# The central pressure in polytropic stars



Another important relation is obtained between the central pressure and the central density.
Substitute $K$ from the mass-radius relation:

$$P_c = K\rho_c^{1+\frac{1}{n}} \quad \rightarrow \quad \left[\frac{GM}{-\xi_1^2(d\theta/d\xi)_{\xi_1}}\right]^{(n-1)}\left(\frac{R}{\xi_1}\right)^{3-n} = \frac{[K(n+1)]^n}{4\pi G}$$

We obtain

$$P_c = \frac{(4\pi G)^{1/n}}{(n+1)}\left[\frac{GM}{M_n}\right]^{\frac{n-1}{n}}\left(\frac{R}{R_n}\right)^{\frac{3-n}{n}}\rho_c^{\frac{n+1}{n}}$$

where $M_n = -\xi_1^2(d\theta/d\xi)_{\xi_1}$ and $R_n = \xi_1$.

Now eliminating $R$, using $D_N \equiv \frac{\rho_c}{\bar{\rho}} = \frac{\rho_c 4\pi R^3}{3M}$, and assembling all $n$-dependent coefficients into one constant $B_n$, we get

$$P_c = (4\pi)^{1/3} B_n G M^{2/3} \rho_c^{4/3}$$

| $n$ | $D_n$ | $M_n$ | $R_n$ | $B_n$ |
|---|---|---|---|---|
| 1.0 | 3.290 | 3.14 | 3.14 | 0.233 |
| 1.5 | 5.991 | 2.71 | 3.65 | 0.206 |
| 2.0 | 11.40 | 2.41 | 4.35 | 0.185 |
| 2.5 | 23.41 | 2.19 | 5.36 | 0.170 |
| 3.0 | 54.18 | 2.02 | 6.90 | 0.157 |
| 3.5 | 152.9 | 1.89 | 9.54 | 0.145 |

The remarkable property of this relation is that it depends on the polytropic equation of state only through the value of $B_n$, which varies very slowly with $n$.

**It therefore constitutes an almost universal relation!**

# The degeneracy pressure in polytropic stars



As a star contracts, the density may become so high that the electrons become degenerate and exert a (much) higher pressure than they would if they behaved classically. Stars that are so compact and dense that their interior pressure is dominated by degenerate electrons are known observationally as white dwarfs. They are the remnants of stellar cores in which hydrogen has been completely converted into helium. In most cases, also helium has been fused into carbon and oxygen.

We discussed the degeneracy pressure in Lecture 7. Let's now add a bit more detail.

The pressure of a completely degenerate electron gas in the non-relativistic limit is

$$P_e = K_{NR}\left(\frac{\rho}{\mu_e}\right)^{5/3} \quad \text{with} \quad K_{NR} = \frac{h^2}{20 m_e m_H^{5/3}}\left(\frac{3}{\pi}\right)^{2/3} = 1.0036 \times 10^{13}\ [\text{cgs}]$$

This corresponds to a polytropic relation with $n$=1.5 (the $\gamma = 5/3$ case). Since in the limit of strong degeneracy the pressure no longer depends on the **temperature**, this degeneracy pressure can hold the star up against gravity, regardless of the temperature. Therefore, a degenerate star does **not** have to be **hot** to be in hydrostatic equilibrium, and it can remain in this state forever even when it cools down. This is the situation in white dwarfs.

A few slides ago we obtained that for $n$=1.5, $R{\sim}M^{-1/3}$, i.e. the stellar radius is inversely proportional to the mass.

# The relativistic degeneracy in polytropic stars



More massive white dwarfs are thus more compact, and therefore have a higher density. Above a certain density the electrons will become relativistic as they are pushed up to higher momenta by the Pauli exclusion principle. The degree of relativity increases with density, and therefore with the mass of the white dwarf, until at a certain mass all the electrons become extremely relativistic, i.e., their speed $v_e \to c$. In this limit the equation of state has changed to (the pressure increases less steeply with density)

$$P_e = K_{ER} \left(\frac{\rho}{\mu_e}\right)^{4/3} \quad \text{with} \quad K_{ER} = \frac{hc}{8m_H^{4/3}} \left(\frac{3}{\pi}\right)^{1/3} = 1.2435 \times 10^{15}\ [\text{cgs}]$$

which is also a polytropic relation but with $n = 3$.

We have already seen above that an $n$=3 polytrope is special in the sense that it has **a unique mass**, which is determined by $K$ and is independent of the radius:

$$M = -\frac{4}{\sqrt{\pi}} \xi_1^2 \left(\frac{d\theta}{d\xi}\right)_{\xi_1} \left(\frac{K}{G}\right)^{3/2}$$

This value corresponds to an upper limit to the mass of a gas sphere in hydrostatic equilibrium that can be supported by degenerate electrons, and thus to the **maximum possible mass** for a white dwarf. Its existence was first found by Subrahmanyan Chandrasekhar in 1931, after whom this limiting mass was named.

# Chandrasekhar mass



-2.01824

$K_{ER} = 1.2435 \times 10^{15}$ [cgs]

A relativistic electron gas has $K = K_{ER}/\mu_e^{4/3}$

Substituting it and other proper numerical values into

$$M = -\frac{4}{\sqrt{\pi}} \xi_1^2 \left(\frac{d\theta}{d\xi}\right)_{\xi_1} \left(\frac{K}{G}\right)^{3/2}$$

$$\mu_e \approx \frac{2}{1+X}$$

we obtain the Chandrasekhar mass

$$M = M_{Ch} = \frac{5.826}{\mu_e^2} M_\odot$$

Thus, for a highly relativistic electron gas, there is only a **single** possible mass which can be in hydrostatic equilibrium.

White dwarfs are typically formed of helium, carbon or oxygen, for which $\mu_e = 2$ and therefore $M_{Ch} = 1.456\ M_\odot$.

This quantity is called the Chandresekhar mass, after Subrahmanyan Chandresekhar, who first derived it. He did the calculation while on his first trip out of India, to start graduate school at Cambridge at age 20… This work earned Chandrasekhar the 1983 Nobel Prize for Physics (which he shared with Fowler for their contributions to the understanding of stellar evolution).

A further increase of the mass (e.g., due to accretion from a companion star) leads to the loss of stability and collapse. This is the cause of supernovae type Ia explosions.

Find the value

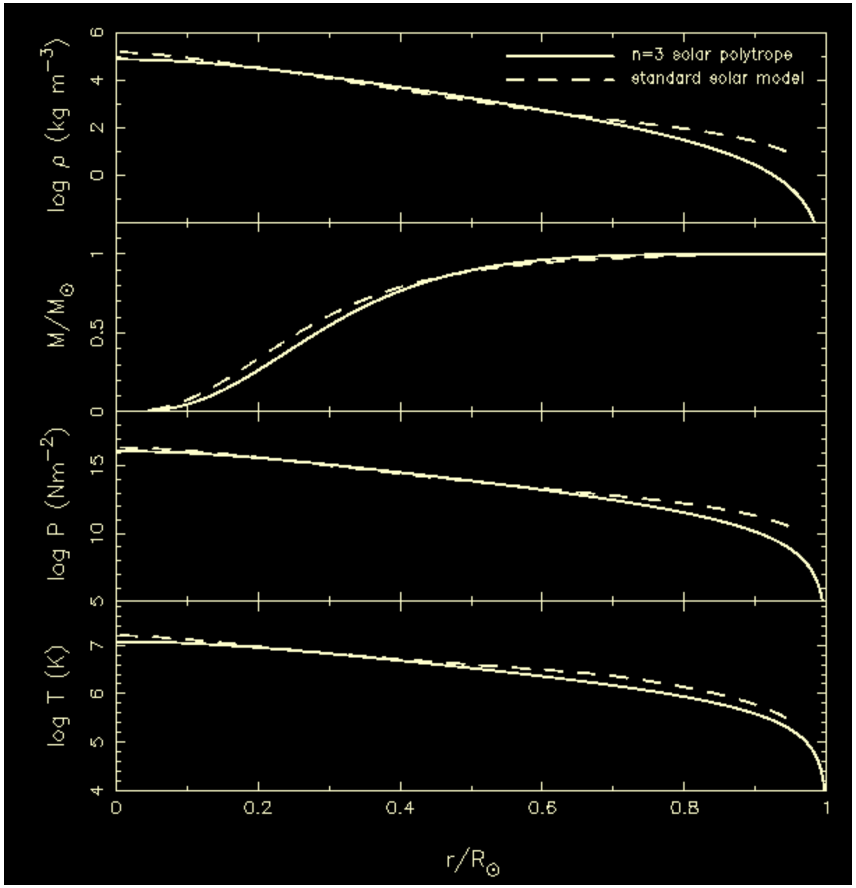
# Comparison with real models



- How do these polytropic models, compare to the results of a detailed solution of the equations of stellar structure? To make this comparison we will take an $n=3$ polytropic model of the Sun (often known as the Eddington Standard Model, a model with the constant fraction of radiation pressure and $\mu$=const), with the co-called Standard Solar Model (SSM - Bahcall 1998, Physics Letters B, 433, 1).
- For this, we need to convert the dimensionless radius $\xi$ and density $\theta$ to actual radius (in cm) and density (in g cm$^{-3}$).
- Polytrope does remarkably well (particularly at the core) considering how simple the physics is.

| Property | n=3 polytrope | SSM |
|---|---|---|
| $\rho_c$ | $7.65 \times 10^1$ g cm$^{-3}$ | $1.52 \times 10^2$ g cm$^{-3}$ |
| $P_c$ | $1.25 \times 10^{17}$ dyn cm$^{-2}$ | $2.34 \times 10^{17}$ dyn cm$^{-2}$ |
| $T_c$ | $1.18 \times 10^7$ K | $1.57 \times 10^7$ K |

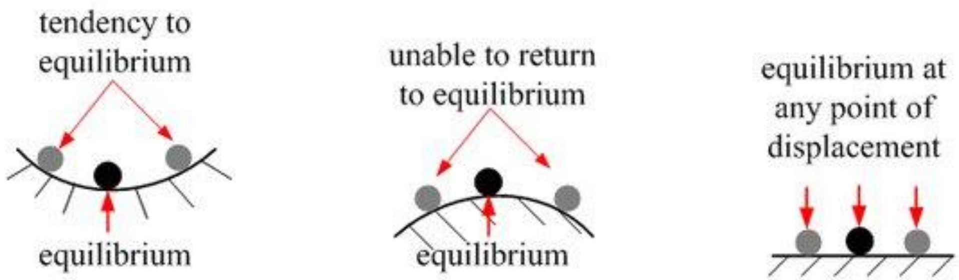
Comparison of numerical solution for an $n = 3$ polytrope of the Sun versus the Standard Solar Model.



# The stability of stars



We have so far considered stars in both hydrostatic and thermal equilibrium (HE & TE). But an important question is whether these equilibria are stable?




(a) stable equilibrium (b) unstable equilibrium (c) neutral equilibrium

A rigorous treatment of this problem is very complicated, so we will only look at a very simplified example to illustrate the principles.

# Dynamical stability of stars



- Suppose a star in hydrostatic equilibrium is compressed on a short timescale, $\tau << \tau_{KH}$, so that the compression can be considered as adiabatic.
- It can be shown that a star that has $\gamma_{ad}>4/3$ everywhere is dynamically stable, and if $\gamma_{ad}=4/3$, it is neutrally stable. However, the situation when $\gamma_{ad}<4/3$ in some part of the star requires further investigation.
- If $\gamma_{ad}<4/3$ in a sufficiently large core, where $P/\rho$ is high, the star becomes unstable. However if $\gamma_{ad}<4/3$ in the outer layers where $P/\rho$ is small, the star as a whole need **not** become unstable.
- Stars dominated by an ideal gas or by non-relativistic degenerate electrons have $\gamma_{ad}=5/3$ and are therefore dynamically stable. However, we have seen that for relativistic particles $\gamma_{ad}\rightarrow 4/3$ and stars dominated by such particles tend towards a neutrally stable state.
- A small disturbance of such a star could either lead to a collapse or an explosion. This is the case if radiation pressure dominates (at high $T$ and low $\rho$), or the pressure of relativistically degenerate electrons (at very high $\rho$).

Overall, if the configuration of a star is to be approximately described by a polytrope (in which case $\gamma$ and $\gamma_{ad}$ are identical), the index $n$ may only vary between 1.5 and 3, or

$$\frac{4}{3} < \gamma_{ad} \leq \frac{5}{3}$$

# Cases of dynamical instability



- We have seen earlier (Lectures 3 and 7) that the contribution of radiation pressure increases with mass and becomes dominant for $M \gtrsim 100\ M_{\odot}$. A gas dominated by radiation pressure has an adiabatic index $\gamma_{ad}$=4/3, or $n$=3, which means that hydrostatic equilibrium in such stars becomes marginally unstable. Therefore, stars much more massive than 100 $M_{\odot}$ should be very unstable, and indeed none are known to exist (while those with M > 50 $M_{\odot}$ indeed show signs of being close to instability, e.g. they lose mass very readily).
- A process that can lead to $\gamma_{ad}<4/3$ is partial ionization (e.g. $H \leftrightarrow H^+ + e^-$). Since this normally occurs in the very outer layers, where $P/\rho$ is small, it does not lead to overall dynamical instability of the star. However, partial ionization is connected to driving oscillations in some kinds of star.
- At very high temperatures two other processes can occur that have a similar effect to ionization:
  - These are pair creation ($\gamma + \gamma \leftrightarrow e^+ + e^-$) and photo-disintegration of nuclei (e.g. $\gamma$ + Fe $\leftrightarrow$ $\alpha$). These processes, that may occur in massive stars in late stages of evolution, also lead to $\gamma_{ad}<4/3$ but now in the core of the star. These processes can lead to a stellar explosion or collapse.

# Summary



- We discussed methods of finding the solution of the equations of stellar structure.
- We have defined a method to relate the internal pressure and density as a function of radius – the polytropic equation of state.
- We derived the Lane-Emden equation.
- We saw how this equation could be numerically integrated in general.
- We derived a number useful relations between stellar parameters.
- There is a theoretical upper limit to the mass of a white dwarf (Chandrasekhar limit). It is confirmed by observations, we do not see WDs with masses $>1.4M_{\odot}$.
- Further increase of the WD mass e.g. as a result of accretion from the companion, will lead to the loss of stability and collapse, causing supernovae type Ia explosions.
- We compared the $n=3$ polytrope with the Standard Solar model, finding quite good agreement considering how simple the input physics was.
- Finally, we discussed cases of dynamical instability of stars.

# Stellar evolution codes



- A stellar evolution code — a piece of software that can construct a model for the interior of a star, and then evolve it over time.
- Stellar evolution codes are often complicated to use. Rich Townsend from the University of Wisconsin-Madison created EZ-Web, a simple, web-based interface to a code that can be used to calculate models over a wide range of masses and metallicities: http://ftp.astro.wisc.edu/~townsend/static.php?ref=ez-web
- Read **carefully** the description of the program on its webpage and play with it.
- To construct and evolve a model, enter parameters into the form, and then submit the calculation request to the server.

## Submit a Calculation

Initial Mass: 1.0
Metallicity: 0.02
Maximum Age: 0
Maximum Number of Steps: 0
Create Detailed Structure Files? ☑
Use CGS units? ☑
Email Address: vitaly.neustroev@oulu.fi

Submit!

# EZ-Web: Output File Formats



Detailed structure files are text (ASCII) files containing one line for each grid point of the model. Each line is divided into 36 columns, containing the following data:

| Column Number | Datum | Description |
|---|---|---|
| 1 | $M_r$ | Lagrangian mass coordinate ($M_\odot$) |
| 2 | $r$ | Radius coordinate ($R_\odot$) |
| 3 | $L_r$ | Luminosity ($L_\odot$) |
| 4 | $P$ | Total pressure (N $m^{-2}$) |
| 5 | $\rho$ | Density (kg $m^{-3}$) |
| 6 | $T$ | Temperature (K) |
| 7 | $U$ | Specific internal energy (J $kg^{-1}$) |
| 8 | $S$ | Specific entropy (J $K^{-1}$ $kg^{-1}$) |
| 9 | $C_P$ | Specific heat at constant pressure (J $K^{-1}$ $kg^{-1}$) |
| 10 | $\Gamma_1$ | First adiabatic exponent |
| 11 | $\nabla_{ad}$ | Adiabatic temperature gradient |
| 12 | $\mu$ | Mean molecular weight (see note below) |
| 13 | $n_e$ | Electron number density ($m^{-3}$) |
| 14 | $P_e$ | Electron pressure (N $m^{-2}$) |
| 15 | $P_r$ | Radiation pressure (N $m^{-2}$) |
| 16 | $\nabla_{rad}$ | Radiative temperature gradient |
| 17 | $\nabla$ | Material temperature gradient |
| 18 | $v_c$ | Convective velocity (m $s^{-1}$) |
| 19 | $\kappa$ | Rosseland mean opacity ($m^2$ $kg^{-1}$) |
| 20 | $\epsilon_{nuc}$ | Power per unit mass from all nuclear reactions, excluding neutrino losses (W $kg^{-1}$) |
| 21 | $\epsilon_{PP}$ | Power per unit mass from PP chain (W $kg^{-1}$) |
| 22 | $\epsilon_{CNO}$ | Power per unit mass from CNO cycle (W $kg^{-1}$) |
| 23 | $\epsilon_{3\alpha}$ | Power per unit mass from triple-alpha reaction (W $kg^{-1}$) |
| 24 | $\epsilon_{\nu,nuc}$ | Power loss per unit mass in nuclear neutrinos (W $kg^{-1}$) |
| 25 | $\epsilon_\nu$ | Power loss per unit mass in non-nuclear neutrinos (W $kg^{-1}$) |
| 26 | $\epsilon_{grav}$ | Power per unit mass from gravitational contraction (W $kg^{-1}$) |
| 27 | $X$ | Hydrogen mass fraction (all ionization stages) |
| 28 | — | Molecular hydrogen mass fraction |
| 29 | $X^+$ | Singly-ionized hydrogen mass fraction |
| 30 | $Y$ | Helium mass fraction (all ionization stages) |
| 31 | $Y^+$ | Singly-ionized helium mass fraction |
| 32 | $Y^{++}$ | Doubly-ionized helium mass fraction |
| 33 | $X_C$ | Carbon mass fraction |
| 34 | $X_N$ | Nitrogren mass fraction |
| 35 | $X_O$ | Oxygen mass fraction |
| 36 | $\Psi$ | Electron degeneracy parameter |

Summary files have the filename 'summary.txt'. They are text (ASCII) files containing one line for each time step. Each line is divided into 23 columns, containing the following data:

| Column Number | Datum | Description |
|---|---|---|
| 1 | $i$ | Step number |
| 2 | $t$ | Age (years) |
| 3 | $M$ | Mass ($M_\odot$) |
| 4 | $\text{Log}_{10} L$ | Luminosity ($L_\odot$) |
| 5 | $\text{Log}_{10} R$ | Radius ($R_\odot$) |
| 6 | $\text{Log}_{10} T_s$ | Surface temperature (K) |
| 7 | $\text{Log}_{10} T_c$ | Central temperature (K) |
| 8 | $\text{Log}_{10} \rho_c$ | Central density (kg $m^{-3}$) |
| 9 | $\text{Log}_{10} P_c$ | Central pressure (N $m^{-2}$) |
| 10 | $\psi_c$ | Central electron degeneracy parameter |
| 11 | $X_c$ | Central hydrogen mass fraction |
| 12 | $Y_c$ | Central helium mass fraction |
| 13 | $X_{C,c}$ | Central carbon mass fraction |
| 14 | $X_{N,c}$ | Central nitrogen mass fraction |
| 15 | $X_{O,c}$ | Central oxygen mass fraction |
| 16 | $\tau_{dyn}$ | Dynamical timescale (seconds) |
| 17 | $\tau_{KH}$ | Kelvin-Helmholtz timescale (years) |
| 18 | $\tau_{nuc}$ | Nuclear timescale (years) |
| 19 | $L_{PP}$ | Luminosity from PP chain ($L_\odot$) |
| 20 | $L_{CNO}$ | Luminosity from CNO cycle ($L_\odot$) |
| 21 | $L_{3\alpha}$ | Luminosity from triple-alpha reactions ($L_\odot$) |
| 22 | $L_Z$ | Luminosity from metal burning ($L_\odot$) |
| 23 | $L_\nu$ | Luminosity of neutrino losses ($L_\odot$) |
| 24 | $M_{He}$ | Mass of helium core ($M_\odot$) |
| 25 | $M_C$ | Mass of carbon core ($M_\odot$) |
| 26 | $M_O$ | Mass of oxygen core ($M_\odot$) |
| 27 | $R_{He}$ | Radius of helium core ($R_\odot$) |
| 28 | $R_C$ | Radius of carbon core ($R_\odot$) |
| 29 | $R_O$ | Radius of oxygen core ($R_\odot$) |

# EZ-Web: Output Files



summary.txt

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 00000 | 0.00000000E+00 | 1.00000000E+00 | -1.54655885E-01 | -5.27369747E-02 | 3.74939401E+00 | 7.12628079E+00 | 1.89376232E+00 | 1.71504221E+01 | -1.72462659E+00 | 6.9 |
| 00001 | 5.00000000E+04 | 1.00000000E+00 | -1.49121128E-01 | -4.77912012E-02 | 3.74830482E+00 | 7.12713579E+00 | 1.89350124E+00 | 1.71510013E+01 | -1.72840433E+00 | 6.9 |
| 00002 | 1.00000000E+05 | 9.99999996E-01 | -1.49130943E-01 | -4.78005418E-02 | 3.74830703E+00 | 7.12713434E+00 | 1.89350183E+00 | 1.71509989E+01 | -1.72840028E+00 | 6.9 |
| 00003 | 1.60000000E+05 | 9.99999992E-01 | -1.49116611E-01 | -4.77976786E-02 | 3.74830918E+00 | 7.12713456E+00 | 1.89350736E+00 | 1.71510028E+01 | -1.72839088E+00 | 6.9 |
| 00004 | 2.32000000E+05 | 9.99999987E-01 | -1.49108696E-01 | -4.77962652E-02 | 3.74831046E+00 | 7.12713431E+00 | 1.89351552E+00 | 1.71510085E+01 | -1.72837393E+00 | 6.9 |
| 00005 | 3.18400000E+05 | 9.99999980E-01 | -1.49105445E-01 | -4.77956457E-02 | 3.74831096E+00 | 7.12713340E+00 | 1.89352734E+00 | 1.71510169E+01 | -1.72834643E+00 | 6.9 |
| 00006 | 4.22080000E+05 | 9.99999973E-01 | -1.49102674E-01 | -4.77956034E-02 | 3.74831163E+00 | 7.12713159E+00 | 1.89354415E+00 | 1.71510288E+01 | -1.72830439E+00 | 6.9 |
| 00007 | 5.46496000E+05 | 9.99999963E-01 | -1.49101674E-01 | -4.77957983E-02 | 3.74831198E+00 | 7.12712863E+00 | 1.89356776E+00 | 1.71510459E+01 | -1.72824262E+00 | 6.9 |
| 00008 | 6.95795200E+05 | 9.99999952E-01 | -1.49100163E-01 | -4.77959634E-02 | 3.74831244E+00 | 7.12712421E+00 | 1.89360055E+00 | 1.71510701E+01 | -1.72815442E+00 | 6.9 |
| 00009 | 8.74954240E+05 | 9.99999939E-01 | -1.49096703E-01 | -4.77957651E-02 | 3.74831320E+00 | 7.12711805E+00 | 1.89364565E+00 | 1.71511041E+01 | -1.72803135E+00 | 6.9 |
| 00010 | 1.08994509E+06 | 9.99999923E-01 | -1.49089337E-01 | -4.77947706E-02 | 3.74831455E+00 | 7.12710948E+00 | 1.89370761E+00 | 1.71511517E+01 | -1.72786022E+00 | 6.9 |
| 00011 | 1.34793411E+06 | 9.99999904E-01 | -1.49075486E-01 | -4.77923941E-02 | 3.74831682E+00 | 7.12709846E+00 | 1.89379152E+00 | 1.71512178E+01 | -1.72762874E+00 | 6.9 |
| 00012 | 1.65752093E+06 | 9.99999881E-01 | -1.49064057E-01 | -4.77877304E-02 | 3.74831735E+00 | 7.12717252E+00 | 1.89360724E+00 | 1.71511389E+01 | -1.72829797E+00 | 6.9 |
| 00013 | 2.02902511E+06 | 9.99999854E-01 | -1.49058031E-01 | -4.77853689E-02 | 3.74831767E+00 | 7.12723844E+00 | 1.89343991E+00 | 1.71510352E+01 | -1.72894882E+00 | 6.9 |
| 00014 | 2.47483013E+06 | 9.99999821E-01 | -1.49066347E-01 | -4.77864553E-02 | 3.74831614E+00 | 7.12728312E+00 | 1.89330108E+00 | 1.71509334E+01 | -1.72946174E+00 | 6.9 |
| 00015 | 3.00979616E+06 | 9.99999781E-01 | -1.49087443E-01 | -4.77911374E-02 | 3.74831320E+00 | 7.12730681E+00 | 1.89320595E+00 | 1.71508522E+01 | -1.72979571E+00 | 6.9 |
| 00016 | 3.65175539E+06 | 9.99999734E-01 | -1.49115460E-01 | -4.77982762E-02 | 3.74830977E+00 | 7.12731248E+00 | 1.89316946E+00 | 1.71508102E+01 | -1.72992448E+00 | 6.9 |
| 00017 | 4.42210647E+06 | 9.99999677E-01 | -1.49138113E-01 | -4.78048977E-02 | 3.74830742E+00 | 7.12730426E+00 | 1.89321072E+00 | 1.71508307E+01 | -1.72981694E+00 | 6.9 |
| 00018 | 5.34652777E+06 | 9.99999609E-01 | -1.49135133E-01 | -4.78057741E-02 | 3.74830860E+00 | 7.12728762E+00 | 1.89335191E+00 | 1.71509410E+01 | -1.72943992E+00 | 6.9 |
| 00019 | 6.45583332E+06 | 9.99999527E-01 | -1.49065456E-01 | -4.77907936E-02 | 3.74831853E+00 | 7.12727168E+00 | 1.89361171E+00 | 1.71511688E+01 | -1.72878195E+00 | 6.9 |
| 00020 | 7.78699998E+06 | 9.99999428E-01 | -1.48885007E-01 | -4.77503749E-02 | 3.74834343E+00 | 7.12726873E+00 | 1.89400408E+00 | 1.71515396E+01 | -1.72785515E+00 | 6.9 |
| 00021 | 9.38439998E+06 | 9.99999310E-01 | -1.48564759E-01 | -4.76816228E-02 | 3.74838912E+00 | 7.12728893E+00 | 1.89454437E+00 | 1.71520782E+01 | -1.72666097E+00 | 6.9 |
| 00022 | 1.13012800E+07 | 9.99999168E-01 | -1.48109770E-01 | -4.75913211E-02 | 3.74845771E+00 | 7.12734294E+00 | 1.89523691E+00 | 1.71527985E+01 | -1.72522940E+00 | 6.9 |
| 00023 | 1.36015360E+07 | 9.99998997E-01 | -1.47552839E-01 | -4.74935587E-02 | 3.74854807E+00 | 7.12743479E+00 | 1.89608781E+00 | 1.71537096E+01 | -1.72356257E+00 | 6.9 |
| 00024 | 1.63618432E+07 | 9.99998792E-01 | -1.46934111E-01 | -4.74018535E-02 | 3.74865690E+00 | 7.12756248E+00 | 1.89710578E+00 | 1.71548169E+01 | -1.72163437E+00 | 6.9 |
| 00025 | 1.96742118E+07 | 9.99998546E-01 | -1.46272479E-01 | -4.73212090E-02 | 3.74878198E+00 | 7.12771910E+00 | 1.89831289E+00 | 1.71561337E+01 | -1.71936924E+00 | 6.9 |
| 00026 | 2.36490542E+07 | 9.99998250E-01 | -1.45560955E-01 | -4.72474872E-02 | 3.74892300E+00 | 7.12789882E+00 | 1.89973001E+00 | 1.71576735E+01 | -1.71669826E+00 | 6.9 |
| 00027 | 2.84188650E+07 | 9.99997894E-01 | -1.44773718E-01 | -4.71711957E-02 | 3.74908167E+00 | 7.12809312E+00 | 1.90139161E+00 | 1.71594598E+01 | -1.71351138E+00 | 6.9 |
| 00028 | 3.41426380E+07 | 9.99997466E-01 | -1.43891012E-01 | -4.70841974E-02 | 3.74925884E+00 | 7.12829385E+00 | 1.90331761E+00 | 1.71615016E+01 | -1.70973537E+00 | 6.9 |
| 00029 | 4.10111656E+07 | 9.99996951E-01 | -1.42901913E-01 | -4.69801836E-02 | 3.74945411E+00 | 7.12848546E+00 | 1.90553278E+00 | 1.71638036E+01 | -1.70526173E+00 | 6.9 |
| 00030 | 4.92533987E+07 | 9.99996332E-01 | -1.41695096E-01 | -4.68920676E-02 | 3.74971176E+00 | 7.12865360E+00 | 1.90804310E+00 | 1.71663509E+01 | -1.70003488E+00 | 6.9 |
| 00031 | 5.91440784E+07 | 9.99995587E-01 | -1.40421745E-01 | -4.68196557E-02 | 3.74999389E+00 | 7.12875249E+00 | 1.91081485E+00 | 1.71690538E+01 | -1.69398795E+00 | 6.9 |
| 00032 | 7.10128941E+07 | 9.99994690E-01 | -1.39167470E-01 | -4.67400411E-02 | 3.75026765E+00 | 7.12866410E+00 | 1.91386574E+00 | 1.71717639E+01 | -1.68673306E+00 | 6.9 |
| 00033 | 8.52554730E+07 | 9.99993612E-01 | -1.38026553E-01 | -4.66552182E-02 | 3.75051047E+00 | 7.12874196E+00 | 1.91648455E+00 | 1.71741536E+01 | -1.68123348E+00 | 6.9 |
| 00034 | 1.02346568E+08 | 9.99992315E-01 | -1.36984433E-01 | -4.65467916E-02 | 3.75071678E+00 | 7.12897797E+00 | 1.91868948E+00 | 1.71762186E+01 | -1.67744215E+00 | 6.9 |
| 00035 | 1.22855881E+08 | 9.99990755E-01 | -1.35983308E-01 | -4.64054584E-02 | 3.75089640E+00 | 7.12932521E+00 | 1.92063044E+00 | 1.71780482E+01 | -1.67484950E+00 | 6.9 |
| 00036 | 1.47467057E+08 | 9.99988878E-01 | -1.34970459E-01 | -4.62265162E-02 | 3.75106014E+00 | 7.12975320E+00 | 1.92245915E+00 | 1.71797475E+01 | -1.67300550E+00 | 6.8 |
| 00037 | 1.77000469E+08 | 9.99986620E-01 | -1.33897983E-01 | -4.60045274E-02 | 3.75121726E+00 | 7.13024626E+00 | 1.92431005E+00 | 1.71814161E+01 | -1.67156214E+00 | 6.8 |
| 00038 | 2.12440562E+08 | 9.99983903E-01 | -1.32719347E-01 | -4.57317009E-02 | 3.75137551E+00 | 7.13080193E+00 | 1.92629885E+00 | 1.71831436E+01 | -1.67027268E+00 | 6.8 |
| 00039 | 2.54968675E+08 | 9.99980632E-01 | -1.31388672E-01 | -4.53979620E-02 | 3.75154131E+00 | 7.13142720E+00 | 1.92852657E+00 | 1.71850093E+01 | -1.66897018E+00 | 6.8 |
| 00040 | 3.06002410E+08 | 9.99976691E-01 | -1.29858094E-01 | -4.49899684E-02 | 3.75171996E+00 | 7.13213593E+00 | 1.93108907E+00 | 1.71870877E+01 | -1.66753684E+00 | 6.7 |
| 00041 | 3.67242892E+08 | 9.99971940E-01 | -1.28074520E-01 | -4.44905117E-02 | 3.75191612E+00 | 7.13294767E+00 | 1.93408672E+00 | 1.71894558E+01 | -1.66587891E+00 | 6.7 |
| 00042 | 4.40731470E+08 | 9.99966207E-01 | -1.25976693E-01 | -4.38782142E-02 | 3.75213443E+00 | 7.13388760E+00 | 1.93763302E+00 | 1.71921994E+01 | -1.66390866E+00 | 6.6 |
| 00043 | 5.28917764E+08 | 9.99959283E-01 | -1.23493337E-01 | -4.31263762E-02 | 3.75237935E+00 | 7.13498638E+00 | 1.94186085E+00 | 1.71954172E+01 | -1.66153290E+00 | 6.6 |
| 00044 | 6.34741317E+08 | 9.99950910E-01 | -1.20538777E-01 | -4.22002572E-02 | 3.75265493E+00 | 7.13628241E+00 | 1.94693238E+00 | 1.71992297E+01 | -1.65864233E+00 | 6.5 |
| 00045 | 7.61729581E+08 | 9.99940767E-01 | -1.17009703E-01 | -4.10561172E-02 | 3.75296513E+00 | 7.13782331E+00 | 1.95304725E+00 | 1.72037852E+01 | -1.65510395E+00 | 6.4 |
| 00046 | 9.14115497E+08 | 9.99928455E-01 | -1.12781748E-01 | -3.96369196E-02 | 3.75331252E+00 | 7.13966811E+00 | 1.96045415E+00 | 1.72092673E+01 | -1.65075035E+00 | 6.3 |
| 00047 | 1.09697860E+09 | 9.99913475E-01 | -1.07686651E-01 | -3.78815662E-02 | 3.75370862E+00 | 7.14189436E+00 | 1.96946964E+00 | 1.72159140E+01 | -1.64537216E+00 | 6.2 |
| 00048 | 1.31641432E+09 | 9.99895190E-01 | -1.01178106E-01 | -3.59651317E-02 | 3.75437754E+00 | 7.14465517E+00 | 1.98051946E+00 | 1.72241022E+01 | -1.63884948E+00 | 6.0 |
| 00049 | 1.54898087E+09 | 9.99875449E-01 | -9.42346845E-02 | -3.38055184E-02 | 3.75503358E+00 | 7.14763343E+00 | 1.99250253E+00 | 1.72329745E+01 | -1.63166363E+00 | 5.8 |
| 00050 | 1.77002222E+09 | 9.99856347E-01 | -8.76008939E-02 | -3.16188235E-02 | 3.75559868E+00 | 7.15051586E+00 | 2.00416026E+00 | 1.72416003E+01 | -1.62456154E+00 | 5.7 |
| 00051 | 1.97990273E+09 | 9.99837891E-01 | -8.12876282E-02 | -2.94185200E-02 | 3.75607685E+00 | 7.15330015E+00 | 2.01548032E+00 | 1.72499691E+01 | -1.61756000E+00 | 5.5 |
| 00052 | 2.18690941E+09 | 9.99819368E-01 | -7.50471407E-02 | -2.71342967E-02 | 3.75649486E+00 | 7.15609533E+00 | 2.02689644E+00 | 1.72584033E+01 | -1.61040174E+00 | 5.4 |
| 00053 | 2.39397010E+09 | 9.99800507E-01 | -6.85584537E-02 | -2.49353339E-02 | 3.75701755E+00 | 7.15898227E+00 | 2.03858853E+00 | 1.72670753E+01 | -1.60310219E+00 | 5.2 |
| 00054 | 2.59545554E+09 | 9.99781826E-01 | -6.20419966E-02 | -2.28210218E-02 | 3.75758951E+00 | 7.16187739E+00 | 2.05023874E+00 | 1.72757417E+01 | -1.59584425E+00 | 5.1 |
| 00055 | 2.78929645E+09 | 9.99763539E-01 | -5.57511042E-02 | -2.06620826E-02 | 3.75808276E+00 | 7.16471797E+00 | 2.06170654E+00 | 1.72842681E+01 | -1.58861828E+00 | 4.9 |
| 00056 | 2.97995906E+09 | 9.99745235E-01 | -4.95424753E-02 | -1.84181578E-02 | 3.75851296E+00 | 7.16756687E+00 | 2.07324477E+00 | 1.72928411E+01 | -1.58126888E+00 | 4.8 |
| 00057 | 3.16956154E+09 | 9.99726708E-01 | -4.33501270E-02 | -1.60708345E-02 | 3.75888738E+00 | 7.17045803E+00 | 2.08498470E+00 | 1.73015585E+01 | -1.57371883E+00 | 4.6 |
| 00058 | 3.35729449E+09 | 9.99708036E-01 | -3.72066497E-02 | -1.36333288E-02 | 3.75920450E+00 | 7.17337945E+00 | 2.09688009E+00 | 1.73103829E+01 | -1.56599733E+00 | 4.5 |
| 00059 | 3.54213729E+09 | 9.99689319E-01 | -3.07757887E-02 | -1.14591319E-02 | 3.75972512E+00 | 7.17637742E+00 | 2.10887939E+00 | 1.73193338E+01 | -1.55835552E+00 | 4.3 |
| 00060 | 3.71808305E+09 | 9.99671193E-01 | -2.46230571E-02 | -9.28414674E-03 | 3.76017581E+00 | 7.17929665E+00 | 2.12056875E+00 | 1.73280453E+01 | -1.55087599E+00 | 4.2 |
| 00061 | 3.88962452E+09 | 9.99653214E-01 | -1.86076170E-02 | -7.04529775E-03 | 3.76056024E+00 | 7.18220225E+00 | 2.13223151E+00 | 1.73367232E+01 | -1.54335760E+00 | 4.0 |
| 00062 | 4.06034558E+09 | 9.99635006E-01 | -1.26041325E-02 | -4.70348384E-03 | 3.76089021E+00 | 7.18515306E+00 | 2.14411498E+00 | 1.73455517E+01 | -1.53562397E+00 | 3.9 |
| 00063 | 4.22989604E+09 | 9.99616602E-01 | -6.62937157E-03 | -2.26652615E-03 | 3.76116542E+00 | 7.18814554E+00 | 2.15620153E+00 | 1.73545170E+01 | -1.52768977E+00 | 3.8 |
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| 00065 | 4.56021189E+09 | 9.99579795E-01 | 5.41789441E-03 | 2.41356253E-03 | 3.76183719E+00 | 7.19422809E+00 | 2.18061413E+00 | 1.73726331E+01 | -1.51171625E+00 | 3.5 |
| 00066 | 4.71603842E+09 | 9.99561978E-01 | 1.12662351E-02 | 4.63670085E-03 | 3.76218771E+00 | 7.19721646E+00 | 2.19255932E+00 | 1.73814907E+01 | -1.50390773E+00 | 3.3 |
| 00067 | 4.86837666E+09 | 9.99544263E-01 | 1.69941478E-02 | 6.92110129E-03 | 3.76247748E+00 | 7.20019856E+00 | 2.20452497E+00 | 1.73903399E+01 | -1.49601796E+00 | 3.2 |
| 00068 | 5.01982629E+09 | 9.99526349E-01 | 2.26980449E-02 | 9.29866539E-03 | 3.76271468E+00 | 7.20322458E+00 | 2.21672172E+00 | 1.73993358E+01 | -1.48789473E+00 | 3.1 |
| 00069 | 5.16982148E+09 | 9.99508301E-01 | 2.83532402E-02 | 1.17579880E-02 | 3.76289881E+00 | 7.20628287E+00 | 2.22911255E+00 | 1.74084503E+01 | -1.47954768E+00 | 2.9 |
| 00070 | 5.31710305E+09 | 9.99490274E-01 | 3.39047944E-02 | 1.42730113E-02 | 3.76302919E+00 | 7.20934914E+00 | 2.24159563E+00 | 1.74176084E+01 | -1.47104777E+00 | 2.8 |
| 00071 | 5.46138618E+09 | 9.99472314E-01 | 3.96478838E-02 | 1.65052977E-02 | 3.76334882E+00 | 7.21246717E+00 | 2.25413119E+00 | 1.74268200E+01 | -1.46264272E+00 | 2.7 |
| 00072 | 5.59887734E+09 | 9.99454916E-01 | 4.51349529E-02 | 1.87227460E-02 | 3.76361186E+00 | 7.21550833E+00 | 2.26638567E+00 | 1.74358012E+01 | -1.45437651E+00 | 2.5 |
| 00073 | 5.73271120E+09 | 9.99437706E-01 | 5.04785809E-02 | 2.09837027E-02 | 3.76381729E+00 | 7.21853534E+00 | 2.27862120E+00 | 1.74447410E+01 | -1.44606649E+00 | 2.4 |
| 00074 | 5.86545094E+09 | 9.99420357E-01 | 5.57791373E-02 | 2.33241155E-02 | 3.76397222E+00 | 7.22160867E+00 | 2.29107313E+00 | 1.74538103E+01 | -1.43756432E+00 | 2.3 |
| 00075 | 5.99691090E+09 | 9.99402893E-01 | 6.10244679E-02 | 2.57371570E-02 | 3.76407704E+00 | 7.22472292E+00 | 2.30373392E+00 | 1.74630002E+01 | -1.42885808E+00 | 2.2 |
| 00076 | 6.12597596E+09 | 9.99385466E-01 | 6.61623761E-02 | 2.81973101E-02 | 3.76413144E+00 | 7.22785719E+00 | 2.31649543E+00 | 1.74722319E+01 | -1.42005037E+00 | 2.0 |
| 00077 | 6.25223979E+09 | 9.99368144E-01 | 7.12869514E-02 | 3.05523844E-02 | 3.76423504E+00 | 7.23102521E+00 | 2.32929997E+00 | 1.74814724E+01 | -1.41131338E+00 | 1.9 |
| 00078 | 6.37467690E+09 | 9.99351082E-01 | 7.64053040E-02 | 3.27336119E-02 | 3.76442402E+00 | 7.23421914E+00 | 2.34202133E+00 | 1.74906321E+01 | -1.40285449E+00 | 1.8 |
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| 00081 | 6.72310421E+09 | 9.99301040E-01 | 9.08224142E-02 | 3.94537906E-02 | 3.76466820E+00 | 7.24392669E+00 | 2.38012814E+00 | 1.75177395E+01 | -1.37841970E+00 | 1.4 |
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| 00084 | 7.05564971E+09 | 9.99251105E-01 | 1.04298680E-01 | 4.64502292E-02 | 3.76453905E+00 | 7.25414349E+00 | 2.41961445E+00 | 1.75451420E+01 | -1.35439823E+00 | 1.0 |

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| 1.00000000E+00 | 8.85479869E-01 | 7.00396738E-01 | 1.04457019E+05 | 1.93082209E-07 | 8.33872385E+03 | 2.37249724E+12 | 1.60510318E+09 | 2.96510225E+08 | 1.3485 |
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| 9.99999100E-01 | 8.79186642E-01 | 7.00396738E-01 | 1.30496479E+09 | 3.11964891E-04 | 3.89763278E+04 | 1.54769961E+13 | 1.67147787E+09 | 6.86232584E+08 | 1.4168 |
| 9.99998638E-01 | 8.78433875E-01 | 7.00396738E-01 | 1.98070932E+09 | 4.17358611E-04 | 4.33208841E+04 | 1.67687188E+13 | 1.67136245E+09 | 6.15136844E+08 | 1.4549 |
| 9.99997944E-01 | 8.77581105E-01 | 7.00396738E-01 | 3.00139660E+09 | 5.53222075E-04 | 4.85736916E+04 | 1.81896776E+13 | 1.67117632E+09 | 5.44801204E+08 | 1.5008 |
| 9.99996896E-01 | 8.76603987E-01 | 7.00396738E-01 | 4.54704972E+09 | 7.27014416E-04 | 5.50143472E+04 | 1.97656317E+13 | 1.67088605E+09 | 4.83830260E+08 | 1.5486 |
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| 9.99992811E-01 | 8.74105804E-01 | 7.00396738E-01 | 1.06115947E+10 | 1.24420086E-03 | 7.28412042E+04 | 2.36449450E+13 | 1.67008748E+09 | 4.15050060E+08 | 1.6060 |
| 9.99988828E-01 | 8.72448516E-01 | 7.00396738E-01 | 1.65605420E+10 | 1.64367621E-03 | 8.49795028E+04 | 2.61979147E+13 | 1.66968812E+09 | 4.11362893E+08 | 1.6010 |
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| 9.99971260E-01 | 8.67834905E-01 | 7.00396738E-01 | 4.31312051E+10 | 3.00657810E-03 | 1.17930398E+05 | 3.34695379E+13 | 1.66935552E+09 | 4.30031378E+08 | 1.5825 |
| 9.99952599E-01 | 8.64651294E-01 | 7.00396738E-01 | 7.18383205E+10 | 4.14550948E-03 | 1.40531118E+05 | 3.85273070E+13 | 1.66926083E+09 | 4.21637314E+08 | 1.5992 |
| 9.99920699E-01 | 8.60685984E-01 | 7.00396738E-01 | 1.21718865E+11 | 5.75334327E-03 | 1.69400119E+05 | 4.47727600E+13 | 1.66902692E+09 | 4.01872862E+08 | 1.6252 |
| 9.99865572E-01 | 8.55713066E-01 | 7.00396738E-01 | 2.09677980E+11 | 8.02876054E-03 | 2.06864809E+05 | 5.25505548E+13 | 1.66867929E+09 | 3.82536939E+08 | 1.6473 |
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| 9.99302235E-01 | 8.31376765E-01 | 7.00396738E-01 | 1.18140756E+12 | 2.27734107E-02 | 4.02259083E+05 | 9.13892360E+13 | 1.66789652E+09 | 3.54953476E+08 | 1.6685 |
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| 9.97901252E-01 | 8.03119671E-01 | 7.00396738E-01 | 3.92955580E+12 | 4.67356585E-02 | 6.46385409E+05 | 1.39506896E+14 | 1.66824232E+09 | 3.48567022E+08 | 1.6697 |
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| 9.09829452E-01 | 5.13397475E-01 | 7.00394711E-01 | 6.01273148E+14 | 1.22627442E+00 | 3.67055418E+06 | 7.43117749E+14 | 1.58232234E+09 | 3.38791400E+08 | 1.6659 |
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| 7.16330700E-01 | 3.56030385E-01 | 6.99902595E-01 | 5.39622934E+15 | 7.21008497E+00 | 5.60622541E+06 | 1.12872513E+15 | 1.43019464E+09 | 3.37381303E+08 | 1.6666 |
| 7.00731155E-01 | 3.48221726E-01 | 6.99752169E-01 | 6.03599994E+15 | 7.88451238E+00 | 5.73352879E+06 | 1.15417350E+15 | 1.42274476E+09 | 3.37211150E+08 | 1.6666 |
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| 5.72078561E-01 | 2.93780591E-01 | 6.96438950E-01 | 1.30875221E+16 | 1.45186075E+01 | 6.73878674E+06 | 1.35603905E+15 | 1.37359662E+09 | 3.35828936E+08 | 1.6669 |
| 5.55893080E-01 | 2.87807441E-01 | 6.95595019E-01 | 1.42233191E+16 | 1.54890047E+01 | 6.86292402E+06 | 1.38108050E+15 | 1.36862299E+09 | 3.35659858E+08 | 1.6669 |
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| 5.23736687E-01 | 2.76323239E-01 | 6.93466041E-01 | 1.66651038E+16 | 1.75079411E+01 | 7.10993371E+06 | 1.43097794E+15 | 1.35936711E+09 | 3.35328445E+08 | 1.6669 |
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# EZ-Web: Limitations



- EZ-Web and the underlying EZ code have a number of limitations which restrict their validity. In some cases, the results can be misleading or inaccurate, and users should be aware of this if using EZ-Web for research purposes.

- As an alternative to EZ-Web, consider using MESA-Web — a web-based interface to the fully-featured MESA stellar evolution code. MESA-Web can produce models which are suitable for detailed scientific investigations: http://user.astro.wisc.edu/~townsend/static.php?ref=mesa-web-submit

# Schematic stellar evolution



THE (TEMPERATURE, DENSITY) DIAGRAM
ZONES OF THE EQUATION OF STATE
ZONES OF NUCLEAR BURNING
EVOLUTION OF A STAR IN THE ($\log P$, $\log \rho$) PLANE

# Introduction



- We have derived all the differential equations that define uniquely the equilibrium properties of a star of a given mass and composition. We know how to solve them.
- Our task now is to combine the knowledge acquired so far into a general picture of the evolution of stars.
- We will consider the schematic evolution of a star, as seen from its centre. The centre is the point with the highest pressure and density, and (usually) the highest temperature, where nuclear burning proceeds fastest. Therefore, the centre is the most evolved part of the star, and it sets the pace of evolution, with the outer layers lagging behind.
- The stellar centre is characterized by the central density $\rho_c$, pressure $P_c$ and temperature $T_c$ and the composition (usually expressed in terms of $\mu$ and/or $\mu_e$). These quantities are related by the equation of state (EOS).
- We can thus represent the evolution of a star by an evolutionary track in the ($P_c$, $\rho_c$) diagram or the ($T_c$, $\rho_c$) diagram.
- Since the only property that distinguishes the evolutionary track of a star from that of any other star of the same composition is its **mass**, we may expect to obtain different lines in the ($T_c$, $\rho_c$) plane for different masses.
- The ($T_c$, $\rho_c$) plane will be divided into zones dominated by different equations of state and different nuclear processes.

# Zones of the equation of state



- As the ranges of density and temperature typical of stellar interiors span many orders of magnitude, logarithmic scales will be used for both.
- By considering the EOS we can derive the evolution of the central temperature. This is obviously crucial for the evolution track of a star because nuclear burning requires $T_c$ to reach certain (high) values.
- We have previously encountered various regimes for the EOS:
  - The most common EOS is that of an ideal gas: $P = \frac{\Re T \rho}{\mu} = K_0 \rho T$
  - If radiation pressure is dominant, then the equation of state changes to $P = \frac{aT^4}{3}$
  - At high densities and relatively low temperatures, the electrons become degenerate, and since their contribution to the pressure is dominant, the EOS is replaced by $P = K_1 \rho^{5/3}$. This is independent of temperature. More accurately: the complete degeneracy implied by this relation is only achieved when $T_c \rightarrow 0$.
  - For still higher densities, when relativistic effects play an important role, the EOS changes to the form $P = K_2 \rho^{4/3}$.

# EOS in the (log $P$, log $\rho$) plane



- The transition from one state to the other is, of course, gradual with the change in density and temperature, but an approximate boundary may be traced in the (log $P$, log $\rho$) plane.
- The boundaries may be defined by the requirement that the pressure obtained from a one EOS be equal to that obtained another. For example, the boundary between the ideal gas zone and the non-relativistic-degeneracy zone may be obtained, $K_0 \rho T = K_1 \rho^{5/3}$, which defines a straight line with a slope of 1.5:

$$\log \rho = 1.5 \log T + \text{constant}.$$

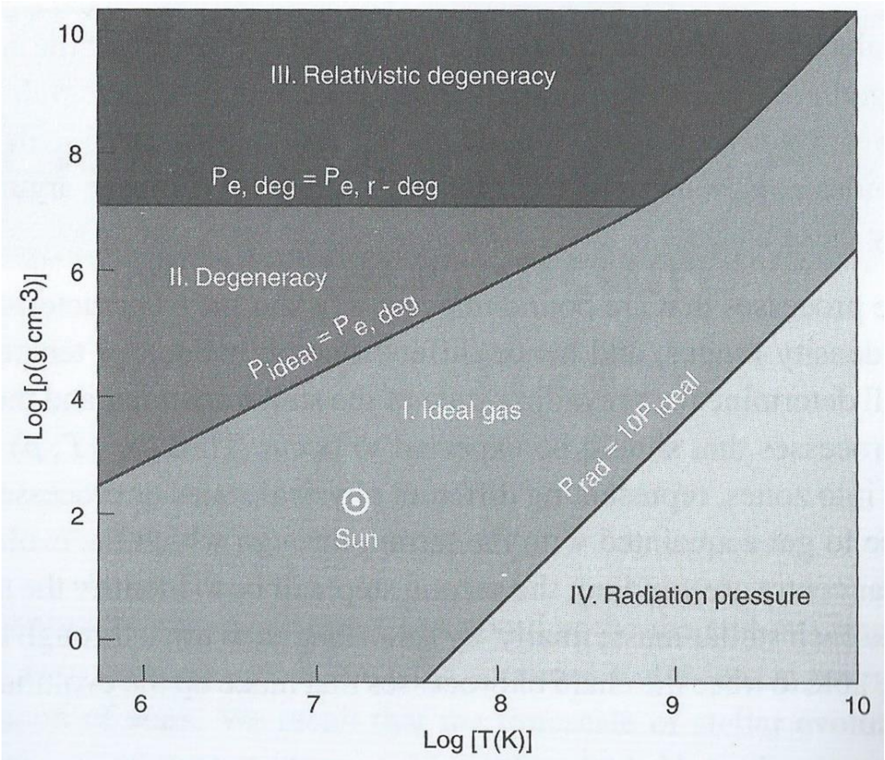



Figure from Prialnik

# Zones of nuclear burning



The nuclear energy generation rate is a sensitive function of the temperature, which can be written as

$$\varepsilon \approx \varepsilon_0 \rho^{\lambda} T^{n}$$

where for most nuclear reactions (those involving two nuclei) $\lambda=1$, while $n$ depends mainly on the masses and charges of the nuclei involved and usually $n$>>1.

For H-burning by the pp-chain, $n\approx4$ and for the CNO-cycle which dominates at somewhat higher temperature, $n\approx18$.

For He-burning by the triple-alpha reaction, $n\approx40$ (and $\lambda=2$ because three particles are involved). For C-burning and O-burning reactions $n$ is even larger.

As discussed in previous lectures, the consequences of this strong temperature sensitivity are that

- each nuclear reaction takes place at a particular, nearly constant temperature, and
- nuclear burning cycles of subsequent heavier elements are well separated in temperature

The threshold given by $\varepsilon \approx \varepsilon_{min}$ is

$$\log \rho = -\frac{n}{\lambda} \log T + \frac{1}{\lambda} \log \frac{\varepsilon_{min}}{\varepsilon_0}$$

On one side of the threshold the rate of nuclear burning may be assumed negligible, and on the other side – considerable.

# Nuclear burning in the (log $P$, log $\rho$) plane

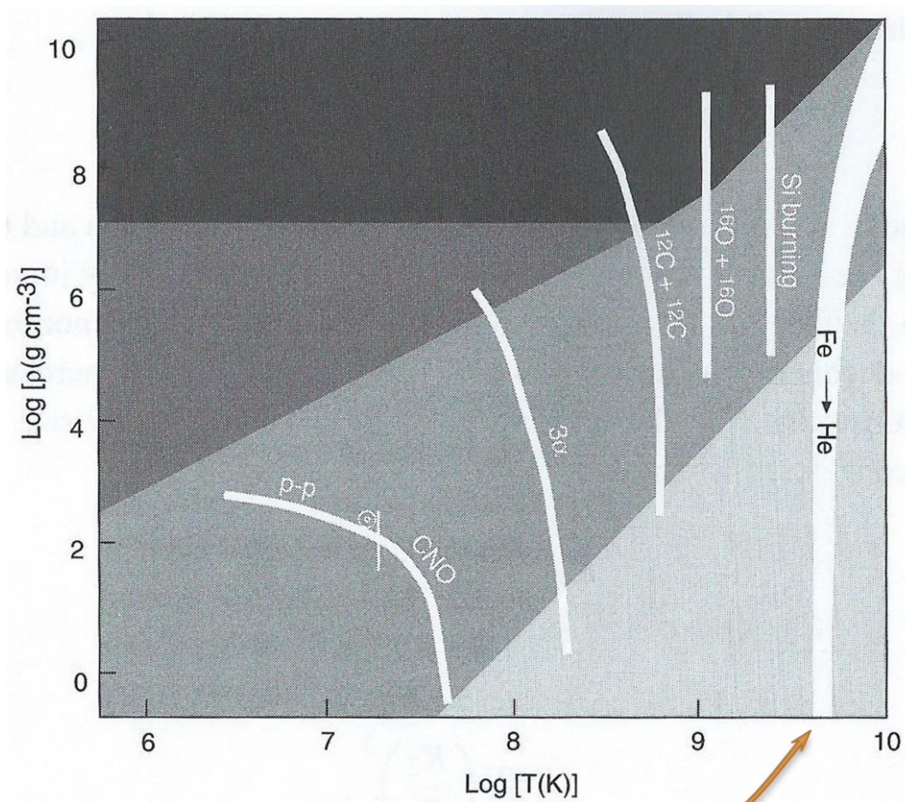


The transformation of hydrogen into the iron group elements comprises five major stages:

- hydrogen burning into helium either by the p-p chain or by the CNO cycle;
- helium burning into carbon by the 3a reaction;
- carbon burning;
- oxygen burning;
- silicon burning.

Nucleosynthesis ends with iron.

Iron nuclei heated to very high temperatures are disintegrated by energetic photons into helium nuclei. This energy **absorbing** process reaches equilibrium, with the relative abundance of iron to helium nuclei determined by the values of temperature and density. A threshold may be defined for the process of iron photodisintegration, as a strip in the (log $P$, log $\rho$) plane, by the requirement that the number of helium and iron nuclei be approximately equal.



# The evolutionary path of the central point



Are the centre of a star of given mass $M$ may assume any combination of temperature and density values, or these values are in some way constrained by $M$ ?

We now regard the ($\log P$, $\log \rho$) plane as a ($\log P_c$, $\log \rho_c$) plane, referring to the stellar centre.

Assuming a polytropic configuration for a star in hydrostatic equilibrium, the central density is related to the central pressure by equation

$$P_c = (4\pi)^{1/3} B_n G M^{2/3} \rho_c^{4/3}$$

This relation is only weakly dependent on the polytropic index $n$, especially for stable configurations, for which $n$ varies between 1.5 and 3 and the coefficient $B_n$ between 0.157 and 0.206 (see the table above), and it is independent of $K$. As we noted before, this relation provides a good approximation to hydrostatic equilibrium for any configuration.

Additionally, $P_c$ is related to $\rho_c$ and $T_c$ by the EOS (we have different ones). Combining each of them with the above relation, we can eliminate $P_c$ to obtain a relation between $\rho_c$ and $T_c$.

For example, for a star of mass $M$, whose central point is found in the ideal gas zone I, we obtain the relation between $\rho_c$ and $T_c$

$$\rho_c = \frac{K_0^3}{4\pi B_n^3 G^3} \frac{T_c^3}{M^2}$$

For a star of given mass, the central density varies as the central temperature cubed.

# The central point in the (log $P$, log $\rho$) plane



- For a star of mass $M$, whose central point is found in the ideal gas zone I, we obtain the relation

$$\rho_c = \frac{K_0^3}{4\pi B_n^3 G^3} \frac{T_c^3}{M^2}$$

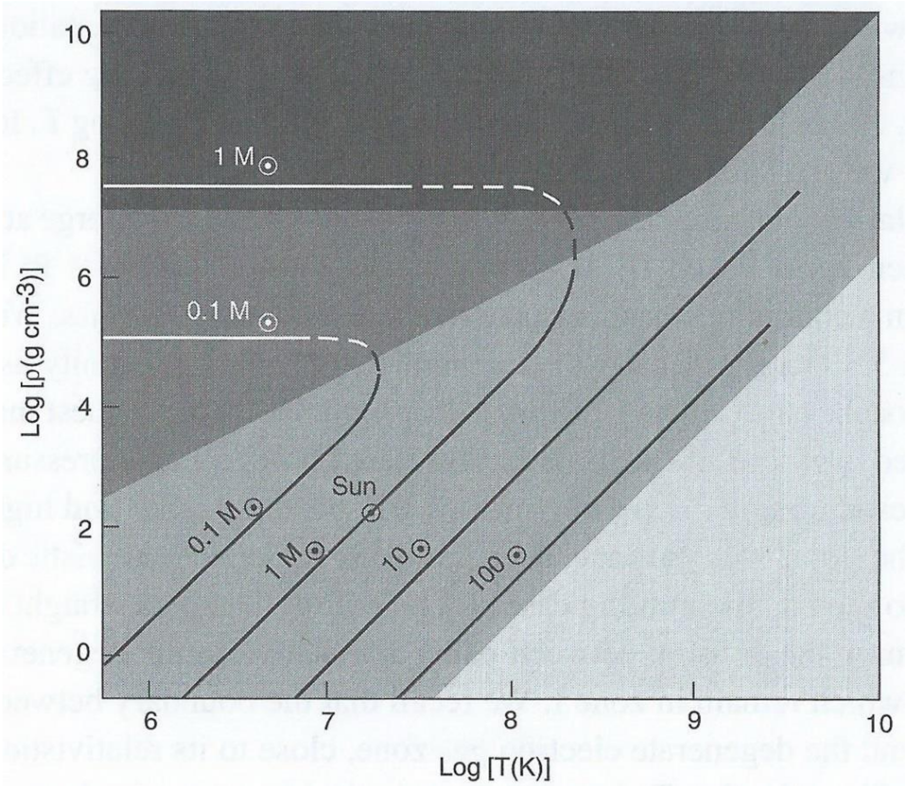
  On logarithmic scales, it becomes a straight line with a slope of 3. Different masses define different parallel lines.

- If at the centre of a star the electrons are strongly but non-relativistically degenerate, the central point is found in zone II . Then the relation is

$$\rho_c = \left(\frac{B_{1.5} G}{K_1}\right)^3 M^2$$

  which replaces the ideal gas relation. Here $\rho_c$ is independent of $T_c$, and the corresponding line in the (log $P_c$, log $\rho_c$) plane is horizontal and increases with mass $M$.

- Zones I and II are the only stable regions in the (log $P$, log $\rho$) plane. Hence, there is no need to consider the others.




For relatively low masses, the relations will merge at the boundary between zones I and II, resulting in a continuous bending path characteristic of each mass.

# Evolution of a star in the (log $P$, log $\rho$) plane



Stars are limited to a rather narrow mass range of 0.1 $M_\odot$ to ~100 $M_\odot$. The lower limit is set by the minimum temperature required for nuclear burning, and the upper limit by the requirement of dynamical stability.