OBSERVATIONAL ASTRONOMY

AUTUMN 2024

Lecture 13

Vitaly Neustroev

Spectroscopy

Spectral analysis is the source of most of our astrophysical knowledge.

Outline

1. General introduction to spectroscopy
2. Practical spectroscopy
3. Spectral reductions (calibration)

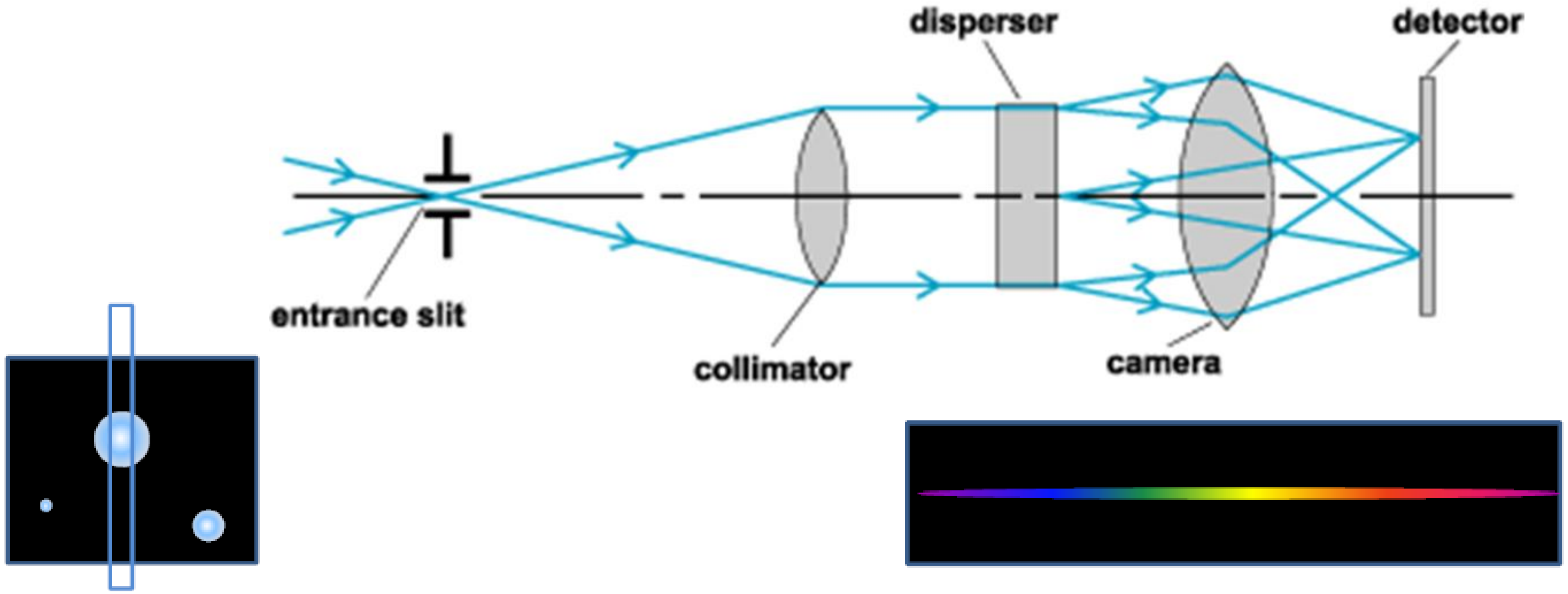
Techniques of Spectroscopy

Spectroscopy (spectrophotometry) is the most informative technique of light analysis, that measures how much light an object produces at various wavelengths of light.

- At X-ray and gamma ray wavelengths detectors have intrinsic energy resolution.
- At lower energies we must use different techniques to separate radiation of different wavelength/energy/frequency spatially.
- These techniques are largely those of interference, so considering radiation as waves.

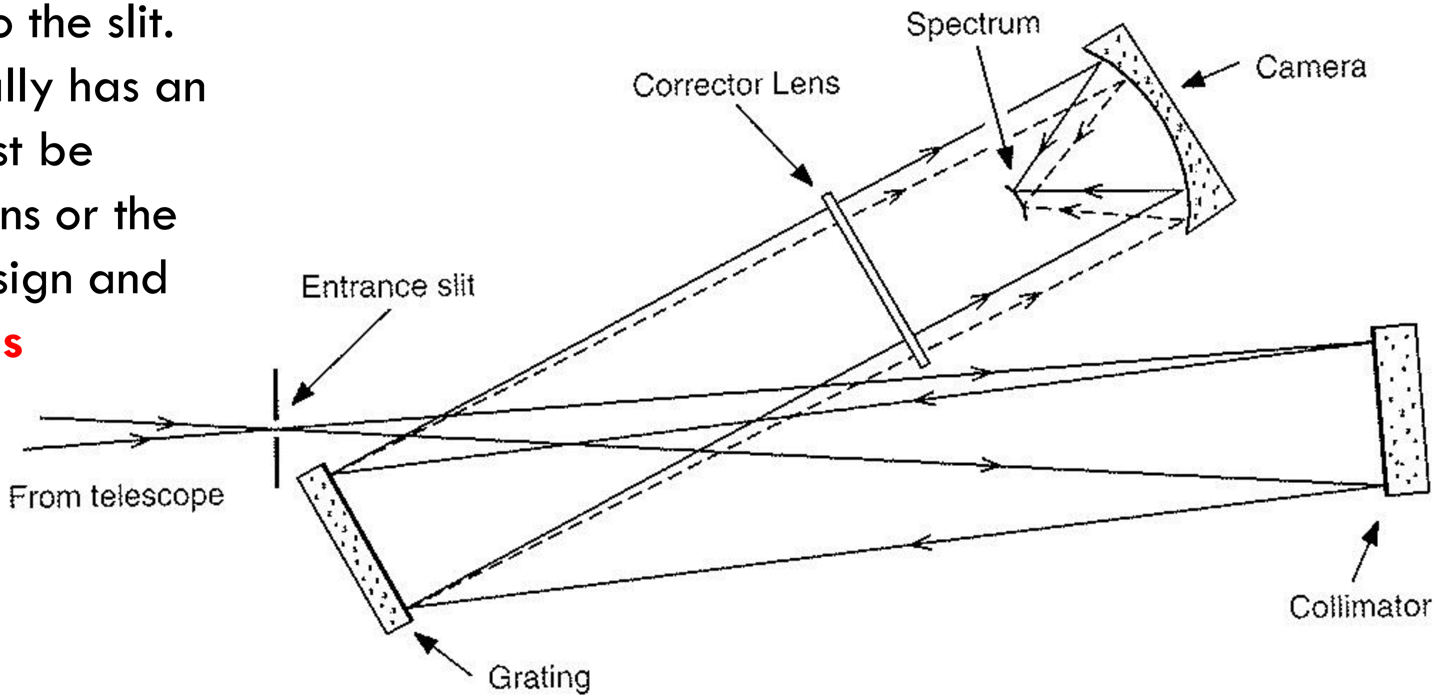
Spectrometers

- All spectrometers have essentially the same basic design, but many different implementations are possible depending on the constraints and choice of spectral disperser.



Spectrometers: main elements

- **Entrance aperture:**
  The image of a target is focused onto the slit. The slit is in the focal plane, and usually has an adjustable width w. The slit width must be matched to either the seeing conditions or the diffraction disc depending on the design and application. A narrower slit **improves** resolution ~1/w ×.

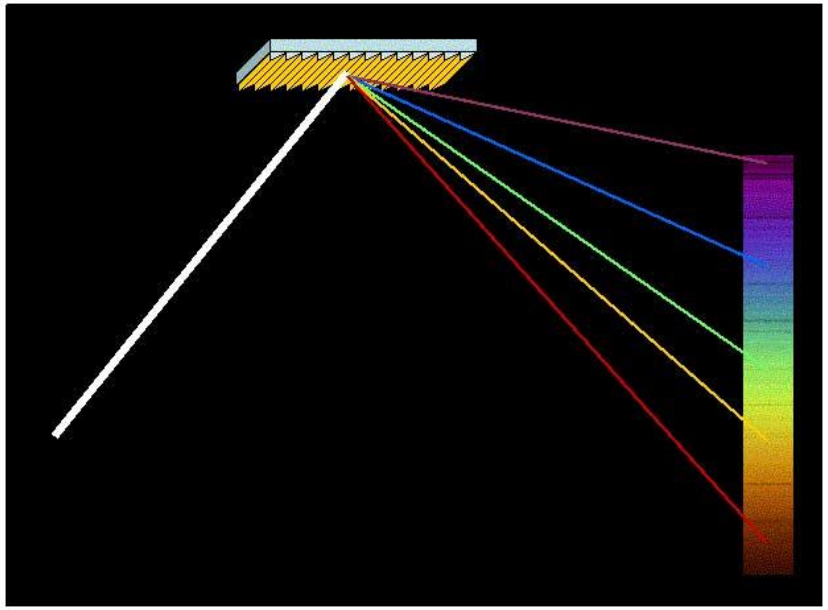


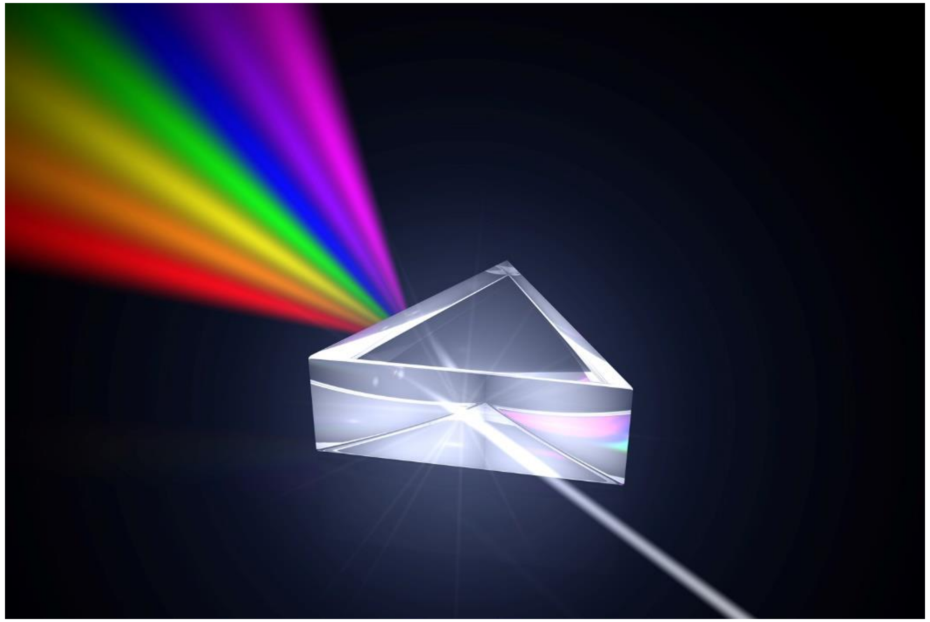
- **Collimator:** makes the rays parallel

- **Disperser** disperses the light into colours: grating or prism, usually on rotating stage so can adjust central wavelength.

- **Camera:** to re-focus parallel output beam from disperser onto focal plane of detector (CCD)

Disperser: Grating vs Prism

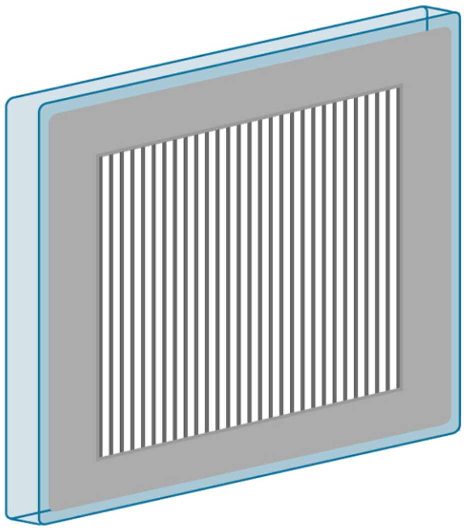




Resolution of a prism is low compared to what is possible with a grating, therefore **grating is usually the primary dispersive element in a modern spectrograph.**

Diffraction Grating (1)

- A diffraction grating is a set of multiple, identical slits (transmitting or reflecting) separated by a distance comparable to the wavelength of light.
- Each slit can be considered as radiating secondary waves (Huygens' secondary wavelets).
- The amplitude at any point on the image side of the slit can be calculated by summing the amplitude contributed by each set of secondary wavelets.



Diffraction Grating (2)

- The theory of Fraunhofer diffraction from a plane grating predicts that the diffracted light is distributed as:

$$I(\theta) = I_0 f_1 f_2,$$

where I is the output intensity leaving the grating in direction ϑ with respect to the normal,
I_0 is the input intensity at the grating,
f_1 is the diffraction pattern for a single grating slit, and
f_2 is the pattern for a set of N identical apertures.

- The two patterns are given by:

$$f_1 = \frac{\sin^2(\pi\alpha)}{(\pi\alpha)^2}, \qquad \alpha = \frac{a \sin\theta}{\lambda}$$

$$f_2 = \frac{\sin^2(N\pi\delta)}{\sin^2(\pi\delta)}, \qquad \delta = \frac{d \sin\theta}{\lambda}$$

where a is the linear width of the apertures (assumed rectangular) and d is the linear separation between them (the groove spacing). We assume normal incidence of the incoming light here.

Diffraction Grating (3)

- Consider monochromatic light.
- Principal maxima ("orders") in the multi-slit pattern occur for $\delta = n$, where n is any integer, i.e. $\sin(\pi\delta)=0$.

$$f_2 = \frac{\sin^2(N\pi\delta)}{\sin^2(\pi\delta)}$$

L'Hôpital's rule $\lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)}$

- Indeed,

$$\lim_{\beta\to\pm n\pi}\frac{\sin(N\beta)}{\sin(\beta)} = \lim_{\beta\to\pm n\pi}\frac{\frac{d}{d\beta}(\sin(N\beta))}{\frac{d}{d\beta}(\sin(\beta))} = \lim_{\beta\to\pm n\pi}\frac{N\cos(N\beta)}{\cos\beta} = N\lim_{\beta\to\pm n\pi}\frac{\cos(N\beta)}{\cos\beta} = N$$

- Thus, the condition for principal maxima $\sin(\pi\delta)=0$

Diffraction Grating (4)

- Consider monochromatic light.
- Principal maxima ("orders") in the multi-slit pattern occur for $\delta = n$, where n is any integer.

  This implies the path difference Δ between adjacent slits will be *n* wavelengths:

Using the law of cosines,

$$r_1^2 = r^2 + \left(\frac{d}{2}\right)^2 - dr\cos\left(\frac{\pi}{2} - \theta\right) = r^2 + \left(\frac{d}{2}\right)^2 - dr\sin\theta$$

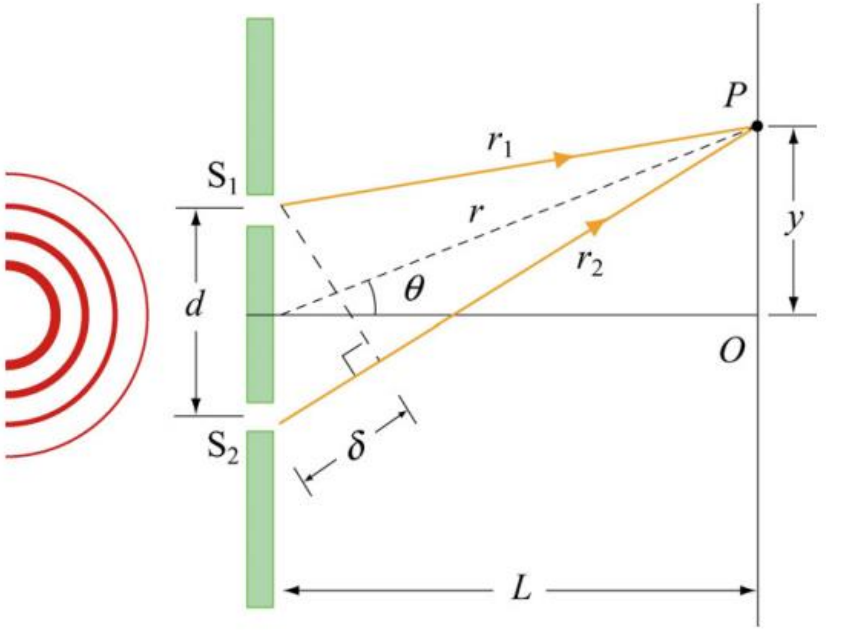
$$r_2^2 = r^2 + \left(\frac{d}{2}\right)^2 - dr\cos\left(\frac{\pi}{2} + \theta\right) = r^2 + \left(\frac{d}{2}\right)^2 + dr\sin\theta$$

Subtracting

$$r_2^2 - r_1^2 = (r_2 + r_1)(r_2 - r_1) = 2dr\sin\theta$$

If L>>d, i.e. the distance to the screen is much greater than the distance between the slits):

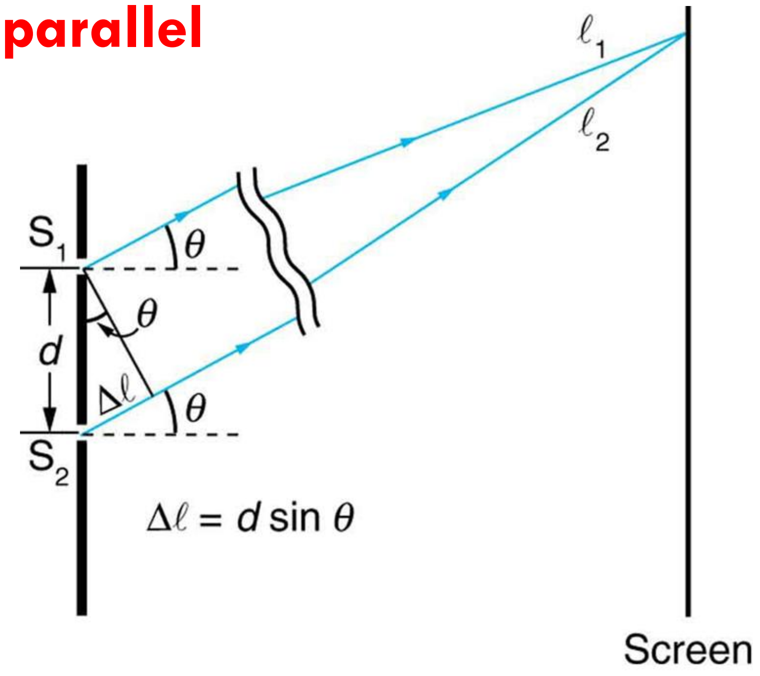
$$\delta = r_2 - r_1 \approx d\sin\theta$$



Diffraction Grating (5)

- Consider monochromatic light.
- Principal maxima ("orders") in the multi-slit pattern occur for $\delta = n$, where n is any integer.
  This implies the path difference Δ between adjacent slits will be n wavelengths.
  If L>>d, the two rays and are essentially treated as being **parallel**

$$\Delta = \Delta l = d \sin \vartheta$$



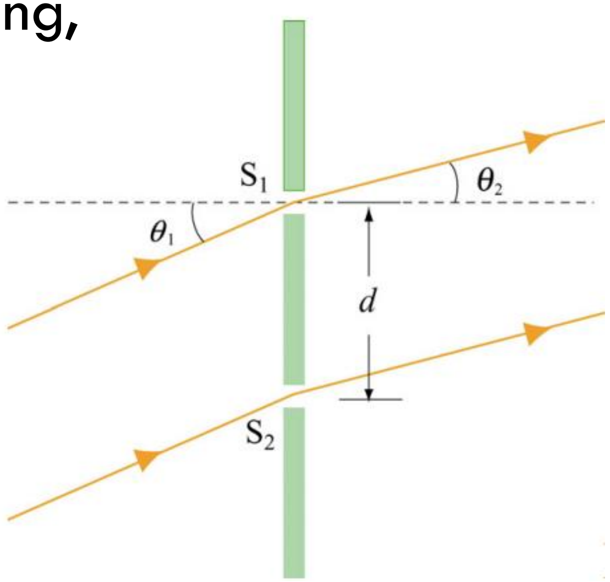
ϑ – is the angle of diffraction

Diffraction Grating (6)

- Consider monochromatic light.
- Principal maxima ("orders") in the multi-slit pattern occur for $\delta = n$, where n is any integer.

If the normal to the plane of the apertures is at an angle ϑ_1 to the incoming radiation, which corresponds to the often utilized inclined grating, then

$$\Delta = d \sin\vartheta_1 + d \sin\vartheta_2$$

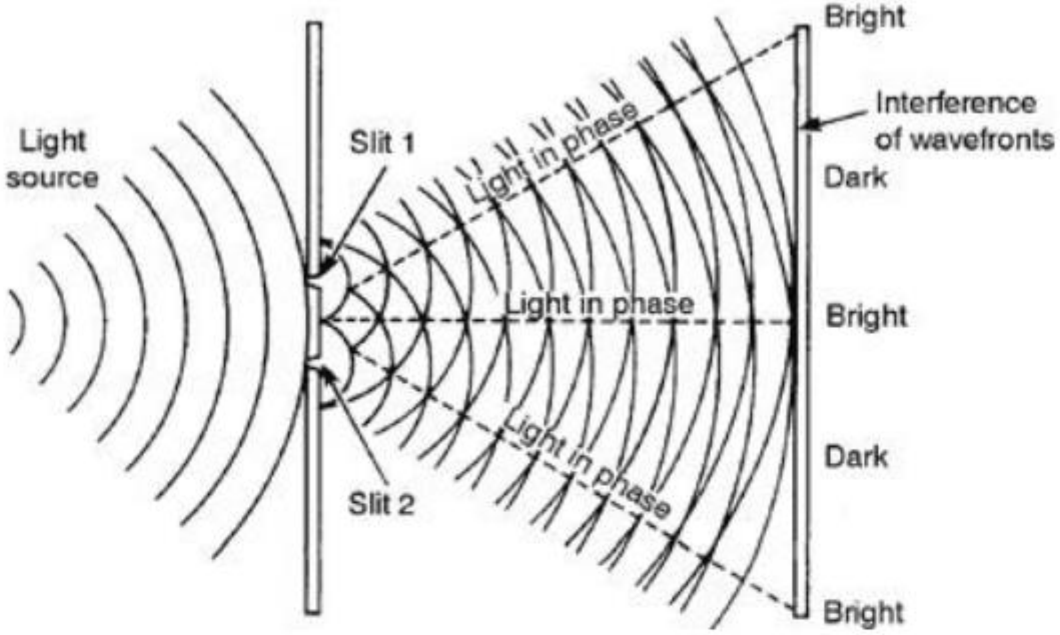


ϑ_1 – is the angle of incidence

ϑ_2 – is the angle of diffraction

Diffraction Grating (7)

- Thus, if the path difference Δ between adjacent slits is n wavelengths, this will produce constructive interference. Maxima in the output intensity occur at a sequence of angles $\sin\theta_n = n\lambda/d$.



Diffraction Grating (8)

- Principal maxima are given by the general grating equation:

$$n\lambda = d(\sin\alpha + \sin\beta)$$

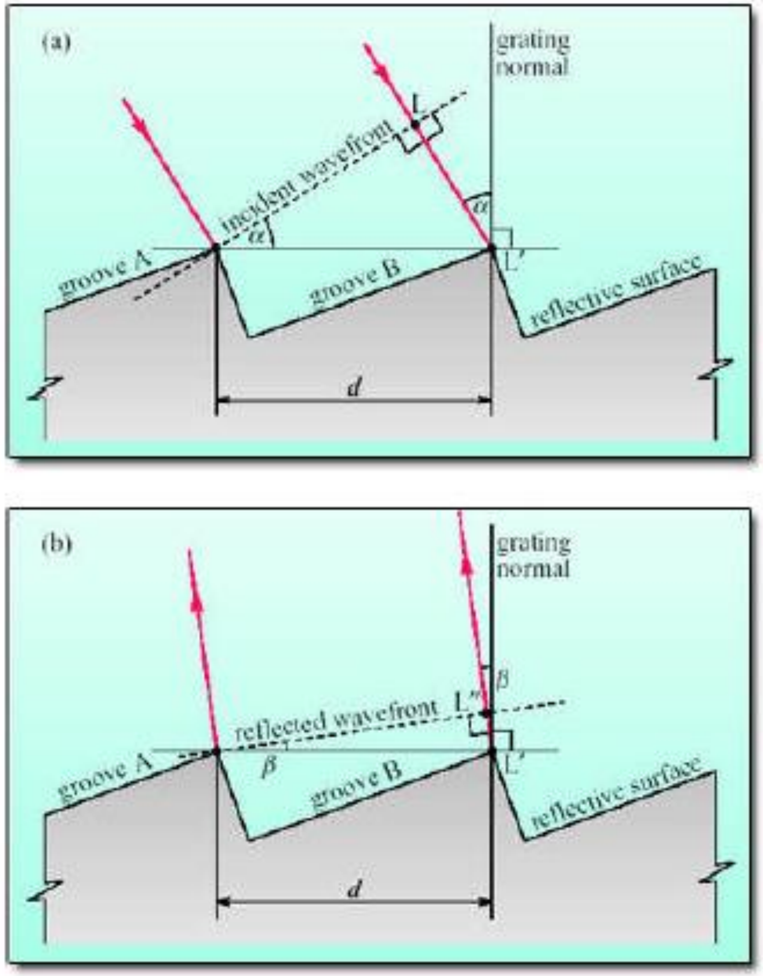
n is an integer representing the order in which the grating is being used.

n is called the order of diffraction.

d – the groove spacing.

α – is the angle of incidence

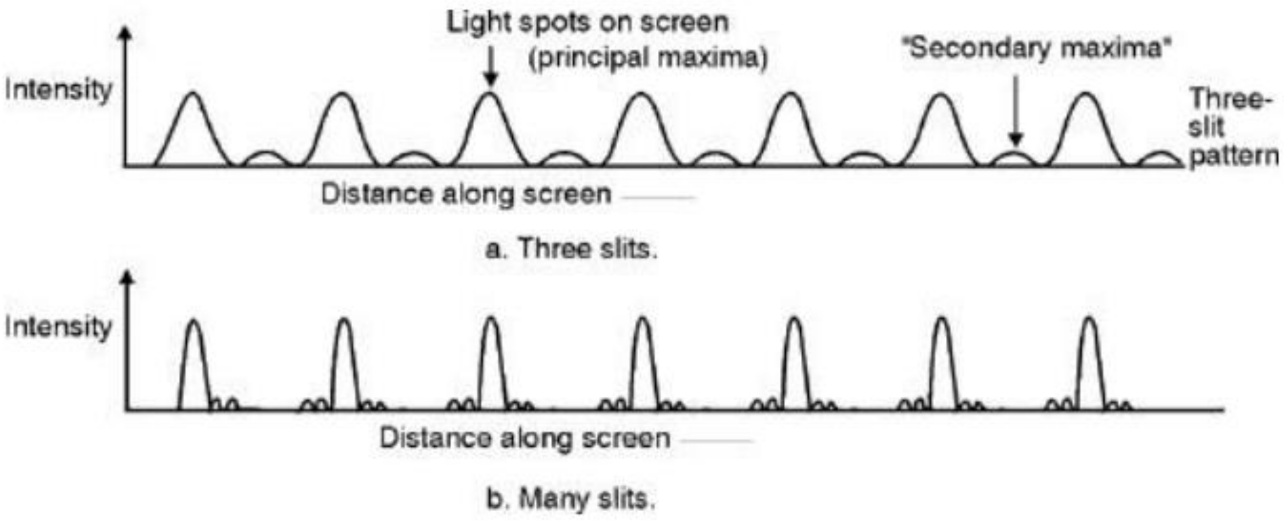
β – is the angle of diffraction



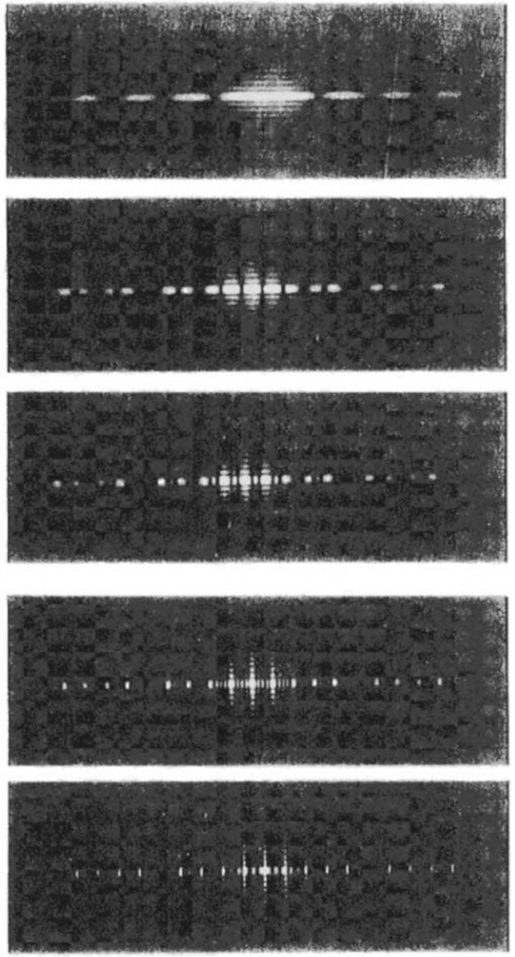
Diffraction Grating (9)

- The single slit diffraction pattern modifies this by affecting the heights of the maxima, the strongest maximum is that at $n = 0$.
- This maximum is of no use to us, because it does not provide any discrimination in wavelength, it is at the same angle for any λ.
- Gratings are designed to concentrate radiation in orders with $n \neq 0$ (note that positive and negative n are equivalent).

Diffraction Grating (10)



The monochromatic multi-slit pattern for 3 slits and a large number of slits. Each peak corresponds to a particular order. The addition of slits **increases the sharpness** and brightness of the peaks **but leaves the locations** of the orders unchanged.



The monochromatic multi-slit pattern for 1 slit (top), 2, 3, 5, and 10 slits (bottom).

Credit: Kitchin C. R.

Resolving power & Spectral resolution

- Spectral resolution for order n is determined by the wavelength shift needed to place the diffraction pattern maximum for $\lambda + \delta\lambda$ on the first minimum in the pattern for λ. The **resolving power** is

$$R = \frac{\lambda}{\delta\lambda} = nN$$

it depends both on the order n and on the total number N of slits illuminated on the grating.

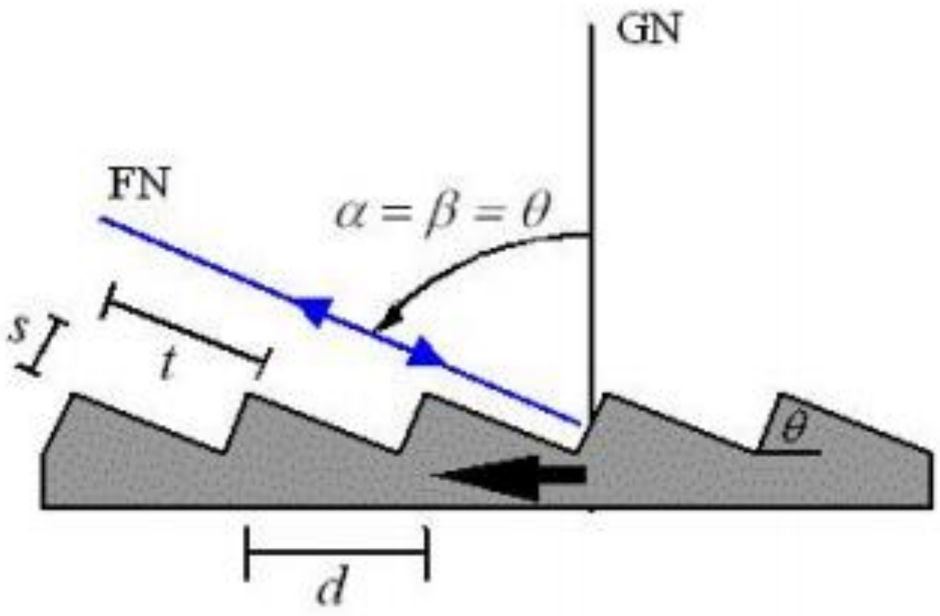
- Astronomers often use the word "dispersion" to refer to $d\lambda/dx$ in the spectrograph focal plane, usually quoted in Å per mm, or Å per pix.
It is inversely related to the resolving power, so lower values correspond to higher resolving power.

Which resolving power to use for your observations?

- Always „the larger the better“ is not the answer.
- High resolution needs a lot of photons, so to get any signal one needs a bright source and/or a large telescope.
- Also, in some cases there is no need for high resolution. If the process you want to study produces velocities of 1000 km/s, there is not much point studying it with resolution of 1 km/s.
- Still, with high resolution you might discover surprising things about your object.

Diffraction Grating: Resolution

- "Echelle" gratings reach very high resolutions by operating at large n $\sim 50 - 100$ and angle of incidence $\alpha \sim 90^{\circ}$. Yield $R > 10^5$.



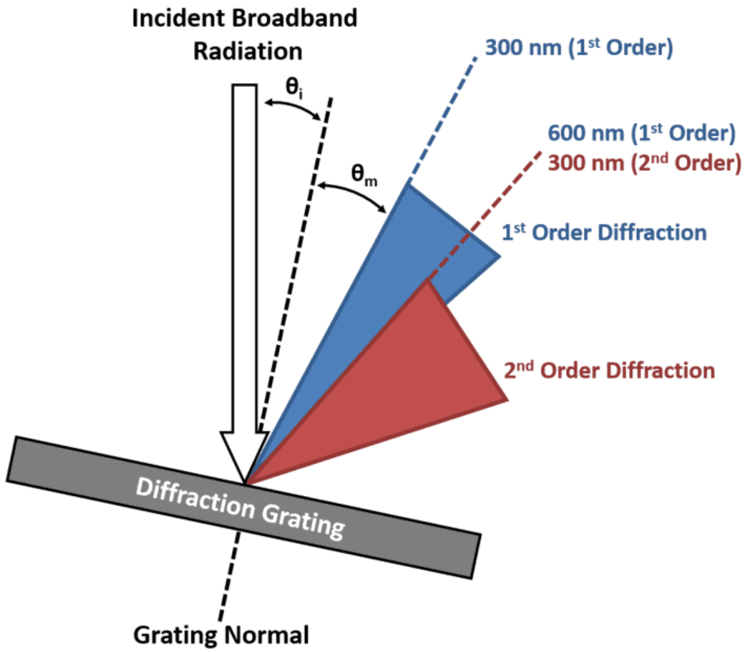
Order overlap

- Red light of a given order is spatially coincident with blue light from a higher order. Wavelength λ_m in order m is superposed on light from wavelength λ_n in order n if

$$\lambda_m = \frac{n\lambda_n}{m}$$

For instance, $\lambda_1 = 10000Å$, $\lambda_2 = 5000Å$, and $\lambda_3 = 3330Å$ are coincident.

Solution: Use "order separating" filters to block out the unwanted orders, (through this becomes difficult for large n).

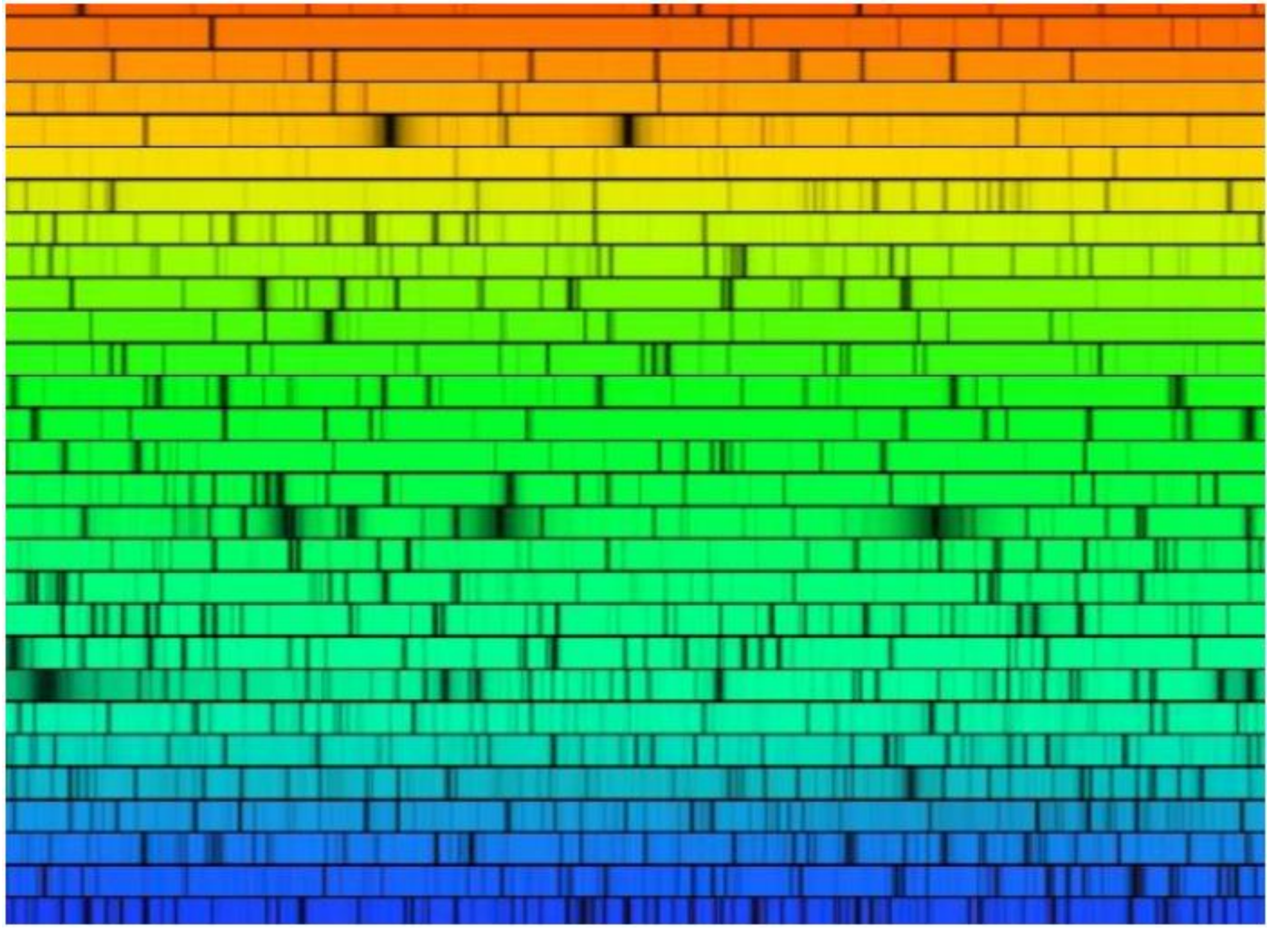


Echelle gratings (1)

- Order overlap is much worse, because adjacent orders differ in wavelength by small amounts (e.g. Order 6 @ 500nm is coincident with order 5 @ 600nm, order 7 @ 429nm, order 8 @ 375nm etc)
- Must separate these orders by cross-dispersion, usually dispersing with a prism at right angles to the grating dispersion.
- Echelle spectrum consists of a number of spectral orders arranged side by side on the detector.
- Echelles can only be used for point sources (stars and quasars) or for small objects, otherwise the light from different orders still overlaps.

Echelle gratings (2)

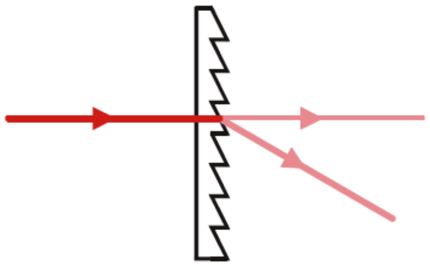
High resolution, optical band solar spectrum



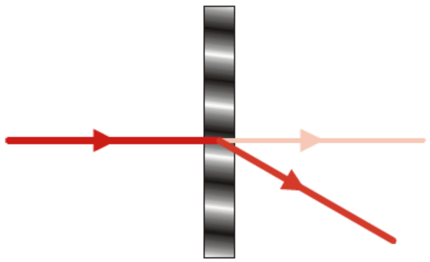
Transmission gratings and grisms

There are different versions of transmission gratings:

- Transmission grating
- Volume Phase Holographic Gratings: **VPH** - use modulations of the index of refraction rather than surface structures to produce dispersion. High efficiency.

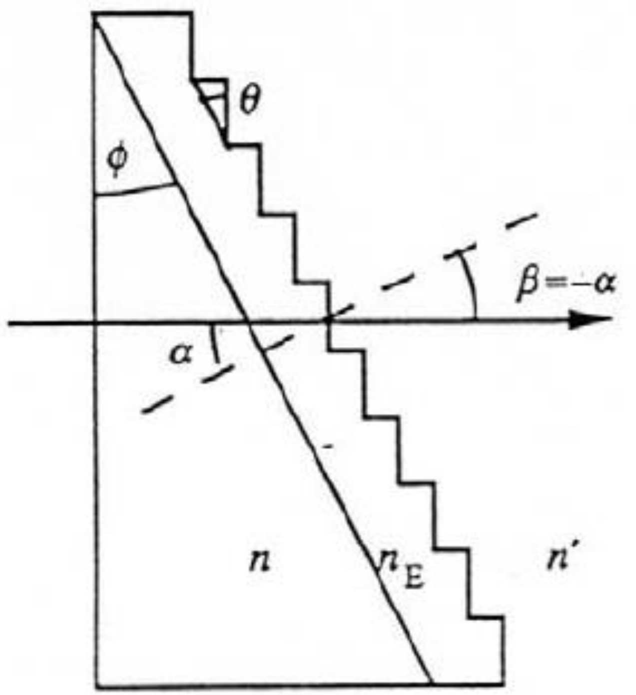


Relief Diffraction Grating



VPH Diffraction Grating

- **Grisms** - a very popular way to convert a camera into a spectrograph is to deposit a transmission grating on the hypotenuse of a right-angled prism and use the deviation of the prism to bring the first order of diffraction on axis.
  - The advantage of a grism is that it can be placed in a filter wheel and treated like another filter. Resolving powers of R ~ 500 – 2000 are practical.



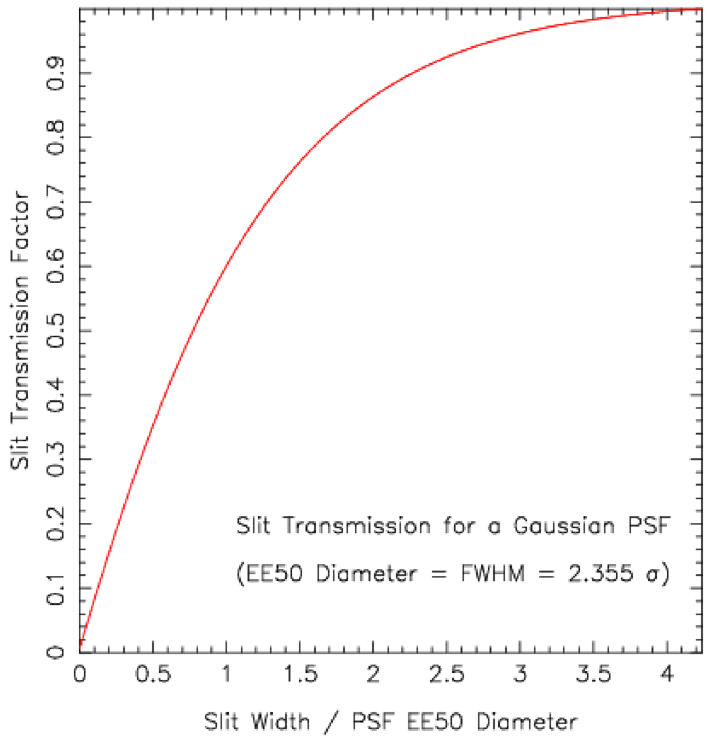
Spectrometer Throughput

- Spectrometer throughput ranges from a few percent to ~50%. The losses accumulate fast. Dispersing elements are usually a big hit, then the losses at multiple surfaces go like (transmission)^n where n is the number of surfaces in the collimator and camera elements (n can be pretty big)

$$0.98^8 \cdot 0.7 \cdot 0.8 = 0.47$$

Camera/collimator with 8 surfaces → 0.98^8; grating → 0.7; ccd → 0.8

- Another throughput issue: slit losses can be very significant!



ALFOSC at the NOT telescope

- UV-optical imaging, low resolution spectroscopy and polarimetry
- 2048x2048 CCD (0.19”/pixel)
- 6.4’x6.4’ field of view (FOV)
- Large selection of broad, intermediate and narrow band filters
- Several grisms:
  - R~200-10,000 (typically 1000)
  - Velocity resolution 30-1500km/s
- Multi Object Spectroscopy with masks
- Imaging polarimetry and spectropolarimetry
- Spectroscopy of objects brighter than R~20
- Imaging of objects brighter than R~23.5