OBSERVATIONAL ASTRONOMY

FALL 2025

Lecture 11

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Aperture photometry (2)

- Assume we have two apertures, one on the star and one on sky. Star aperture includes sky as well, and our estimate of the star intensity is the difference between the two.
- Signal in the sky aperture is:

$$n_{sky} = \beta\, N_{sky}$$

- Signal in the star aperture is:

$$n_{*+sky} = \beta\, N_{sky} + N_{star}$$

Noise on the measurements

- Noise on the measurements has two components, photon noise which is given by the square root of the number of photons, and readout noise, which is determined by the readout noise and by the number of pixels in the aperture. The noise components add in quadrature:

$$\sigma_{sky}^2 = n_{sky} + n_{pix}\,\sigma_R^2$$

$$\sigma_{*+sky}^2 = n_{*+sky} + n_{pix}\,\sigma_R^2$$

$$n_* \approx n_{*+sky} - n_{sky}$$

$$\sigma_*^2 = n_{*+sky} + n_{sky} + 2\,n_{pix}\,\sigma_R^2$$

$$\sigma_*^2 = 2\,\beta\,N_{sky} + N_{star} + 2\,n_{pix}\,\sigma_R^2$$

Signal to Noise ratio

$$S/N = n_* \,/\, \sigma_* = N_{star} \,/\, \sigma_*$$

$$\frac{S}{N} = \frac{N_{star}}{\sqrt{2\,\beta N_{sky} + N_{star} + 2\,n_{pix}\sigma_R^{\;2}}}$$

$N_{star} = \eta\ \varepsilon_{atm}\ \varepsilon_{tel}\ \varepsilon_{filt}\ \varepsilon_{win}\ \varepsilon_{geom}\ \phi_*\ \Delta\lambda\ A\ t$
$N_{sky} = \eta\ \varepsilon_{atm}\ \varepsilon_{tel}\ \varepsilon_{filt}\ \varepsilon_{win}\ \varepsilon_{geom}\ \phi_{sky}\ \Delta\lambda\ A\ t$

If exposure time is short then readout noise ($\sigma_R \sim 10$) will **dominate**, especially when seeing is bad.

Note: seeing comes in with n_{pix} term

What is ignored in this S/N eqn?

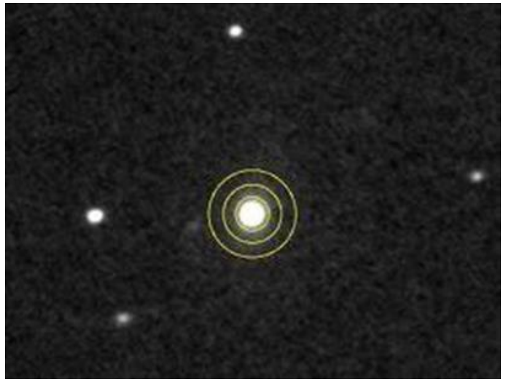
- Bias level/structure correction
- Flat-fielding errors
- Charge Transfer Efficiency (CTE)
  0.99999/pixel transfer
- Non-linearity when approaching full well
- Scale changes in focal plane
- A zillion other potential problems

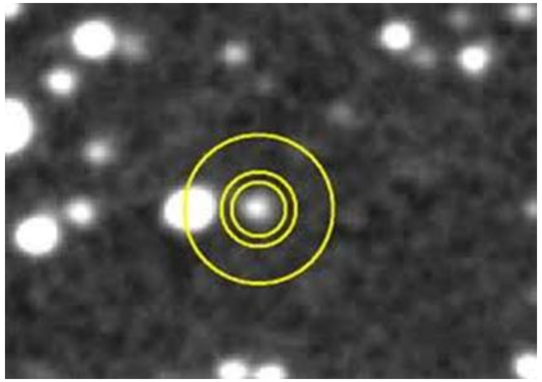
Improving the Signal to Noise (1)

- Larger Sky Aperture – Increasing the sky aperture and scaling it to the size of the object aperture, or using several sky apertures and averaging them, reduces the noise to:

$$\sigma_*^2 = \zeta\, \beta\, N_{sky} + N_{star} + \zeta\, n_{pix*}\, \sigma_R^2$$

where $\zeta = (1 + n_{pix*} / n_{pix\_sky})$, and n_{pix*} and n_{pix_sky} are the number of pixels in the star and sky apertures respectively. In practice, the sky aperture is often an annulus around the star aperture. Must be careful that stars do not get in the sky aperture!





Improving the Signal to Noise (2)

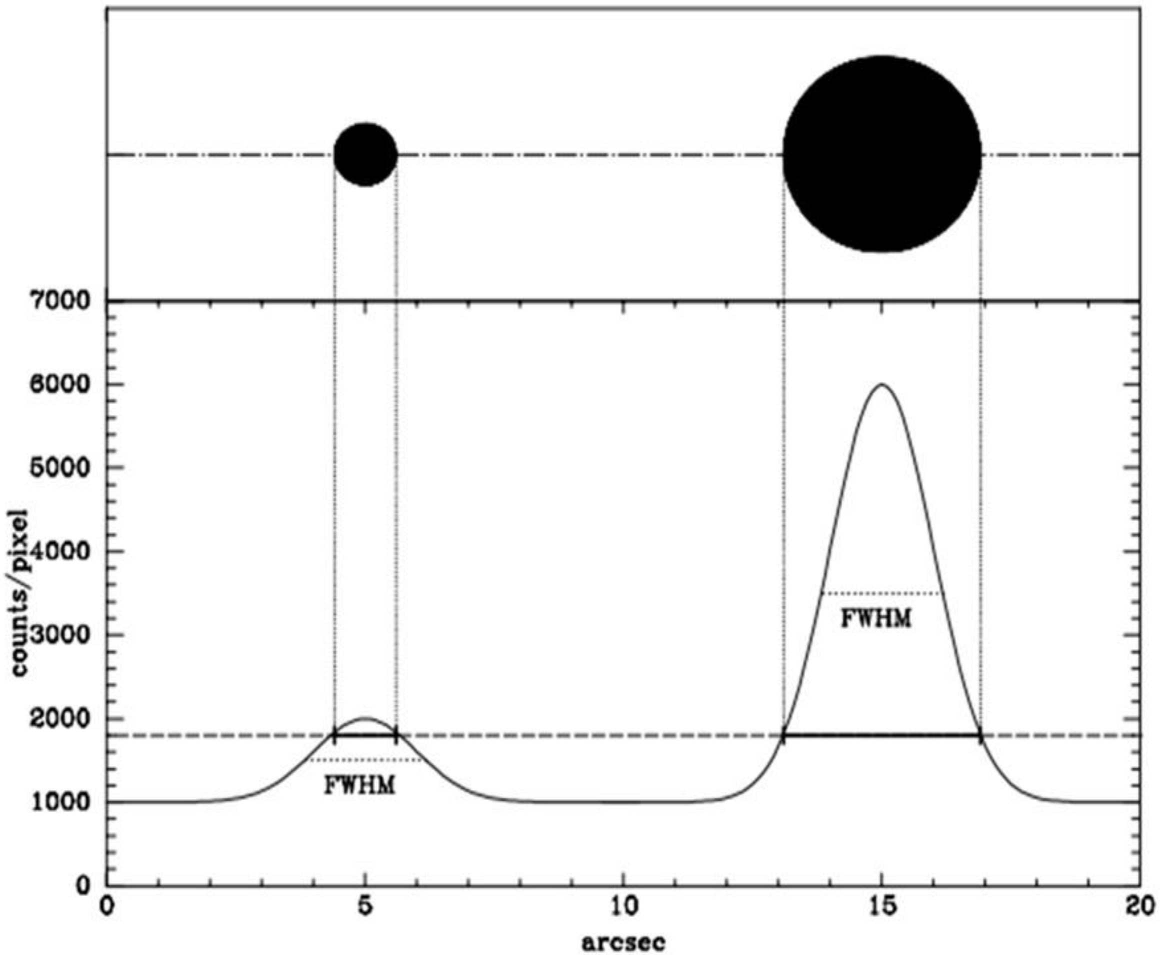
- Smaller object aperture – reducing the object aperture reduces both sky noise and readout noise. However, you lose signal. The problem is if you are comparing the signal in different images, and fluctuations in image size (seeing) cause the amount of signal you lose to vary, then this introduces systematic errors in the brightness measured (photometry).

Solution – Aperture Correction

Aperture Correction (1)

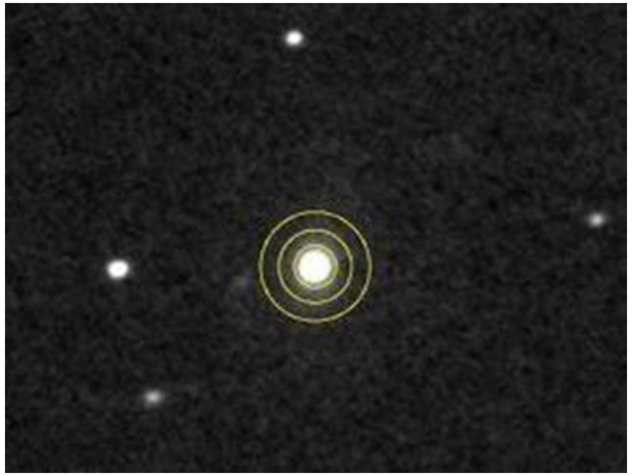
- The **point spread function** (PSF) is the shape of the CCD image of a point (unresolved) source of light.
- Since the PSF is the shape of a point of light on the CCD, and since all stars are points, then all stars have exactly the same shape and size on the CCD, if aberrations are not significant.
- The PSF does not have an edge. The intensity of the star fades smoothly to zero with increasing radius, but there is no place that we could call an "edge".

Aperture Correction (2)



Brighter stars may look bigger, but that is caused by the following effect:

the shape of the faint and bright star are exactly the same, we are simply looking at a larger diameter at a **given intensity** for a bright star than for a faint star.



Aperture Correction (3)

- If we want to measure all the light from a star, how far out in radius do we have to go?
  - One logical answer might be: as big as possible, to get "all" the light from the star. This is **not** a good answer.
  - Reducing the object aperture **reduces** both sky noise and readout noise.

- **But**, a small aperture will only encompass a **fraction** of the total light from the star! However, if the seeing were constant, any aperture would measure the same fraction of light for any star, and when comparing one star with another the effect would cancel out.

Aperture Correction (4)

- The problem is that seeing is not constant. A small aperture might measure 0.5 of the total light from a star on one CCD image, then, if the seeing worsens, the same size aperture might measure only 0.4 of the light from the star on the next CCD image.
- Seeing affects mostly the inner Gaussian core of the image. Using an aperture 4 to 10 times the diameter of the typical FWHM will get most of the light. In this size aperture, reasonable variations in the seeing will not result in measurable variations in measured counts.

Aperture Correction (5)

- However, for faint objects, an aperture 4 times the FWHM will contain a lot of sky signal. This will result in a low S/N ratio.
- Aperture Correction: If we measure the bright object in a small aperture (say radius = 1 FWHM) and also in a bigger aperture which gets "all" the light (say 4 FWHM) we can easily find the ratio of light in the small to large aperture (which we express as a magnitude difference).

Aperture Correction (6)

- The aperture correction is defined as:

$$\Delta = m_{inst}(4\ \text{FWHM}) - m_{inst}(1\ \text{FWHM})$$

(Δ is always a negative number)

- How do we use the aperture correction?

$$\text{total} = m_{inst}(1\ \text{FWHM}) + \Delta$$

- "Total" is our estimate of the total instrumental magnitude in the faint star, m_{inst}(1 FWHM) is the measured magnitude in the small aperture for the faint star, and Δ is the aperture correction derived from a bright star in the same frame.

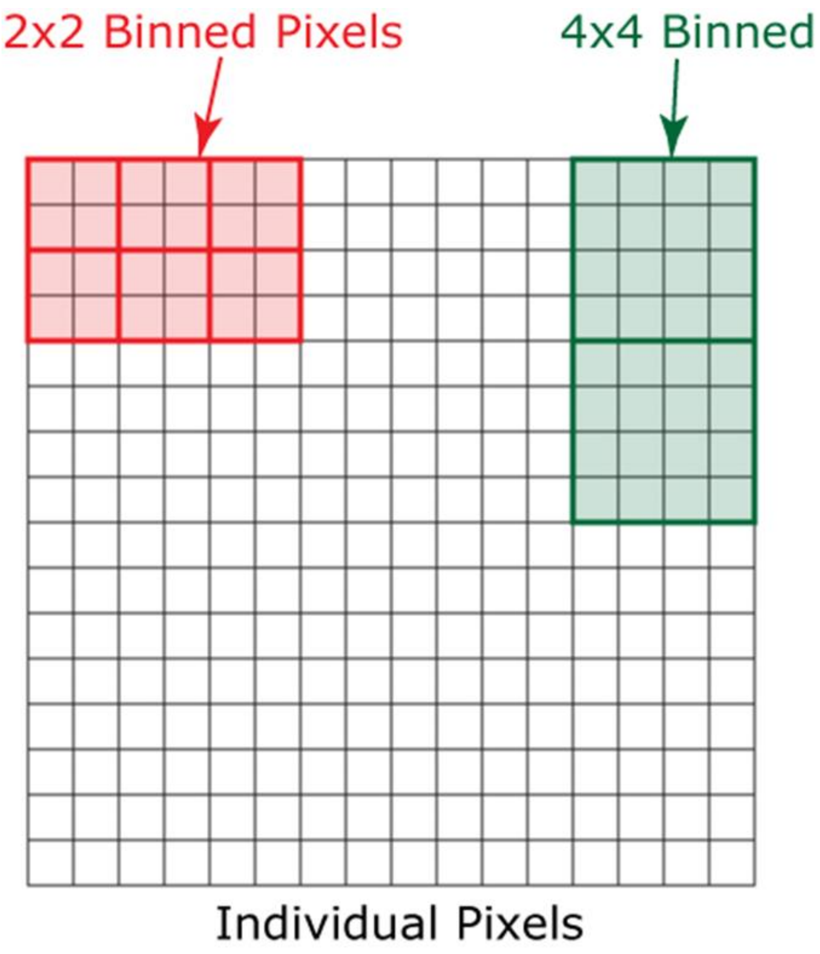
Aperture Correction (7)

- There must be an optimum aperture size that gives the maximum S/N.
- The optimum size of the small aperture has been studied by several authors.
- The optimum aperture seems to be achieved when the measurement aperture has a diameter about $1.4 \times$ FWHM of the PSF. At this aperture, the aperture correction is about -0.3 mag.
- However, the S/N does not appear to be too sensitive to the exact small aperture size.

Improving the Signal to Noise (3)

- On-chip binning – Most CCDs have the option of *binning*: combining a set of adjacent pixels into a single pixel produced as output. For example, a square of 4 pixels on the CCD chip might be reported as one pixel containing their combined value. But you only have to read the output capacitor out once and you only get one lot of readout noise.

On-chip binning



- This way you reduce readout noise at the expense of resolution. Resolution should always be smaller than the characteristic size of the star images.

Profile Fitting (1)

- Profile fitting is used most commonly in crowded fields, where it is difficult or impossible to define a sky aperture free of stars (or galaxies).
- It does however offer an advantage in precision even in sparse fields, because it weights the data more correctly.

Profile Fitting (2)

- Basic assumption is that the intensity profile (which is in principle a 2 dimensional function) is the same for all stars in a particular CCD image.
- Intensity profile is determined by seeing or by diffraction, or occasionally by aberrations.
- If it is determined by aberrations you need to be very careful, because the assumption that the profile is the same at all positions on the CCD may **not** be correct.

Profile Fitting (3)

- From a set of isolated, comparatively bright (but not saturated) stars in the frame, determine the image profile, this is called the **Point Spread Function (PSF).**
- For ground based data an empirical approximation to the PSF is the Moffat function:

$$f(r) = C_i \left(1 + r^2/R_0^2\right)^{-\beta} + B_i \quad (r < r_{max})$$

$$f(r) = B_i \ (r > r_{max})$$

R_0 is the characteristic radius of the star image,
r is the distance from the centre of the image,
β describes the overall shape of the PSF,
B_i is the background in the region of star i, and
C_i is the relative brightness of star i.

Profile Fitting (4)

$$\text{FWHM} = 2\,R_0\sqrt{2^{1/\beta} - 1}$$

- Fit this function for each of the stars in the image to the data, using a least squares or similar technique.
- For each star determine B_i and C_i.
  R_0 and β are constant within an image.

Profile Fitting (5)

- Then we have a set of scaling factors, which can be converted to a relative magnitude.
- We need aperture photometry of one star, either from this CCD frame or from another, this can be a bright isolated star with high S/N, this gives the magnitudes of all of the stars in the frame.
- The fit gives the correct weighting, rather than adding in lots of pixels with very little signal, S/N from profile fitting is usually at least a factor of 2 higher than from aperture photometry.
- Profile fitting can cope with fields in which stars are close or their images even overlap.

Profile Fitting (6)

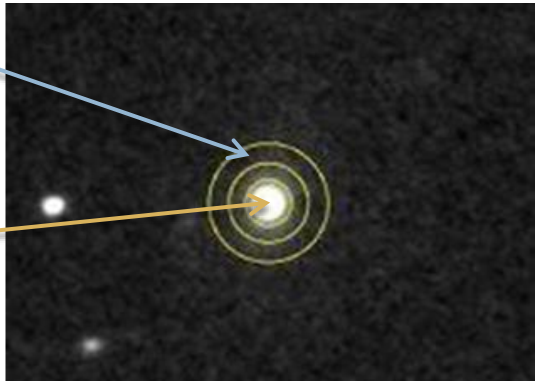
- For ground-based data the PSF is determined by the seeing, and must be redetermined for each CCD image.
- For space based (e.g. Hubble Space Telescope) data the PSF is fixed, and is often available as part of the standard calibration data produced with the observations.
  It still depends upon the passband (filter).

Procedures for photometry

Extracting Photometric Data
Differential (Relative) Photometry
Instrumental Magnitudes
Extinction

Extracting Photometric Data

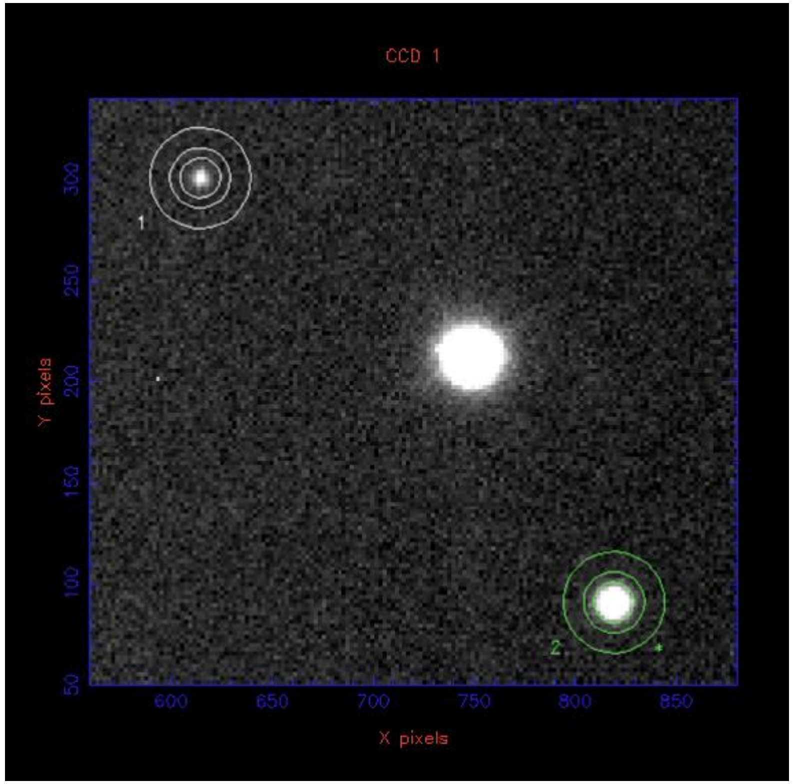
- Determining the number of counts from a source in an image is usually a three-step process (we follow a technique known as aperture photometry):
  1. To measure the centre of the source, which we shall assume is a star.
  2. To estimate the sky background at the position of the star.
  3. To calculate the total amount of light received from the star.
- The sky annulus is unlikely to contain counts from the sky alone. There will also be contributions from cosmic rays, hot pixels, faint stars, and the wings of the PSF of the central star. All of these will add a positive skew to the histogram of pixel values in the annulus. The mean of these pixel values will then not be an accurate representation of the sky background.
  Instead, the sky level is usually determined using a more robust estimator, such as **the median**.



- The total signal from the star can then be calculated by summing the counts from each pixel that falls inside the aperture (usually a **circle** or **ellipse**), and then subtracting the determined sky background from each pixel.

Differential (Relative) Photometry

- Even if the intrinsic brightness of the star is constant, the number of counts we detect might change, due to transparency variations or seeing variations. To obtain an accurate light-curve we need to correct for these effects, using a **comparison star**.
- This is a second star which is known (or assumed) to have a constant flux. We assume that the comparison star is affected in the same way as our target star by seeing and transparency variations (this is a very good assumption).
- Therefore, if transparency or seeing variations cause the counts from the target star, N_t, to halve, they will also cause the counts from the comparison star, N_c, to halve.
  The ratio N_t/N_c is therefore corrected for transparency and seeing variations. Correcting aperture photometry this way is known as differential or relative photometry.



Courtesy of Vik Dhillon

Calibrated Magnitudes (1)

- Once the sky-subtracted signal from an object, measured in **counts**, is extracted from an image, it is useful to convert it to a calibrated **magnitude** in a photometric system:
  - Calculate the instrumental magnitude, from the counts per second.
  - Determine the extinction coefficient, and correct the instrumental magnitude to the above-atmosphere value.
  - Repeat the above steps for a standard star and use the resulting above-atmosphere instrumental magnitude of the standard star to calculate the zero point.
  - Use the zero point to transform the above-atmosphere instrumental magnitude of the target star to the required photometric system.

Procedures for photometry

- If we have standard stars in the CCD field that we are observing, then its fairly easy to calibrate, as we can just use the N_t values as our measure of intensity.
- If not then we need to observe standard stars in separate CCD frames, and as the PSF will vary between different frames, we need to find a true measure of the brightness of the stars.

$$N_t = C_i \int_0^{rmax} 2\pi r \left(1 + r^2/R_0^2 \right)^{-\beta} dr$$

r_{max} is chosen so that we get all of the light.

(The Moffat function, see the previous lecture)

Instrumental Magnitudes (1)

- We have the sky-subtracted signal from our target object, in counts, N_t. We convert this to an instrumental magnitude, using the formula:

$$m_{inst} = -2.5 \log_{10} (N_t / t_{exp})$$

where t_{exp} is the exposure time of the image in seconds.

- The instrumental magnitude depends on the characteristics of the telescope, instrument, filter and detector used to obtain the data.
- We need to compare the instrumental magnitudes of stars of known magnitude with their true magnitudes, to calculate the offset, and thus to calculate the true magnitudes of all of the stars in the frame.
- The relationship between instrumental magnitudes and calibrated magnitudes can be understood as follows. The counts per second N_t / t_{exp} is proportional to the flux, F_λ. Hence

$$m_{inst} = -2.5 \log_{10} (c\, F_\lambda) = -2.5 \log_{10} (F_\lambda) + c'$$

Instrumental Magnitudes (2)

- Therefore, instrumental magnitudes are offset from calibrated magnitudes by a constant:

$$m_{calib} = m_{inst} + m_{zp}$$

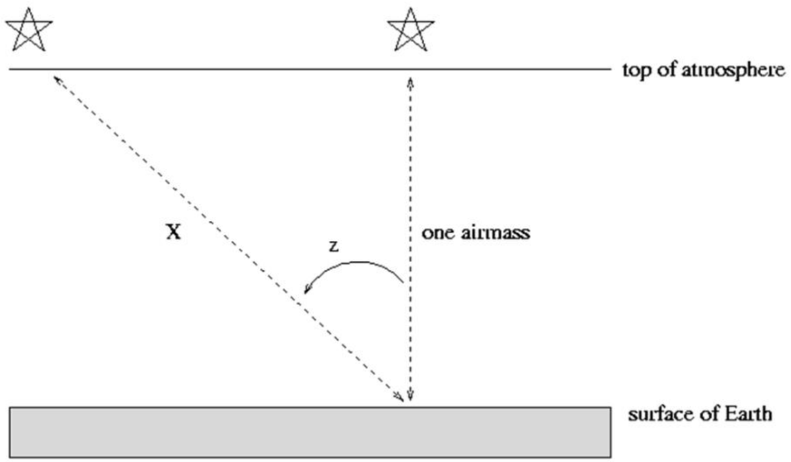
  where the constant, m_{zp}, is known as the **zero-point**.

- The zero-point depends upon the telescope and filter used. We can understand this because if we used a larger telescope to observe a star, the instrumental magnitude would change, but the calibrated magnitude must not!
- An object with a calibrated magnitude equal to the zero-point gives one count-per-second at the telescope.
- For example, suppose the zero-point of a telescope/filter combination is m_{zp} =19.0. If we observe a star with m_{calib} =19.0, then it follows that, for this star m_{inst} =0. By definition then, this star gives one count-per-second.

$$m_{inst} = -2.5 \log_{10} (N_t / t_{exp})$$

Atmospheric absorption

- The next step is to convert the instrumental magnitude, which is measured on the surface of the Earth, to the instrumental magnitude that would be observed above the atmosphere.
- Atmospheric absorption is proportional to the airmass, which is proportional to the secant of the angular distance from the zenith. Strictly this assumes a plane parallel atmosphere, but this is a good approximation for $z < 70°$.



$$X = \sec Z = [\sin \varphi \sin \delta + \cos \varphi \cos \delta \cos h]^{-1}$$

φ is the latitude of the observatory, h is the hour angle of the source, and δ is the declination of the source.

Atmospheric extinction

- The effect of atmospheric extinction on photometry is usually expressed as:

$$m_{obs} = m_{true} + k(\lambda)\sec Z = m_{true} + k(\lambda)\,X$$

Here, m_{true} is the magnitude of the source outside the Earth's atmosphere, m_{obs} is the magnitude observed,

$k(\lambda)$ is the "extinction coefficient" [magnitudes per unit airmass].

- The dominant source of extinction in the atmosphere is Rayleigh scattering by air molecules. This mechanism is proportional to λ^{-4}, which means that extinction is much higher in the blue than in the red.
- The extinction coefficients $k(\lambda)$ have been carefully measured and tabulated for a number of observatories.

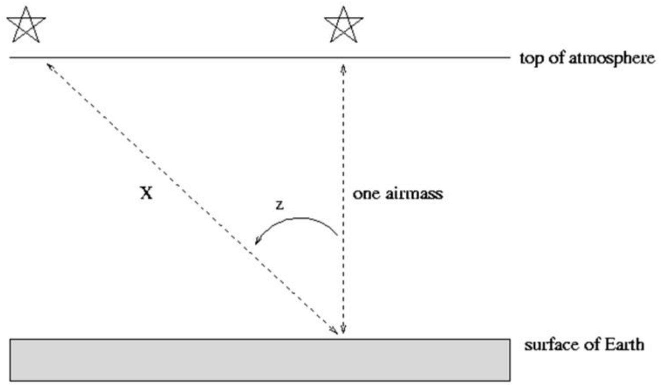
The extinction coefficients on a typical (undusty) night on La Palma

Filter	λc (Å)	k
U	3660	0.55
B	4380	0.25
V	5450	0.15
R	6410	0.09
I	7980	0.06

Extinction coefficient (1)

- However, the extinction can vary from night to night depending on the conditions in the atmosphere, e.g. dust blown over from the Sahara can increase the extinction on La Palma during the summer by up to 1 magnitude.
- How do we find k in practice?
  if we plot instrumental magnitudes vs airmass for a particular star, k is just the slope of the line that passes through the observed points:

$$k = \frac{\Delta m_{inst}}{\Delta X}$$



Extinction coefficient (2)

- Even two measurements of the instrumental magnitude of a star at two different zenith distances is enough to estimate *k* (although less accurate): subtracting $m_{z1} = m_0 + k \sec Z_1$ from $m_{z2} = m_0 + k \sec Z_2$ eliminates m_0, allowing k to be derived.
- For more accurate photometry: observe a set of standard stars (of known magnitude) at different airmass.

!

- Note that no explicit extinction correction is required when performing differential photometry: the target and comparison stars are always observed at the same airmass and hence suffer the same extinction. Hence, the variation due to extinction present in the comparison star is removed from the target star.