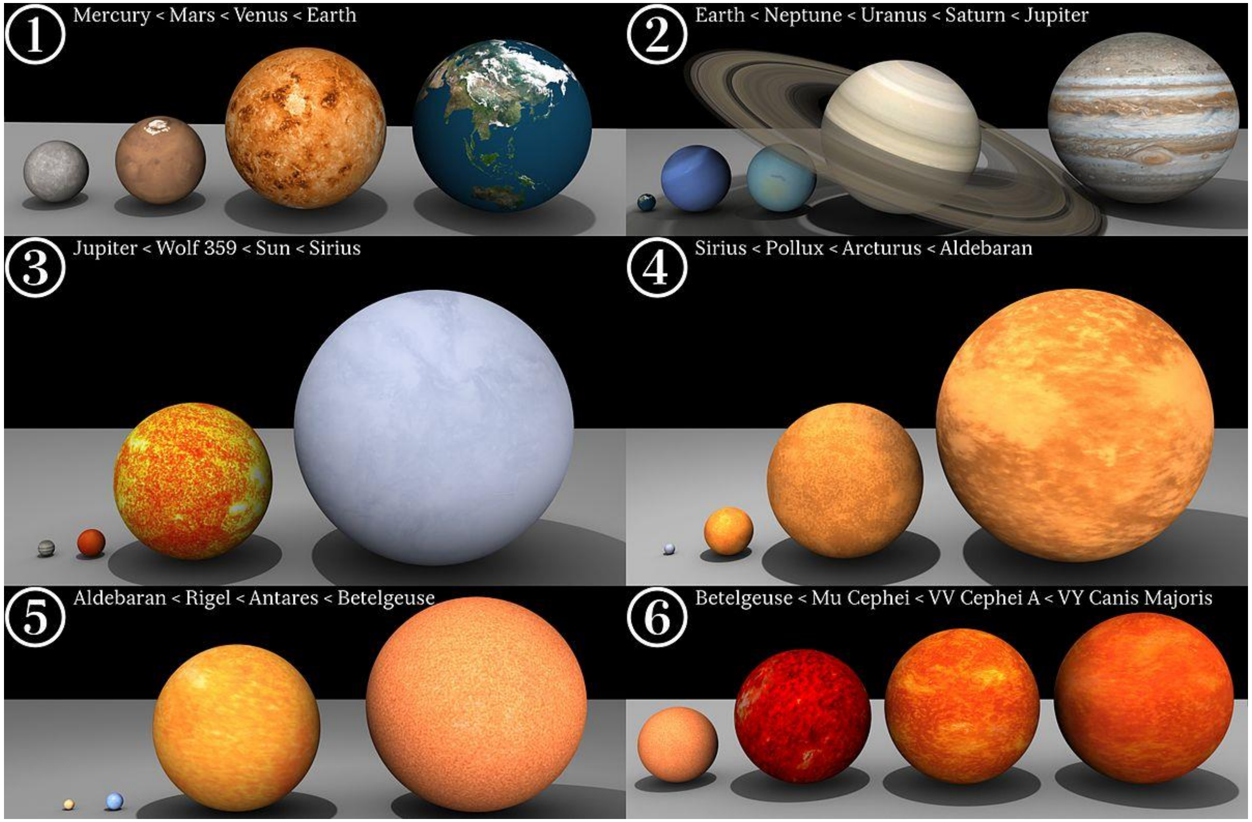
Are all the stars the same?

- **No!**

- Luminosity
  $10^{-6}\ L_\odot < L < 10^{6}\ L_\odot$:
  factor of 10^{12} in L
- Radius
  $10^{-5}\ R_\odot < R < 10^{3}\ R_\odot$:
  factor of 10^{8} in R
- Mass
  $10^{-2}\ M_\odot < M < 10^{2}\ M_\odot$:
  factor of 10^{4} in M
- Temperature
  10^{3} K $< T_{eff} < 10^{5}$ K:
  factor of 10^{2} in T_{eff}

  (but note that in neutron stars T_{eff} can be higher than 10^{7} K)



Spectral Lines

- Spectral lines originate in a stellar atmosphere – a thin, tenuous transition zone between (invisible) stellar interior and (essentially vacuum) exterior.

- Stellar interiors are effectively **invisible** to external observers (apart for e.g. astroseismology) so all the information we receive from stars originates from their atmospheres. Understanding how radiation interacts with matter affecting the emergent line and continuous spectrum is a part of this course. We will discuss it later.

Spectral Types

Morgan-Keenan (M-K) classification scheme orders stars via “OBAFGKM” spectral classes using ratios of spectral line strength.

O-types have the highest T_{eff}’s. OBA stars are “early-type” star, whilst cooler stars are “late-type”.

Spectral classes are each subdivided into (up to) ten divisions – e.g. O2 .. O9, B0, B1 .. B9, A0, A1 .. etc

Table 15.1. *MK spectral classes.*

MK spectral class	Class characteristics
O	Hot stars with He II absorption
B	He I absorption; H developing later
A	Very strong H, decreasing later; Ca II increasing
F	Ca II stronger; H weaker; metals developing
G	Ca II strong; Fe and other metals strong; H weaker
K	Strong metallic lines; CH and CN bands developing
M	Very red; TiO bands developing strongly

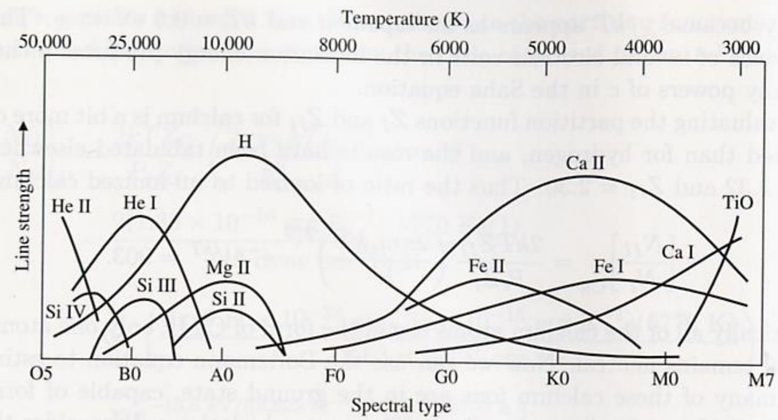


Figure 8.9 The dependence of spectral line strengths on temperature.

Luminosity Class classification

- Luminosity class information is often added, based upon spectral line widths:

Ia	Most luminous supergiants
Ib	Less luminous supergiants
II	Luminous giant
III	Normal giants
IV	Subgiants
V	Main sequence stars (dwarfs)
VI	Subdwarfs
VII	White dwarfs

- Dwarfs have high pressures (large line widths) and supergiants have lower pressures (smaller line widths).

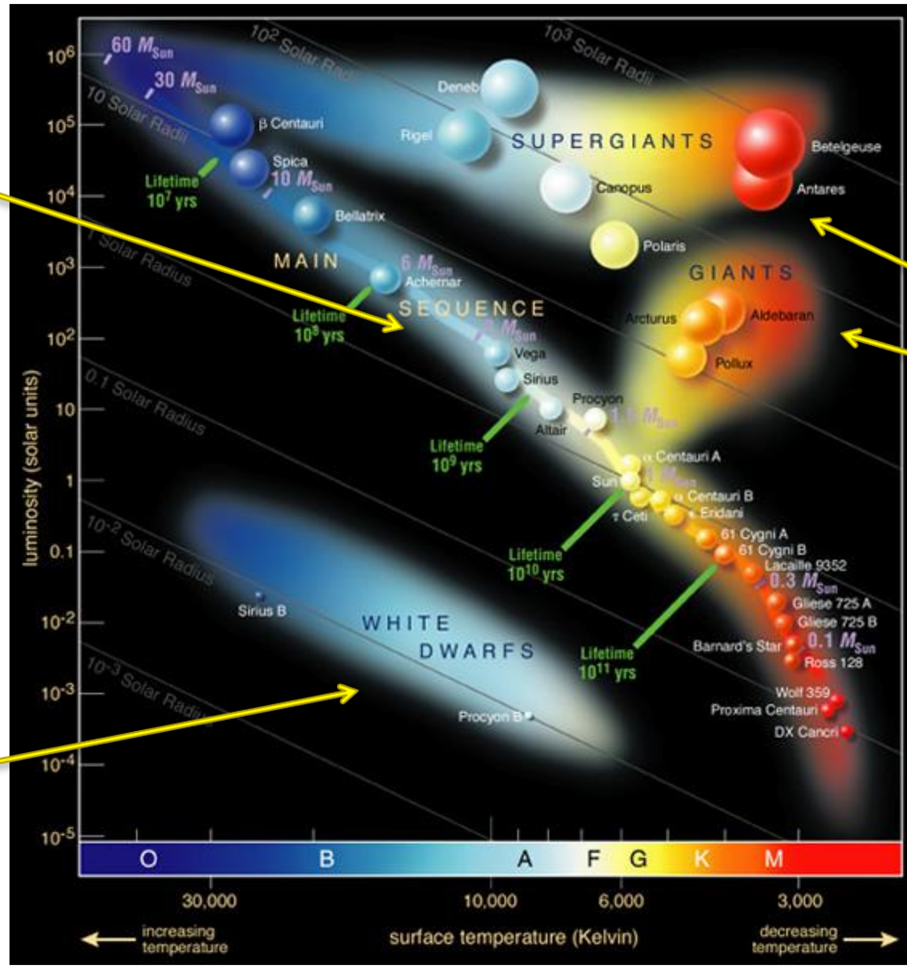
Some properties of representative stars

Star	Sp	T_{eff} (K)	$L/L_\odot$	$R/R_\odot$	$M/M_\odot$	ρ (g/cm^3)	Remarks
α Sco A (Antares)	M0 Ib	3300	34000	530	20	$1.7 \cdot 10^{-7}$	Supergiant
α Boo (Arcturus)	K2 III	3970	130	26	4	$3.2 \cdot 10^{-4}$	Giant
η Ori	B1 V	23000	13000	7	14	0.052	
α CMa A (Sirius)	A1 V	9700	60	2.4	3.3	0.81	
Sun	G2 V	5800	1	1	1	1.4	Main sequence
Barnard's Star	M5 V	3000	0.015	0.5	0.38	4.3	
α CMa B (Sirius B)	A5 VII	8200	0.003	0.03	0.96	$7.7 \cdot 10^{4}$	White dwarf

Hertzsprung-Russell (HR) diagram

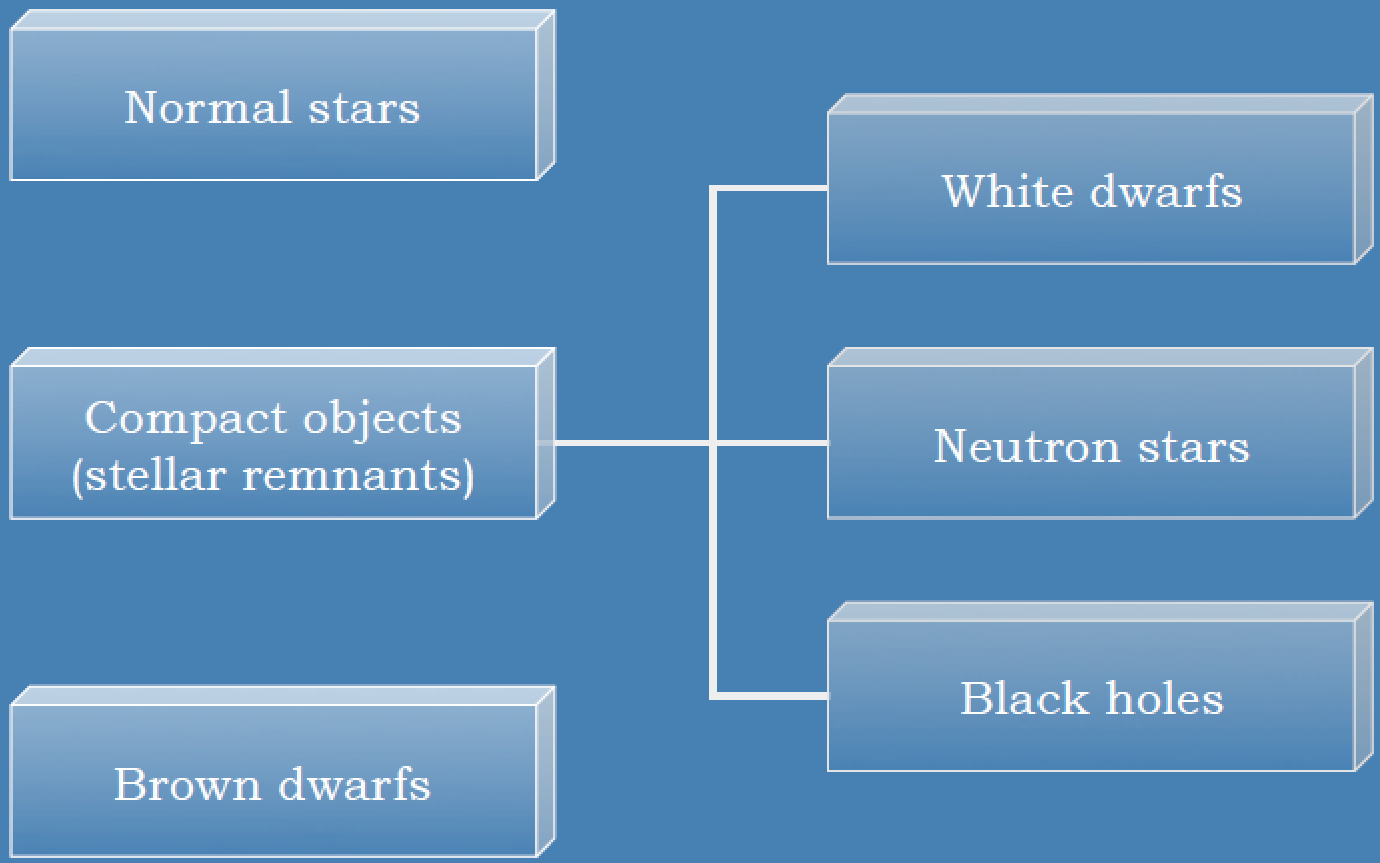
Most of the stars lie on the Main Sequence, with increasing L as T increases

A relatively hot star can have very low luminosity, if its radius is very small (0.01 $R_{\odot}$): White Dwarfs



A relatively cool star can be quite luminous if it has a large enough radius (10-100 $R_{\odot}$): Red Giants and Supergiants

Physical classification of stars



Brown dwarfs

- **Definition**
  Brown dwarfs are objects having insufficient mass to sustain normal hydrogen burning

- Masses of the brown dwarfs are ranging from 13 to 70 M_{Jup} (0.01 to 0.07 $M_{\odot}$)
- The brown dwarfs are cold (T_{eff} < 1400 K) and dim objects, which can be detected by infrared observation
- There are many uncertainties both theoretical and observational concerning these objects

White dwarfs

- White dwarfs consist of degenerate matter and may be treated as graveyard for the stars with initial masses $\leq 8\ M_{\odot}$
- Typical values:
  - $R \sim 10^{-2}\ R_{\odot} \sim R_{\oplus}$
  - $M \sim (0.3 \div 1)\ M_{\odot}$
  - $L \sim (10^{-2} \div 10^{-3})\ L_{\odot}$
  - $\rho \sim (10^{5} \div 10^{6})\ g/cm^{3}$

Limiting mass
(Chandrasekhar limit)
$M_{WD} < 1.4\ M_{\odot}$

Neutron stars

- Neutron stars consist of neutron "fluid" and originate from the evolution of massive stars with $M > 8\ M_\odot$
- Typical values:
  - $R \sim 10\text{-}15$ km
  - $M \sim (1 \div 2)\ M_\odot$
  - $\rho \sim 10^{15}$ g/cm^3

Limiting mass
(Oppenheimer-Volkoff limit)
$M_{NS} < (2 \div 3)\ M_\odot$

Black holes

- Schwarzschild (gravitational) radius: $r_g = \frac{2GM}{c^2}$

- Values of Schwarzschild radius:
  - for Earth is 0.9 cm
  - for the Sun is 3 km

- Supermassive black holes (not stars) are found in the centres of many galaxies as well as in the centre of our own Galaxy, Milky Way.
  Typical values:
  - $M \sim (10^6 \div 10^9)\ M_\odot$
  - $R \sim (10^{11} \div 10^{14})$ cm $\sim (0.01 \div 10)$ AU
    So radii range approximately from $R_\odot$ to the Saturn distance

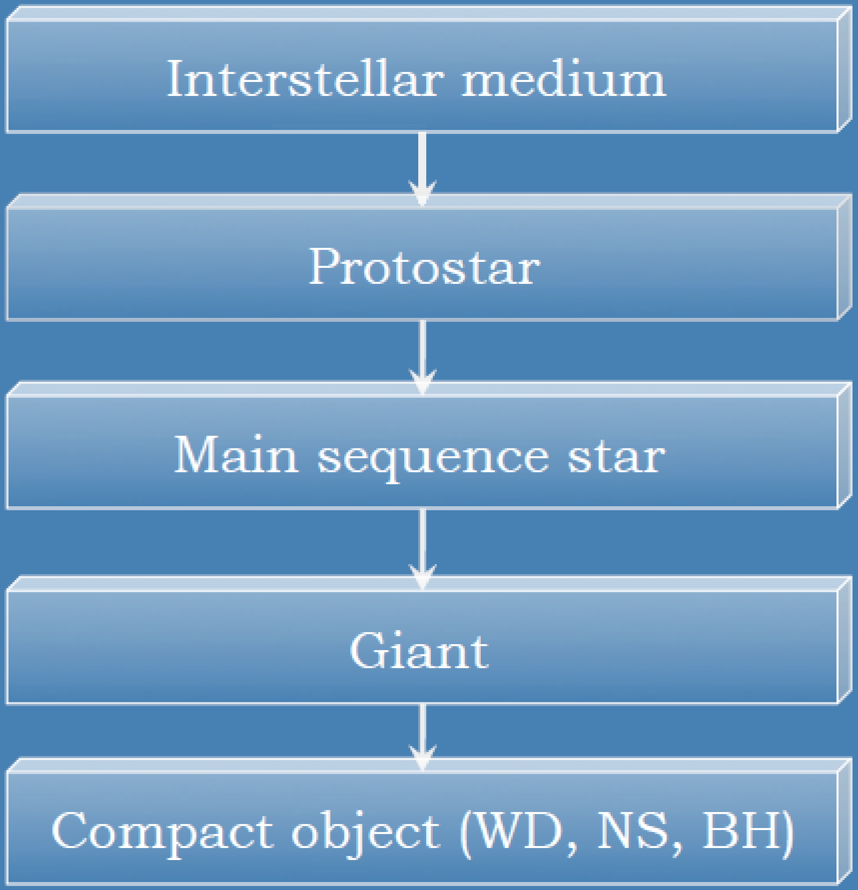
Physics involved

- The subject of stellar interiors covers a very broad area of physics and is expanding even further into fields that were previously not considered as a part of the subject matter of stellar structure.

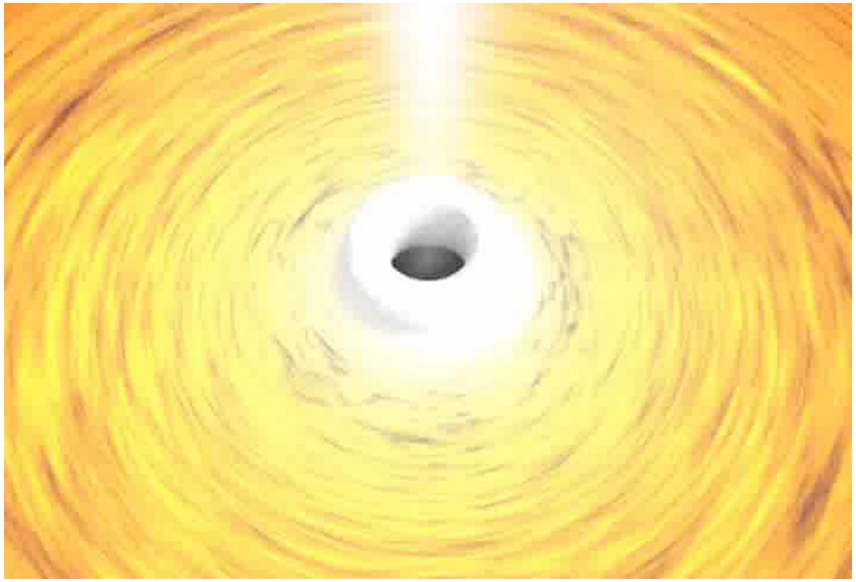
- Mechanics
- Thermodynamics
- Radiation theory
- Relativity
- Atomic physics
- Nuclear physics
- Hydrodynamics
- Solid state physics

Stellar timeline

Stars are like people in that they are born, grow up, mature, and die.

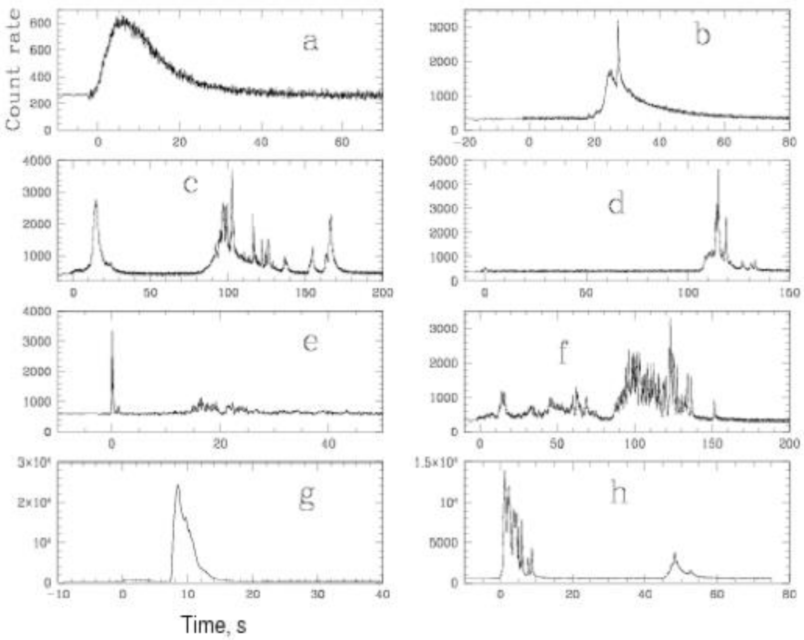


Stellar formation

Standard	Exotic
	
In interstellar gas clouds	In a gaseous disc around a supermassive black hole

Stellar collapse

- Evolution of massive stars, with $M > 20\ M_{\odot}$, ends up in huge eruption observed as supernova or hypernova.
- The core of the star collapses directly into a compact object and two extremely energetic jets of plasma are emitted from its rotational poles.
- In a supernova, the collapse results in a neutron star.
- In a hypernova, a black hole is formed, and a gamma-ray burst might be produced.



Equilibrium in stellar interiors

BASIC ASSUMPTIONS
MASS CONSERVATION
HYDROSTATIC EQUILIBRIUM
VIRIAL THEOREM
STELLAR TIME-SCALES

Introduction and recap (1)

Definition of **a star** as an object:

- Bound by self-gravity
- Radiates energy that is primarily released by nuclear fusion reactions in the stellar interior

Other energy sources are dominant during star formation and stellar death:

- **Star formation** - before the interior is hot enough for significant fusion, gravitational potential energy is radiated as the radius of the forming star contracts.
  Protostellar or pre-main-sequence evolution.
- **Stellar death** - remnants of stars (white dwarfs and neutron stars) radiate stored thermal energy and slowly cool down. Sometimes refer to these objects as stars but more frequently as *stellar remnants*.

Introduction and recap (2)

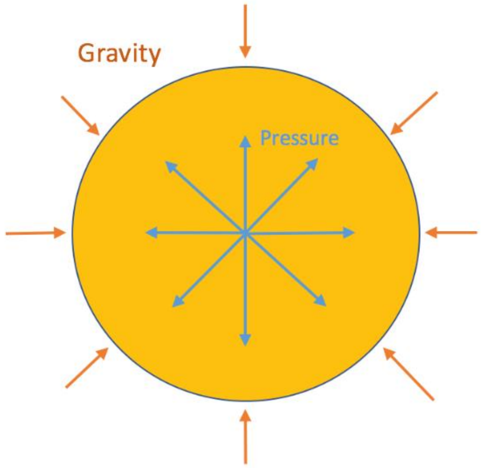
With this definition:

- **planets** are not stars - no nuclear fusion.
- objects in which release of gravitational potential energy is always greater than fusion are not stars either – these are called **brown dwarfs.**

Distinction between brown dwarfs and planets is less clear, most people reserve "planet" to mean very low mass bodies in orbit around a star.

Irrespective of what we call them, physics of stars, planets, stellar remnants is similar.
Balance between:

- **Gravity**
- **Pressure**



Basic assumptions (1)

What are the **main** physical processes which **determine** the structure of **stars**?

- Stars are **held together** by **gravitation** – attraction exerted on each part of the star by all other parts
- Collapse is **resisted** by internal thermal **pressure**.
- These two forces play the principal role in determining stellar structure – they must be (at least almost) in balance. If they are not, the star will explode or collapse on very short (dynamical) time-scale. Since stars do seem to be rather stable on time-scale of millennium, the balance is good.
- Stars **continually radiating** energy into space. As they do not seem to cool dramatically on the civilization lifetime-scale, an energy source must exist (we will see later that thermal energy is not enough).
- Theory must describe - **origin of energy** and **transport to surface**.

Basic assumptions (2)

We make two fundamental assumptions :

1. Neglect the rate of change of properties – assume constant with time.
2. All stars are spherical and symmetric about their centres. Thus, all quantities (e.g., density, temperature, pressure) depend only on the distance from the centre of the star - radius r.

Density as function of radius is $\rho(r)$.

If m is the mass interior to r, then: $m(r) = \int_0^r 4\pi r^2 \rho(r)\, dr$

Differential form of this equation is: $dm = 4\pi r^2 \rho dr$

Two equivalent ways of describing the star:

- Properties as $f(r)$: e.g. temperature $T(r)$
- Properties as $f(m)$: e.g. $T(m)$

Second way often more convenient: over its lifetime, a star's radius will change by many orders of magnitude, while its mass will remain relatively constant. Moreover, the amount of nuclear reactions occuring inside a star depends on ρ and T, not where it is in the star. Thus, a better and more natural way to treat stellar structure is to write radius as a function of mass, i.e.

$$\frac{dr}{dm} = \frac{1}{4\pi r^2 \rho}$$

We will start with these assumptions and later reconsider their validity.

Stellar structure

For our stars – which are isolated, static, and spherically symmetric – there are **four** basic equations to describe structure. All physical quantities depend on the distance from the centre of the star alone

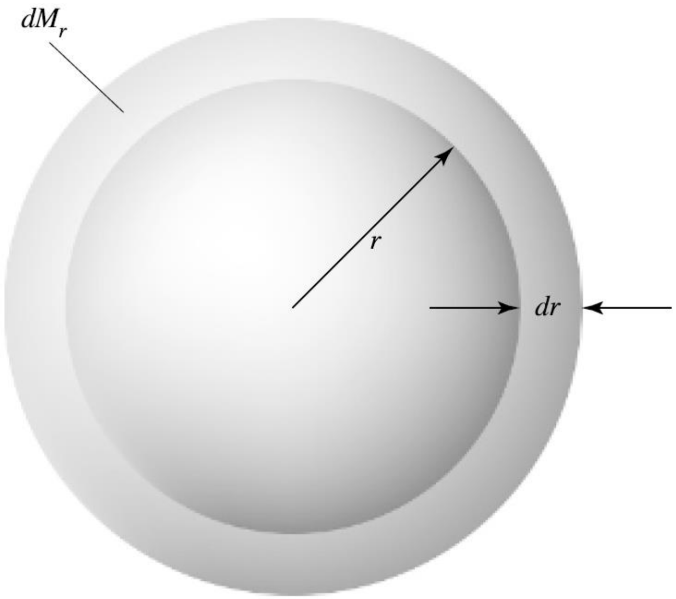
- **Conservation of mass**
- **Equation of hydrostatic equilibrium**: at each radius, forces due to pressure differences balance gravity
- **Conservation of energy**: at each radius, the change in the energy flux equals the local rate of energy release
- **Equation of energy transport**: relation between the energy flux and the local gradient of temperature

These basic equations supplemented with

- **Equation of state** (pressure of a gas as a function of its density and temperature)
- **Opacity** (how opaque the gas is to the radiation field)
- Nuclear energy generation rate as $f(\rho,T)$.

Equation of mass conservation

Mass $m(r)$ contained within a star of radius r is determined by the density of the gas $\rho(r)$.



Consider a thin shell inside the star with radius r and outer radius $r+dr$:

$$dV = 4\pi r^2 dr$$

$$dM = dV\rho(r) = 4\pi r^2 \rho(r) dr$$

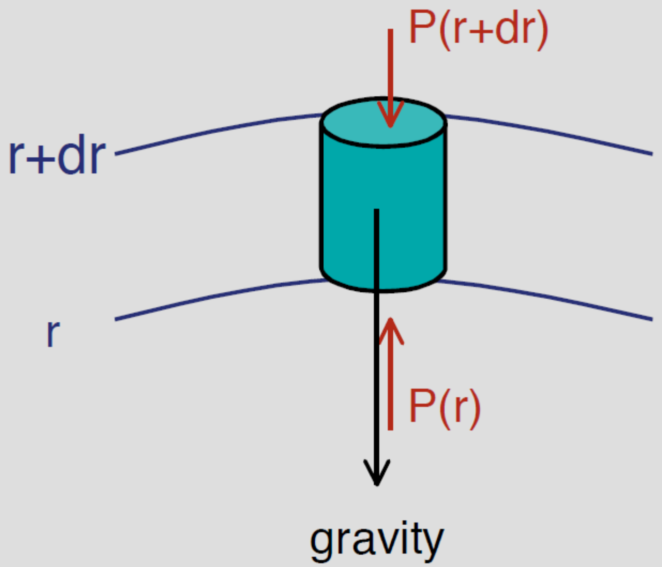
In the limit where $dr \rightarrow 0$:

$$\frac{dM}{dr} = 4\pi r^2 \rho(r)$$

This is the **equation of mass conservation**.

Hydrostatic equilibrium (1)

- Balance between gravity and gradient of internal pressure is known as hydrostatic equilibrium.



- Consider a small cylindrical element between radius r and radius $r+dr$ in the star.
  - Its surface area = ds
  - Mass of the element: $dm = \rho(r)\, ds\, dr$
  - Mass of gas in the star at smaller radii: m = m(r)

Hydrostatic equilibrium (2)

Consider forces acting in radial direction:

- Outward force: pressure exerted by stellar material on the bottom face:

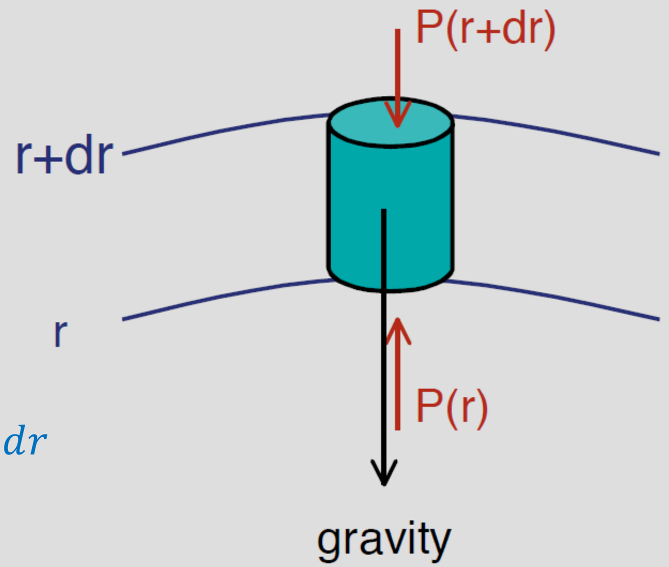
$$F_{P,b} = P(r)ds$$

- Inward forces:
  - Gravity (gravitational attraction of all stellar material lying within r):

$$F_g = \frac{Gm}{r^2}dm = \frac{Gm}{r^2}\rho(r)\,ds\,dr$$

  - Pressure exerted by stellar material on the top face:

$$F_{P,t} = P(r+dr)ds$$



- In hydrostatic equilibrium:

$$F_{P,b} = F_{P,t} + F_g$$

$$P(r)ds = P(r+dr)ds + \frac{Gm}{r^2}\rho(r)\,ds\,dr$$

Hydrostatic equilibrium (3)

$$P(r)ds = P(r+dr)ds + \frac{Gm}{r^2}\rho(r)\,ds\,dr$$

$$\Rightarrow \quad P(r+dr) - P(r) = -\frac{Gm}{r^2}\rho(r)\,dr$$

If we consider an infinitesimal element, we write for $dr \rightarrow 0$

$$\frac{P(r+dr) - P(r)}{dr} = \frac{dP(r)}{dr}$$

Hence rearranging above, we get **the equation of hydrostatic equilibrium:**

$$\frac{dP(r)}{dr} = -\frac{Gm}{r^2}\rho(r)$$

Hydrostatic equilibrium (4)

The equation of hydrostatic equilibrium:

$$\frac{dP(r)}{dr} = -\frac{Gm}{r^2}\rho(r)$$

Combining it with the equation of mass conservation, we obtain an **alternate** form of hydrostatic equilibrium equation, in which enclosed mass m is used as the dependent variable:

$$\frac{dP(r)}{dm} = \frac{dP(r)}{dr} \times \frac{dr}{dm} = -\frac{Gm}{r^2}\rho \times \frac{1}{4\pi r^2 \rho}$$

$$\frac{dP(r)}{dm} = -\frac{Gm}{4\pi r^4}$$

$$\frac{dr}{dm} = \frac{1}{4\pi r^2 \rho}$$

Hydrostatic equilibrium (5)

Properties of the equation of hydrostatic equilibrium: $\frac{dP(r)}{dr} = -\frac{Gm}{r^2}\rho(r)$

1) Pressure always **decreases** outward
2) Pressure gradient vanishes at $r = 0$
3) Condition at surface of star: $P = 0$ (to a good first approximation)

(2) and (3) are **boundary conditions** for the hydrostatic equilibrium equation.

Accuracy of hydrostatic assumption (1)

We have assumed that the gravity and pressure forces are balanced – how valid is that ?

Consider the case where the outward and inward forces are not equal,
there will be a resultant force acting on the element which will give rise to an acceleration *a:*

$$P(r+dr)ds + \frac{Gm}{r^2}\rho(r)\,ds\,dr - P(r)ds = dm \times a = a\,\rho(r)\,ds\,dr$$

[Applying Newton's second law (F=ma) to the cylinder]

acceleration = 0 everywhere if star **static**

$$\frac{dP(r)}{dr} + \frac{Gm}{r^2}\rho(r) = a\,\rho(r)$$

Now acceleration due to gravity is $g = \frac{Gm}{r^2}$

$$\frac{dP(r)}{dr} + g\rho(r) = a\rho(r)$$

This is a generalized form of the equation of hydrostatic support.

Accuracy of hydrostatic assumption (2)

$$\frac{dP(r)}{dr} + g\rho(r) = a\rho(r)$$

Now suppose there is a resultant force on the element (LHS≠0).

Suppose their sum is small fraction of gravitational term (β): $\beta g\rho(r) = a\rho(r)$

Hence there is an inward acceleration of

$$a = \beta g$$

Assuming it begins at rest, the spatial displacement d after a time t is

$$d = \frac{1}{2}at^2 = \frac{1}{2}\beta g t^2$$

Calculate!

Accuracy of spherical symmetry assumption

Stars are rotating gaseous bodies – to what extent are they flattened at the poles?

If so, departures from spherical symmetry must be accounted for.

Consider mass m near the surface of a star of mass M and radius r.

Element will be acted on by centrifugal force $F_c = m\omega^2 r$, where ω= angular velocity of the star.

There will be **no** departure from spherical symmetry provided that

$$\frac{F_c}{F_g} = m\omega^2 r / \frac{GMm}{r^2} \ll 1 \quad \text{or} \quad \omega^2 \ll \frac{GM}{r^3}$$

Solar rotation period is about P ≈ 27 days.

Angular velocity $\omega = 2\pi/P \approx 2.7\times10^{-6}$ s^{-1} $\qquad F_c/F_g \sim 2\times10^{-5}$

...even rotation rates much faster than that of the Sun are negligibly small to influence star's structure.

Accuracy of spherical symmetry assumption

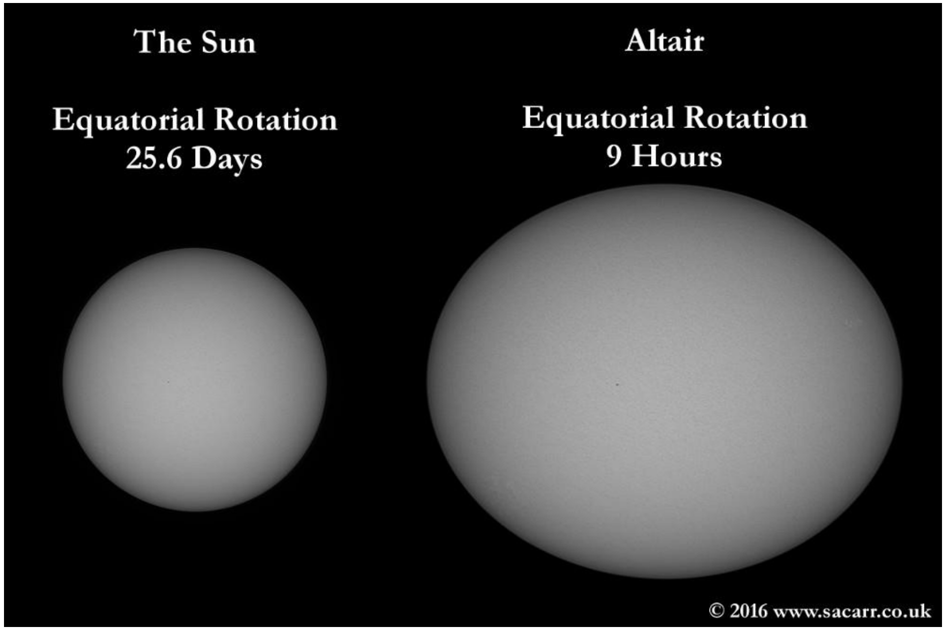
$V = (4/3)\,\pi\, R^3$
$\omega = 2\pi/P$

In terms of mean density, we get

$$\omega^2 \ll \frac{GM}{r^3} \approx \pi G \bar{\rho} \quad \Rightarrow \quad \bar{\rho} \gg \frac{4\pi}{GP^2} \approx \frac{1.9 \times 10^8}{P^2}$$

For the majority of stars, departures from spherical symmetry can be **ignored**.

However, some stars do rotate rapidly and rotational effects must be included in the structure equations –
can change the output of models.



Accuracy of spherical symmetry assumption

Isolation?

In the Solar neighborhood, distances between stars are enormous: e.g. Sun's nearest stellar companion is Proxima Centauri at $d = 1.3$ pc. Ratio of Solar radius to this distance is:

$$\frac{R_{sun}}{d} \approx 2 \times 10^{-8}$$

Two important implications:

- Can ignore the gravitational field and radiation of other stars when considering stellar structure.
- Stars (almost) never collide with each other.

Once star has formed, initial conditions rather than interactions with other stars determine evolution.

However, stars in double systems are elongated due to gravitational attraction.