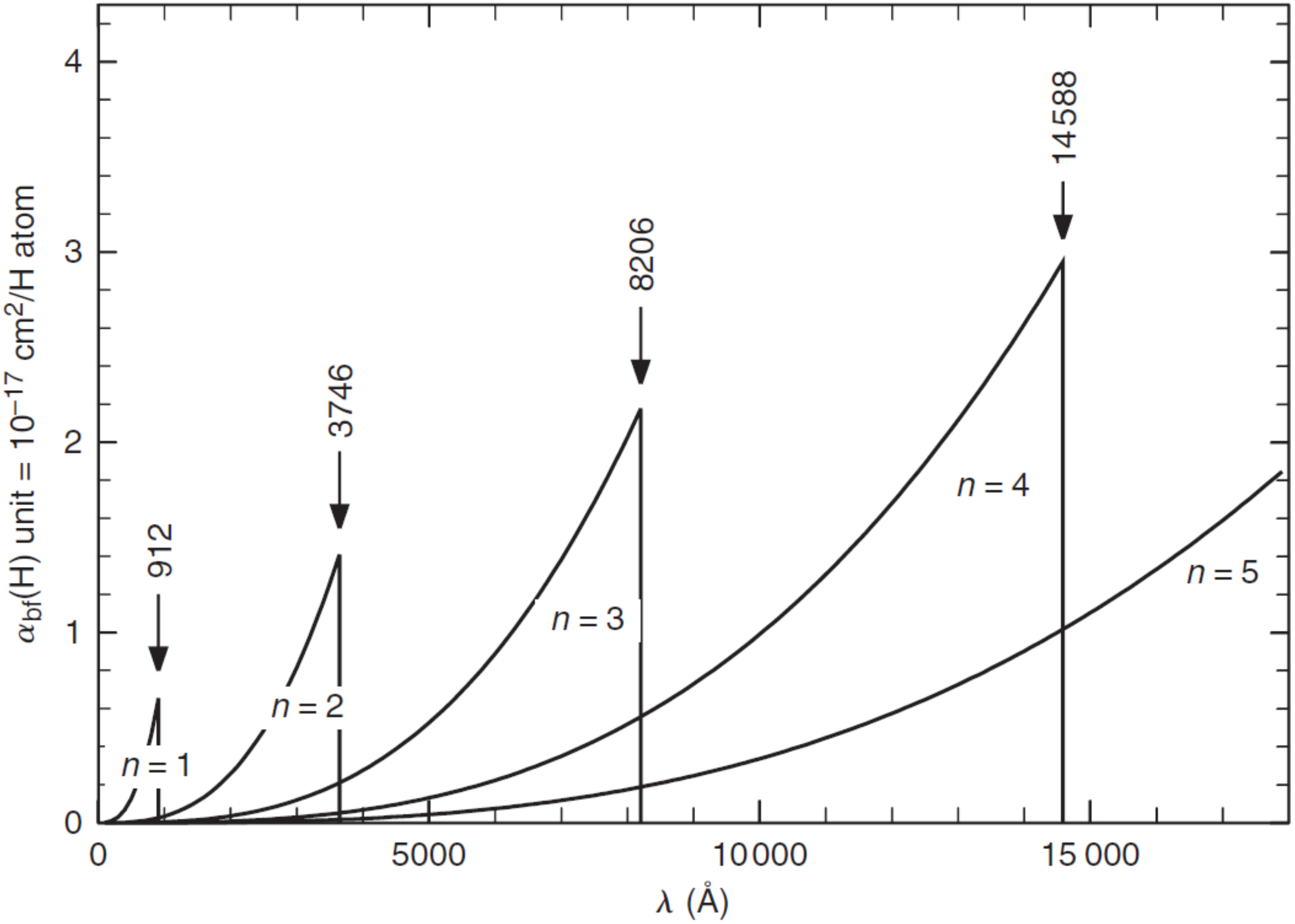
$a_0=1.0449\times10^{-26}$ for λ in angstroms

The photoionization threshold is $E_n=h\nu_{nc}$, so σ_n decreases with ν (increases with λ).

$$h\nu_{nc} = \chi_{ion} - \chi_n$$

For H, at the threshold $\sigma_{1c}=6.3\times10^{-18}$ cm² The total absorption coefficient for H is:

$$\alpha_{bf}^{H}(\lambda) = \sum_{n>\sqrt{\chi_{ion}/hc}}^{\infty} \sigma_{nc}(\lambda) N_n$$



The bound–free absorption coefficient for hydrogen increases with n.

Example: Lyman continuum

Gaunt factor ≈ 1

$\sigma_{bf}(\mathrm{H}) = a_0 \frac{\lambda^3}{n^5} G_{bf}$ cm^2 per neutral H atom $a_0 = 1.0449 \times 10^{-26}$ for λ in angstroms

For H, at the photoionization threshold, $\sigma_{1c} = 6.3\mathrm{x}10^{-18}$ cm^2

$$\tau_\lambda = \int_0^S \kappa_\lambda \rho ds = \int_0^S \sigma_\lambda n ds$$

Absorption by interstellar medium (ISM) at the Lyman edge:

$$\tau_\lambda = \int_0^S \sigma_{1c}(\lambda)\ N_{ISM} ds = \bar{N}_{ISM}\ \sigma_{1c} S$$

$\bar{N}_{ISM} \approx 1\mathrm{cm}^3$ but all the H atoms are in the ground state.

$$\tau_\lambda = 1 \text{ at } \quad S = \frac{1}{\bar{N}_{ISM}\ \sigma_{1c}} = 1.5 \times 10^{17}\mathrm{cm} = \frac{1}{20}\mathrm{pc}$$

Impossible to observe distant objects at λ< 912Å

Free-free absorption coefficient (1)

- The free-free continuous absorption coefficient for H is much smaller than the bound-free coefficient.
- When a free electron collides with a proton, its orbit (**unbound**) is altered. A photon may be absorbed during such a collision, the orbital energy of the electron being increased by the photon energy.
- The strength of the absorption depends on the electron velocity (slower electrons are more likely to absorb a photon because a slow encounter increases the probability of a photon passing by during the collision.
- We adopt a Maxwellian distribution.
- Kramers (1923):

Rydberg constant

$$d\sigma_{ff}(\mathrm{H}) = \frac{2}{3\sqrt{3}} \frac{h^2 e^2 \overset{\text{Rydberg constant}}{R}}{\pi m_e^3} \frac{\lambda^3}{c^3\, v} dv$$

Cross section for the fraction of electrons in the velocity interval

Free-free absorption coefficient (2)

- Integrate over velocity:

$$\sigma_{ff}(\mathrm{H}) = \frac{2}{3\sqrt{3}} \frac{h^2 e^2 R}{\pi m_e^3} \frac{\boldsymbol{\lambda^3}}{c^3} \left(\frac{2m_e}{\pi \kappa \boldsymbol{T}}\right)^{1/2}$$

Gaunt factor

- The total absorption coefficient for H is: $\kappa_{ff}^{H} = \frac{\sigma_{ff} G_{ff} N_i N_e}{N}$

  where the number density of electrons, ions and neutral Hydrogen are N_e, N_i and N, respectively.

- $N_i N_e / N$ can be substituted:

$$\kappa_{ff}^{H} = \sigma_{ff} G_{ff} \lambda^3 \frac{\log e}{2\Theta \mathrm{I}} 10^{-\Theta \mathrm{I}}$$

  where $\mathrm{I}=hcR$, $R=2\pi^2 me^4/h^3c$

- This absorption process is the inverse of Bremsstrahlung emission.

Wavelength dependence of α(H)

- Consider the H absorption coefficient α (per atom) for T=5040K (Θ=5040/T=1). Let us compare the value of α in the Balmer (n=2) to Lyman (n=1) continua at 912Å:

$$\frac{\alpha(Balmer)}{\alpha(Lyman)} = \frac{\sigma_{i2}}{\sigma_{i1}}\frac{N_2}{N_1} = \frac{\sigma_{i2}g_2}{\sigma_{i1}g_1}e^{-(10.2eV/kT)} = \frac{\sigma_{i2}g_2}{\sigma_{i1}g_1}10^{-(10.2\times5040/T)}$$

- From above, $\sigma_n \propto n^{-5}$ and $g_n=2n^2$ so $\frac{\alpha(Balmer)}{\alpha(Lyman)} = \frac{2^{-5}\times 8}{1\times 2}6.3\times10^{-11} \approx 8\times10^{-12}$

- There is a huge difference in hydrogen absorption coefficient at 912Å (Lyman edge) at T=5040K.

- Similar calculations at T=25200K (Θ=5040/T=0.2) give

$$\frac{\alpha(Balmer)}{\alpha(Lyman)} = \frac{2^{-5}\times 8}{1\times 2}0.009 = 0.001$$

- Hydrogen absorption coefficient is very T sensitive!

Wavelength dependence of α(H)

- Consider the H absorption coefficient α (per atom) for T=5040K (Θ=5040/T=1). What about the value of α in the Paschen (n=3) to Balmer (n=2) continua at 3647Å?

$$\frac{\alpha(+)}{\alpha(-)} = \frac{\sigma_u}{\sigma_l}\frac{N_u}{N_l} = \frac{\sigma_u g_u}{\sigma_l g_l} e^{-(\chi_{ul}/kT)} = \frac{\sigma_u g_u}{\sigma_l g_l} 10^{-(\chi_{ul}\times 5040/T)}$$

Transition between levels u and l:

$$\chi_{ul} = C\left(\frac{1}{u^2} - \frac{1}{l^2}\right)$$

where $C=\chi_{\text{ion}}$=-13.6 eV

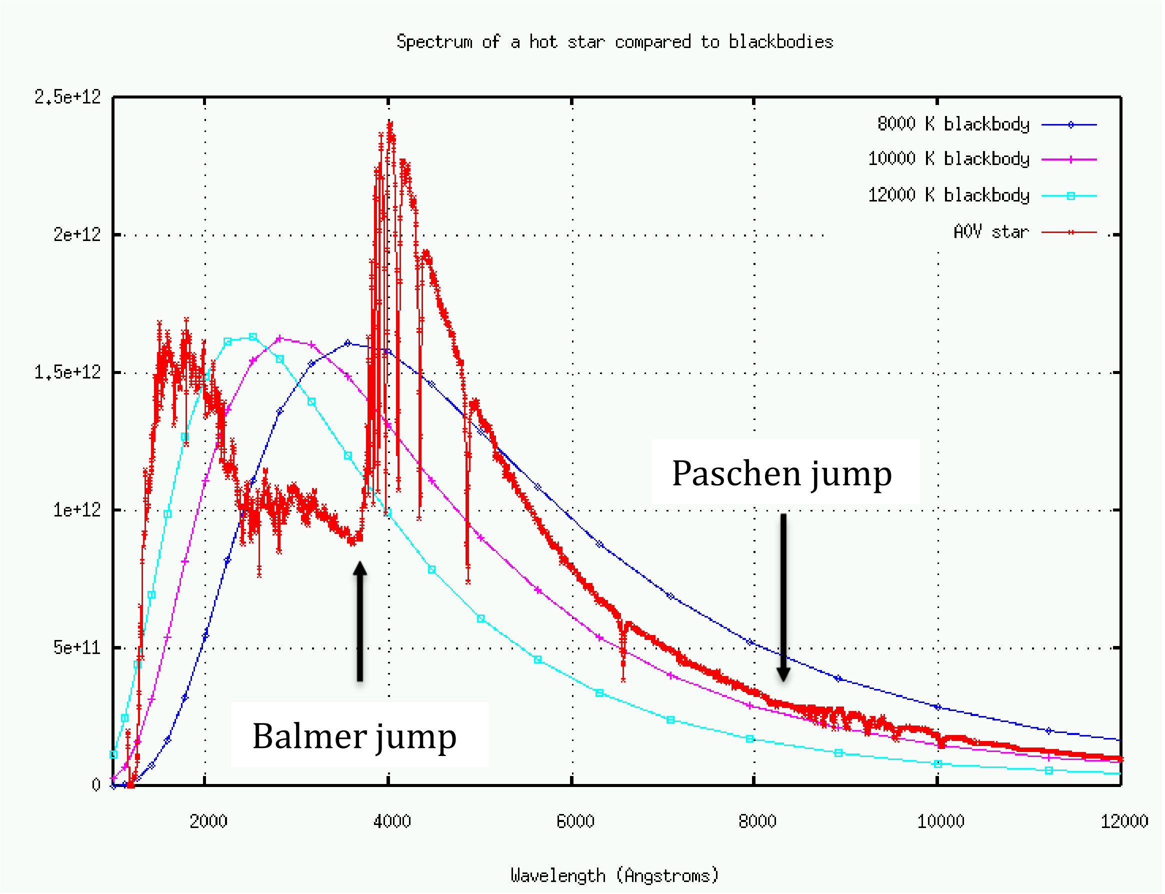
- From above, $\sigma_n \propto n^{-5}$ and $g_n = 2n^2$ so $$\frac{\alpha(Paschen)}{\alpha(Balmer)} \overset{?}{=} 0.004$$

- There is a huge difference with Lyman edge (8×10^{-12}).
  Still, Balmer jump is notable.

- Obviously, all the following jumps will be less and less prominent.

Wavelength dependence of $\alpha(H)$

- There is a huge difference between Lyman edge (8×10^{-12}) and Balmer jump (0.004). Still, Balmer jump is notable.
- Obviously, all the following jumps are less and less prominent.

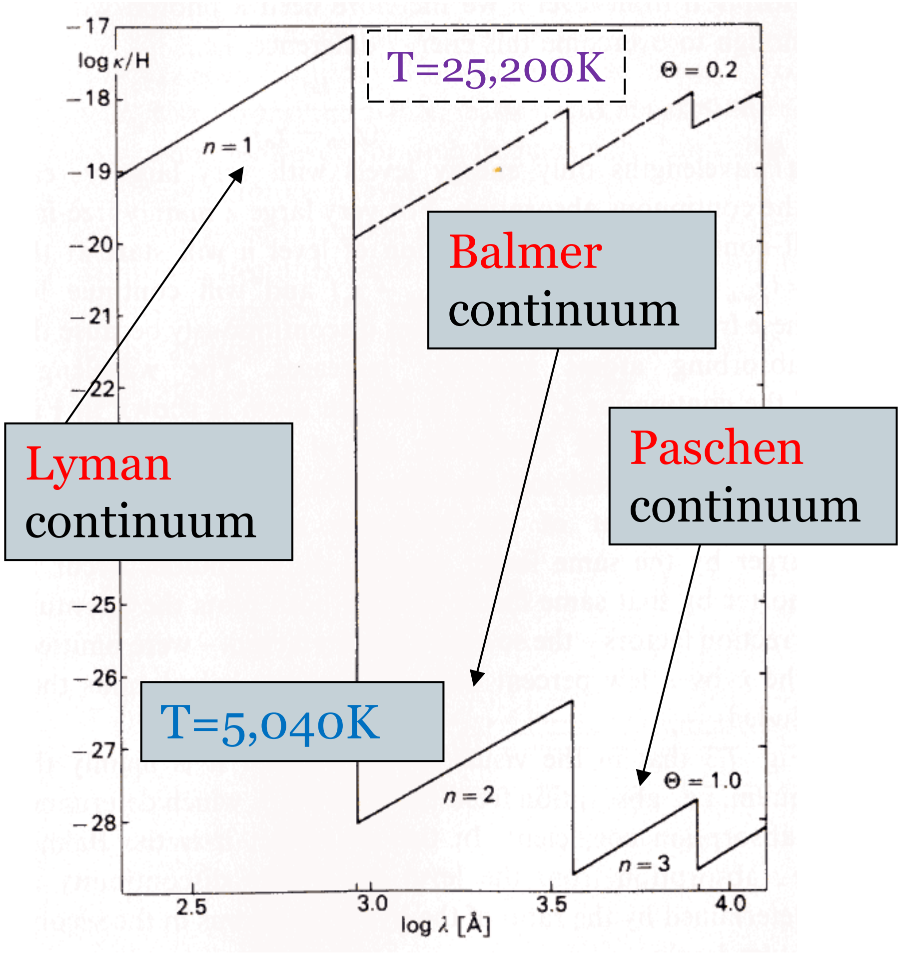


Paschen jump

Balmer jump

Wavelength dependence of α(H)

- Primarily, the Paschen continuum (absorption from n=3) determines the H absorption coefficient in the visual (3647Å<λ<8205Å).
- For He⁺, the ionization energy is larger by a factor of $Z^2=4$ than that of the H atom. All discontinuities occur at wavelengths shorter by a factor of 4, i.e. 228Å instead of 912Å for the He⁺ Lyman continuum.



T=25,200K

Balmer continuum

Lyman continuum

Paschen continuum

T=5,040K

Negative hydrogen ion H^-

- The H atom is capable of holding a second electron in a bound state (binding energy 0.754eV). All photons with λ<1.64μm have sufficient energy to ionize the H^- ion back to neutral H atom plus a free electron. The extra electrons needed to form H^- come from ionized metals (such as Ca^+).
- For Solar-like stars, it turns out that H^- is the dominant continuum opacity source at optical wavelengths. In early-type stars H^- is too highly ionized to play a role, whilst in late-type stars there are too few free electrons (since no ionized metals).

Importance of H^- in the Sun (1)

We can use the Saha equation to derive the relative population of $N(H^-)$ in the Sun ($u^-=1$, $T=5777$K, $\chi_{\text{ion}}=0.754$ eV),

$$\log\frac{N^+}{N^0} = \log\frac{u^+}{u^0} + \log 2 + \frac{5}{2}\log T - \chi_{ion}\Theta - \log P_e - 0.48$$

$$\log\frac{N(H^0)}{N(H^-)} = \log\frac{2}{1} + \log 2 + 9.40 - 0.66 - 1.18 - 0.48 = +7.68$$

So, only **2 out of 10^8** hydrogen atoms is in the form of H^-.

Why then the H^- absorption coefficient so important?
Recall, only H atoms in the 3rd quantum level (n=3, Paschen continuum) can contribute to the visual continuous opacity. From the Boltzmann formula

$$\log N(H_{n=3})/N(H_{n=1}) = \log 2(3)^2/2(1)^2 - 5040/5777 \times 12.1 = -9.6$$

i.e. $N_H(n=3)/N_H(n=1)=2.4\text{x}10^{-10}$ for the Sun. We can now compare the number of H^- ions and H atoms in the Paschen continuum:

$$\log N(H_{n=3})/N(H^-) = 2.4\times 10^{-10}/2.1\times 10^{-8} = 0.01$$

Importance of H^- in the Sun (2)

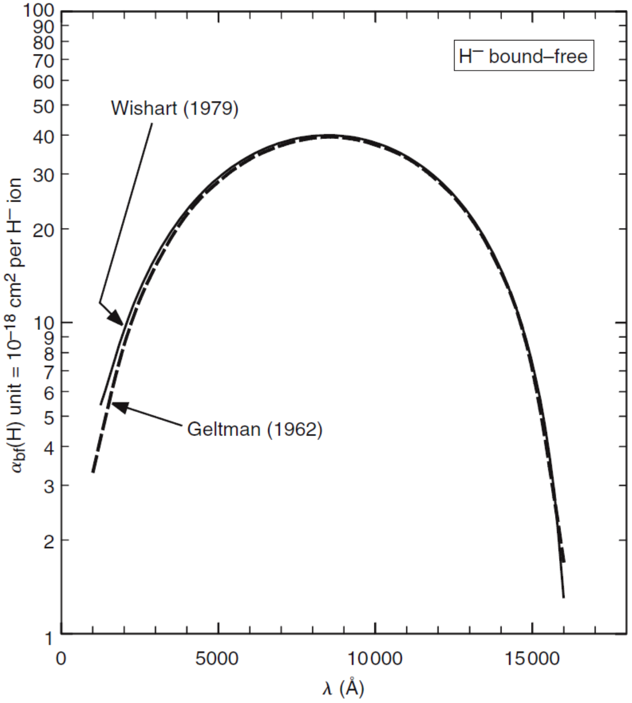
The atomic absorption coefficients per absorbing atom are comparable, so we expect H^- b-f absorption to be 100 times more important than the H Paschen continuum for the Sun.

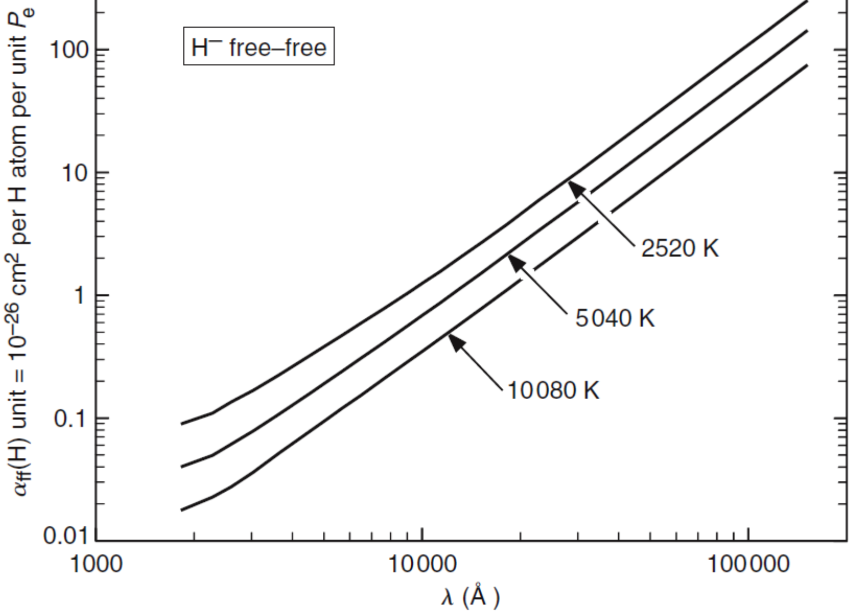
The Balmer continuum (n=2) cannot so easily be neglected and does contribute to the opacity at shorter wavelengths.

Note: For early type stars (A and earlier) we find $N_H(n=3)/N(H^-) \gg 1$ so absorption of neutral H is much more important than H^-. This is why such stars have very strong discontinuities in the Balmer & Paschen limits. We will discuss the importance of the Balmer jump shortly.

H^- continuous opacity

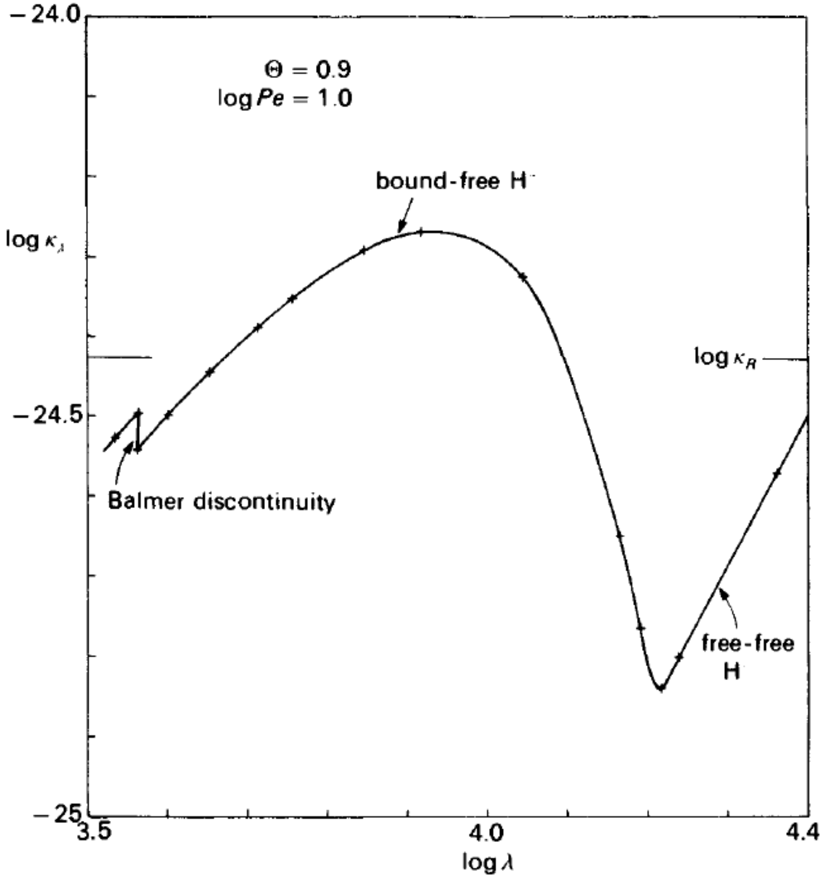
The bound-free H^- absorption can occur for $\lambda<16500$Å, with a different behaviour from H, reaching a maximum at 8000Å, and decreasing towards the ultraviolet. At longer wavelengths, there is only free-free H^- absorption (with a $\nu^{-3} \propto \lambda^3$ dependence).





Hydrogen continuous opacity

- We have identified H^- (bound-free) in the visual and H^- (free-free) in the IR as principal sources of opacity in the Sun.
- The H Balmer continuum shortward of the 3647Å Balmer jump is an additional contributor.
- What observational evidence is there that this is true for the Sun, and what other forms of opacity play a role in other stars?

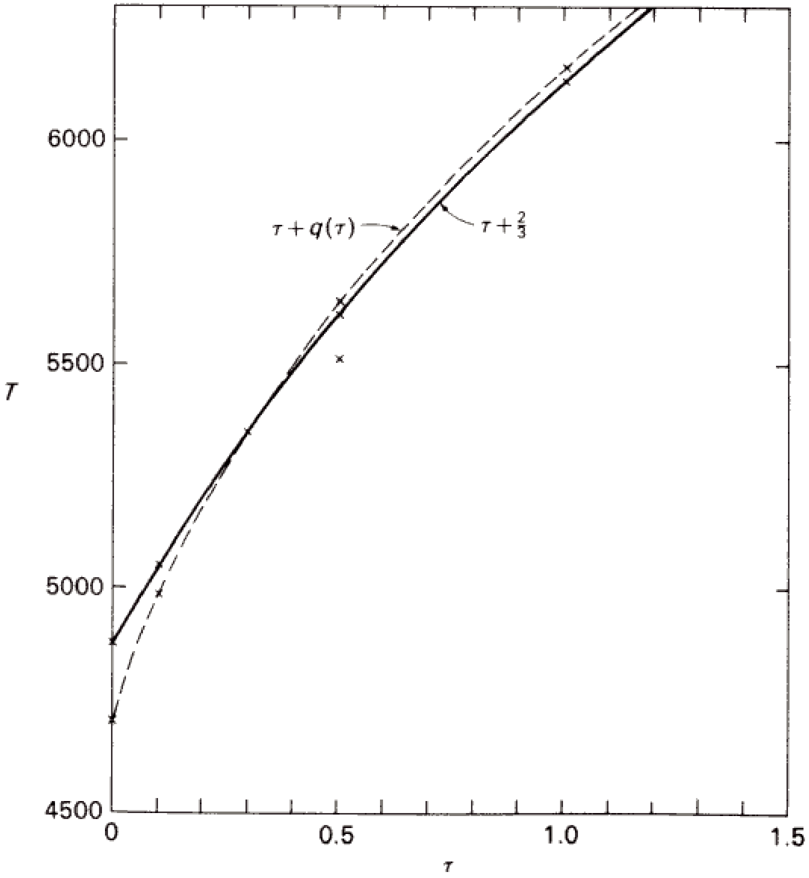


$T(\tau_\lambda)$ from Eddington approximation

We can use the observed limb darkening of the Sun at different λ to derive the depth dependence of the source function, $S_\lambda(\tau_\lambda)$.

Assuming LTE, $S_\lambda(\tau_\lambda)=B_\lambda[T(\tau_\lambda)]$ we can obtain the temperature as a function of τ_λ.

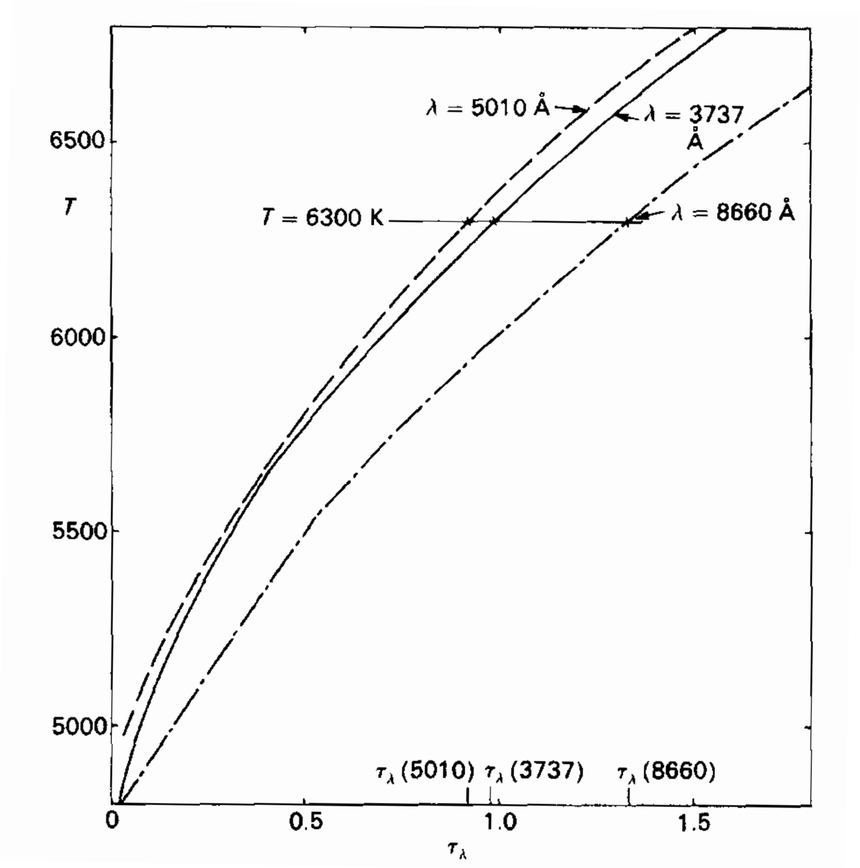
Recall from radiative equilibrium (assuming the Eddington approximation), $T(\tau_\lambda)$ can be obtained for a **grey** atmosphere.



$T(\tau_\lambda)$ from limb darkening

Limb darkening observations of the Sun at different wavelengths (via imaging using suitable filters) to derive $T(\tau_\lambda)$ at various wavelengths (e.g. 3737, 5010 & 8660 Å shown here).

The horizontal line shown at T=6300 K connects points which correspond to the same ***geometrical*** depth, so it is possible to derive the wavelength dependence of τ_λ.



Confirmation of H^-

The wavelength dependence of τ_λ (and hence κ_λ **or** α_λ) can be observationally derived for the Sun – the optical and IR dependence **agrees** remarkably well with the theoretical absorption coefficient for b-f and f-f H^-.