# Observational Astronomy

## Problems Set 3: Solutions.

1. Which has a greater energy flux, 10 photons cm$^{-2}$ s$^{-1}$ at 10 Å or 10$^5$ photons cm$^{-2}$ s$^{-1}$ at 5000 Å?

   Answer:

   $F_\lambda = N \times h\nu = N \times hc / \lambda$ : $F_{10Å} = 1.99\times10^{-8}$ erg cm$^{-2}$ s$^{-1}$ $F_{5000Å} = 3.97\times10^{-7}$ erg cm$^{-2}$ s$^{-1}$

   Answer: 10$^5$ photons cm$^{-2}$ s$^{-1}$ at 5000 Å are larger ($3.97\times10^{-7} > 1.99\times10^{-8}$ erg cm$^{-2}$ s$^{-1}$)

   Or $F_{5000Å} / F_{10Å} = N_{5000Å} / N_{10Å} \times 10Å / 5000Å = 10^5 / 10 \times 10 / 5000 = 20$

2. It is often claimed that stellar magnitude errors can be taken as fractional errors of photometric accuracy. Although this is not quite correct but close to it. Prove it.

   Solution:

$$F \sim 2.512^{-m} = 10^{-0.4m}$$

   From Statistics: $\sigma_q = \left|\frac{dq}{dx}\right| \sigma_x$ →

$$\sigma_F = \left|\frac{dF}{dm}\right| \sigma_m = \left|\frac{2.512^{-m}}{dm}\right| \sigma_m = \ln 2.512 \cdot 2.512^{-m} \sigma_m = 0.921\, F\, \sigma_m$$

   or

$$\sigma_F = \left|\frac{10^{-0.4m}}{dm}\right| \sigma_m = 10^{-0.4m} \cdot \ln 10 \cdot 0.4 = 0.921 \cdot 10^{-0.4m} \sigma_m = 0.921\, F\, \sigma_m$$

$$\frac{\sigma_F}{F} = 0.921 \sigma_m$$

3. A star has a measured $I$-band magnitude of 22.0. How many photons per second are detected from this star by the William Herschel Telescope on La Palma (4.2 m diameter), assuming that the telescope and imaging optics have a throughput of 60%, the detector has a quantum efficiency of 80%, the sky has a brightness of 20 magnitudes per square arcsec, and the seeing is 1 arcsec . Estimate the exposure time required to detect the star at a signal-to-noise ratio of 20.

   Solution:

   How many photons per second are detected from this star by the William Herschel Telescope:

   From Lecture 10, slide 411:

   $N_{star} = \eta\, \varepsilon_{atm}\, \varepsilon_{tel}\, \varepsilon_{filt}\, \varepsilon_{win}\, \varepsilon_{geom}\, \phi\, \Delta\lambda\, A\, t = \eta\, \varepsilon\, \phi_{star}\, \Delta\lambda\, A\, t$

   $\eta=0.8$

   $\varepsilon = \varepsilon_{atm}\, \varepsilon_{tel}\, \varepsilon_{filt}\, \varepsilon_{win} = 0.6$

   $\Delta\lambda=1500$ Å

   $A=\pi D^2/4=138544$ cm$^2$

   For simplicity, we can assume that $\varepsilon_{geom} = 1.0$

However, the WHT telescope is of a Ritchey Chretien Cassegrain system, it has a secondary mirror with the diameter 1.0 m (e.g, https://www.ing.iac.es/PR/wht_info/whtoptics.html). Then from Lecture 10, slide 410, $\varepsilon_{geom}$ =0.94

$\phi_{star}$=F/hν=Fλ/hc= $F_0$λ/hc×$2.512^{-m}$=7.18×$10^{-7}$ photons $s^{-1}$ $cm^{-2}$ $Å^{-1}$

Thus, $N_{star}$ = 0.8×0.6×0.94× 7.18×$10^{-7}$ × 1500 × 138544

Answer:

from the star $N_{star}$~67 phot/sec (~72 if $\varepsilon_{geom}$ =1.0)

from the sky $N_{sky}$~425 phot/sec from square arcsec (~452 if $\varepsilon_{geom}$ =1.0)

Estimate the exposure time required to detect the star at a signal-to-noise ratio of 20.

From Lecture 9, slide 360:

S/N = $\frac{N_* t}{\sqrt{N_* t + 2N_{sky} t}} \rightarrow \frac{N_* \sqrt{t}}{\sqrt{N_* + 2N_{sky}}}$ → t=81 sec (~76 sec if $\varepsilon_{geom}$ =1.0)

4. Calculate the flux $F_\lambda$ of a star (in erg $s^{-1}$ $cm^{-2}$ $Å^{-1}$) having Vega magnitude $R$=15 and AB magnitude r=15 ($\lambda_c$ = 6156 Å).

   Answer:

   Vega: $F=F_0*2.512^{-15}$ =2.18×$10^{-15}$ erg $s^{-1}cm^{-2}$ $Å^{-1}$

   $F_0$= 2.177×$10^{-9}$ erg $s^{-1}cm^{-2}$ $Å^{-1}$ (from Table in slide 397, Lecture 10)

   AB (method 1):

   $m = -2.5\, log\, F_\nu - 48.585;$ $F_\nu$ [ergs $s^{-1}$ $cm^{-2}$ $Hz^{-1}$ ]=$10^{-8}\frac{\lambda[Å]^2}{c[\text{cm s}^{-1}]}$ $F_\lambda$ [ergs $s^{-1}$ $cm^{-2}$ $Å^{-1}$] (slide 385, Lec. 9)

   $F_\nu$ = 3.68×$10^{-26}$ ergs $s^{-1}$ $cm^{-2}$ $Hz^{-1}$

   $F_\lambda$ =2.87×$10^{-15}$ erg $s^{-1}cm^{-2}$ $Å^{-1}$

   AB (method 2):

   In AB magnitudes, mag 0 has a flux of 3631 Jy (slide 385, Lecture 9)

   Then $F_\nu$ [Jy] = 3631*$2.512^{-15}$ = 3.63×$10^{-3}$ Jy

   slide 399, Lecture 10: $F_\lambda$ [erg $s^{-1}$ $cm^{-2}$ $Å^{-1}$]= 3.00×$10^{-5}$ $\lambda^{-2}$ $F_\nu$ [Jy] = 2.87×$10^{-15}$ erg $s^{-1}cm^{-2}$ $Å^{-1}$

5. What fraction of the photons in the $V$ band of a bright star would be absorbed by the atmosphere if one were to observe the star at an airmass of 2.5, and at the zenith (airmass = 1)? Assume that the atmospheric extinction k(λ) in the $V$ band is 0.15 mag $airmass^{-1}$.

   Answer: 13% absorbed at the zenith and 29% at the airmass of 2.5

   (Lecture 12, Slide 443): $m_{obs}$ - $m_{true}$ = k(λ) X

   (1-$2.512^{-0.15*1}$)*100% and (1-$2.512^{-0.15*2.5}$)*100%

6. In making differential observations, explain why you should know the colours of the variable and comparison stars.

   Short answer (but you had to elaborate it!): There is a colour term in the accurate formula, caused by the variation in spectral profile of the stars and the filter response over the passband (Slides 446-447, Lecture 12).