Observational Astronomy

Problem Set 1: Solutions

1. If the seeing at a good observatory site is 0.5 arc sec, at which wavelength do observations on a 20 cm (diameter) telescope become diffraction-limited instead of seeing limited?

Solution: From lecture 2 (slide 73):

According to Rayleigh criterion, the angular resolution is given by $\vartheta \cong 1.22\, \lambda \,/\, D$ (radians), where λ is wavelength and D is the diameter of the aperture.

We need to find at which wavelength ϑ reaches $0.5'' = 2.424 \cdot 10^{-6}$ rad: $\lambda \cong \vartheta \cdot D/1.22 = 3974$ Å

Thus, at $\lambda \gtrsim 4000$ Å the angular resolution becomes larger than the seeing (so diffraction-limited rather than seeing limited).

2. What is the expected value of the Fried parameter at a wavelength of 5500 Å if the observed seeing is 0.6 arcsec? What is the corresponding value of r_0 at 1.6 microns in the infrared assuming Kolmogorov turbulence?

Solution: From Lecture 4 (slide 192):

Full Width Half Maximum of the point spread function due to atmospheric turbulence (the seeing) is given by $\beta = 0.98\, \lambda \,/\, r_0$.

Then r_0 = 18.5 cm for 5500 Å and 53.9 cm for 1.6 microns.

3. A set of 13 measurements are made on a physical quantity. The following values are obtained: 0, 1, 2, 3, …, 11, 12. Estimate the mean value $\langle x \rangle$, the RMS spread σ_x and the accuracy of the mean $\sigma_{\langle x \rangle}$.

Answer: $\langle x \rangle = 6$; $\sigma_x = 3.89$; $\sigma_{\langle x \rangle} = 1.08$

4. A new set of 36 measurements are made with the result that the values

0, 1, 2, …, 5, 6, 7, …, 11, 12

occur 0, 1, 2, …, 5, 6, 5, …, 1, 0 times respectively.

Estimate $\langle x \rangle$, σ_x, $\sigma_{\langle x \rangle}$, *median* and *mode*.

Answer: $\langle x \rangle = 6$; $\sigma_x = 2.45$; $\sigma_{\langle x \rangle} = 0.41$; median=mode=6

5. Four separate groups of astronomers obtained the following estimates of the temperature of a white dwarf: 15000 ± 1000 K, 14000 ± 500 K, 14400 ± 800 K and 20000 ± 5000 K.
What is the weighted mean of these estimates? Which of the measurements then can be considered the best measurement (closest to the weighted mean and with the smallest error)? What would have been the best estimate if you had neglected the accuracies of the individual measurements?

Answer: weighted mean = 14282 ± 389 K; mean = 15850 K
Then the best measurement, closest to the weighted mean and with smallest error is 14000 ± 500 K (14400 ± 800 K is also ok). Neglecting the accuracies: 15000 ± 1000 K

6. If an object is placed at a distance ***p*** from a lens and an image is formed at a distance ***q*** from the lens, the lens's focal length ***f*** can be found as

$$f = \frac{pq}{p+q}$$

Suppose that p and q are measured as ***p*** = 1450 ± 0.5 and ***q*** = 652.5 ± 2, both in centimetres. Find ***f*** and the uncertainty σ_f.

Answer: *f* = 450 ± 0.95