OBSERVATIONAL ASTRONOMY

AUTUMN 2024

Lecture 11

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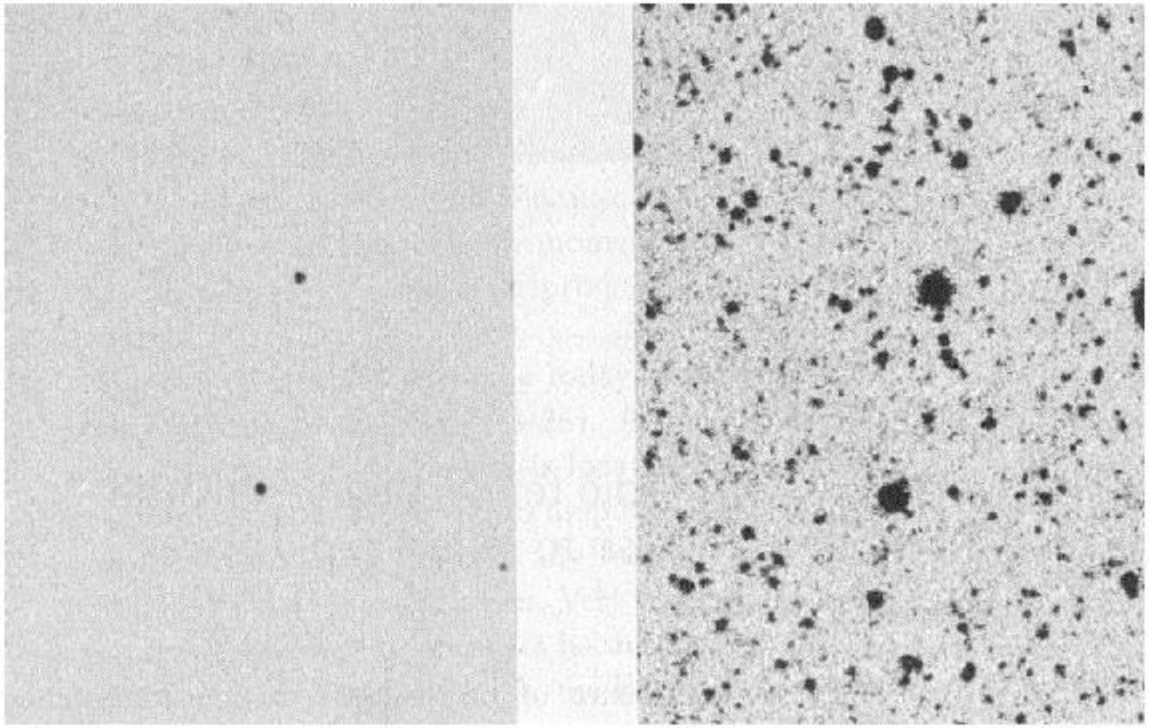
Practical Photometry: S/N (2)



Physical limitations on the precision of photometric measurements (1)

From lecture 9 (slides 353+):

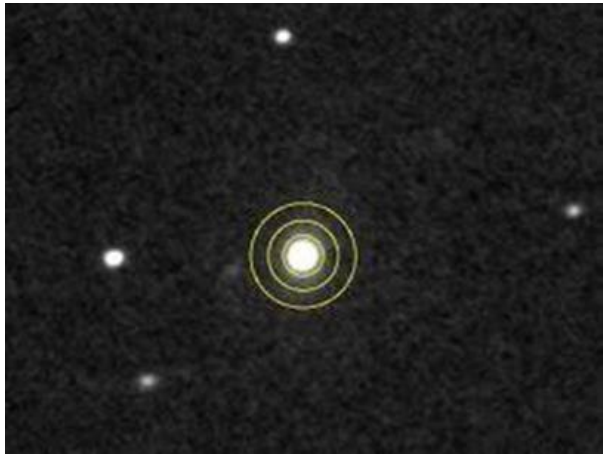
- To calculate the *Output* Signal-To-Noise Ratio of an observation we need to know the signal, and all sources of noise. These are:
  - Photon noise (shot noise) from the signal;
  - Photon noise from the sky background under the signal;
  - Photon noise from the sky background measurement to be subtracted off;
  - Readout noise from all sources;
  - Fixed pattern noise;
  - Bias noise;
  - Dark current noise.

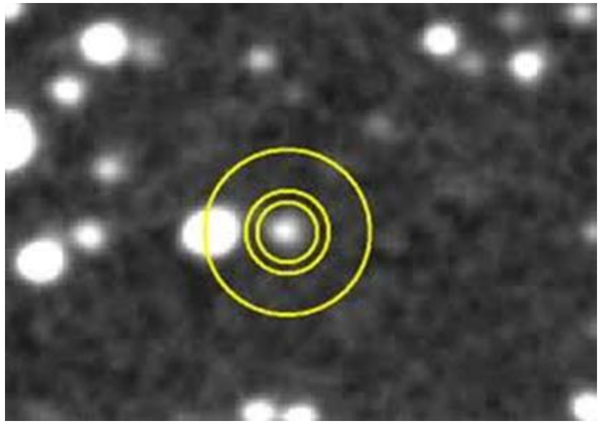
Physical limitations on the precision of photometric measurements (2)

- Detective quantum efficiency= DQE $=[\frac{SNR_{out}}{SNR_{in}}]^2$
- We observe a star on a CCD detector, and process the data in the simplest way possible.

- $Reduced\ Frame = \frac{Object\ Frame - Bias\ Frame}{Flat\ Frame - Bias\ Frame}$

Physical limitations on the precision of photometric measurements (3)





The upper is good, the bottom is bad

- An area centred on the star is defined to be the object area, and is large enough to contain all of the photons from that star.
- An equal area some distance away, which is found to be free of stars, is defined as the sky background area, and the sky background is measured from that.

Aperture photometry (1)

- There are a number of parameters we need to take into account to calculate the signal which reaches the detector:
  - **t** – exposure time
  - **D** – diameter of the telescope
  - S_{sky} [photons / (cm^2 arcsec^2 second)] – brightness of the sky
  - **η** – quantum efficiency of a detector (QE)
  - φ_* [photons / (cm^2 second Å)] – the source flux to be measured
  - Star is observed in a circular aperture of area **β** square arcseconds which covers **n_{pix}** pixels
  - Sky background is determined from a circular aperture of the same size
  - Readout noise is **σ_R** electrons
  - We observe a star of magnitude V in the V filter

!

Signal calculation (1)

- We start from the number of photons incident upon the top of the atmosphere of the Earth from this star:

$$N_* = \phi_* \, \Delta\lambda \, A \text{ photons/second}$$

incident upon the top of the atmosphere in photometric (clear) conditions

$\Delta\lambda$ is the filter passband in Å
φ_* is the flux from a star in photons $s^{-1} cm^{-2}$ Å^{-1}
A is the telescope collecting area in centimetres^2

Signal calculation (2)

- That's at the top of the atmosphere. There are a number of efficiency factors we need to multiply by to calculate the signal which reaches the detector:
  - Atmospheric transmission ε_{atm} (~0.88 for a star at the zenith, in the V filter).
  - Telescope reflection efficiency ε_{tel} (~0.92 per mirror = 0.846 for a Cassegrain telescope)
  - Filter transmission ε_{filt} (~0.85 for a broadband filter)
  - CCD Responsive Quantum Efficiency η (~0.75)
  - Cryostat entrance window efficiency ε_{win} (~0.95)

Signal calculation (3)

- There is also a geometric efficiency factor as part of the aperture of the telescope is blocked by the secondary mirror.

For a ***D*** metre aperture telescope with a ***d*** metre secondary mirror:

$$\varepsilon_{\text{geom}} = \frac{\pi D^2 - \pi d^2}{\pi D^2} = \frac{D^2 - d^2}{D^2}$$

For example, if ***D***=2.0m and ***d***=0.6m, then $\varepsilon = 0.91$

Signal calculation (4)

- For a star the number of photons which is detected is given by:

$$N_{star} = \eta\ \varepsilon_{atm}\ \varepsilon_{tel}\ \varepsilon_{filt}\ \varepsilon_{win}\ \varepsilon_{geom}\ \phi_{*}\ \Delta\lambda\ A\ t$$

t is the exposure time in seconds,
$\Delta\lambda$ = 870 Å for the V band.

For example, for a star of magnitude V=23 on a 2 metre telescope with the efficiencies we have quoted:

$$N_{star} = 2.5\ t$$

Signal calculation (5)

- In the absence of sky background and readout noise it would be simple, we would integrate for 1000 seconds, detect 2500 photons, and have a signal to noise ratio of 50. But sky and readout noise are significant.
- Every square arcsecond of sky gives:

$$N_{sky} = \eta\, \varepsilon_{atm}\, \varepsilon_{tel}\, \varepsilon_{filt}\, \varepsilon_{win}\, \varepsilon_{geom}\, \phi_{sky}\, \Delta\lambda\, A\, t$$

φ_{sky} is the flux from the sky in photons $s^{-1}cm^{-2}\,Å^{-1}arcsec^{-1}$

$\approx$10 t photons from the dark sky (V$\approx$21.5)

Aperture photometry (2)

- Assume we have two apertures, one on the star and one on sky. Star aperture includes sky as well, and our estimate of the star intensity is the difference between the two.
- Signal in the sky aperture is:

$$n_{sky} = \beta\, N_{sky}$$

- Signal in the star aperture is:

$$n_{*+sky} = \beta\, N_{sky} + N_{star}$$

Noise on the measurements

- Noise on the measurements has two components, photon noise which is given by the square root of the number of photons, and readout noise, which is determined by the readout noise and by the number of pixels in the aperture. The noise components add in quadrature:

$$\sigma_{sky}^2 = n_{sky} + n_{pix}\, \sigma_R^2$$

$$\sigma_{*+sky}^2 = n_{*+sky} + n_{pix}\, \sigma_R^2$$

$$n_* \approx n_{*+sky} - n_{sky}$$

$$\sigma_*^2 = n_{*+sky} + n_{sky} + 2\, n_{pix}\, \sigma_R^2$$

$$\sigma_*^2 = 2\, \beta\, N_{sky} + N_{star} + 2\, n_{pix}\, \sigma_R^2$$

Signal to Noise ratio

$$S/N = n_* \,/\, \sigma_* = N_{star} \,/\, \sigma_*$$

$$\frac{S}{N} = \frac{N_{star}}{\sqrt{2\,\beta N_{sky} + N_{star} + 2\,n_{pix}\sigma_R{}^2}}$$

$N_{star} = \eta\, \varepsilon_{atm}\, \varepsilon_{tel}\, \varepsilon_{filt}\, \varepsilon_{win}\, \varepsilon_{geom}\, \phi_*\, \Delta\lambda\, A\, t$
$N_{sky} = \eta\, \varepsilon_{atm}\, \varepsilon_{tel}\, \varepsilon_{filt}\, \varepsilon_{win}\, \varepsilon_{geom}\, \phi_{sky}\, \Delta\lambda\, A\, t$

If exposure time is short then readout noise ($\sigma_R \sim 10$) will **dominate**, especially when seeing is bad.

Note: seeing comes in with n_{pix} term

What is ignored in this S/N eqn?

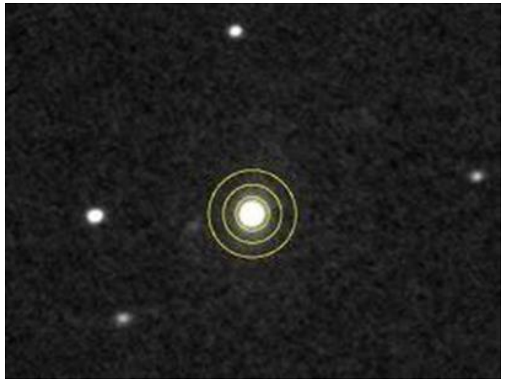
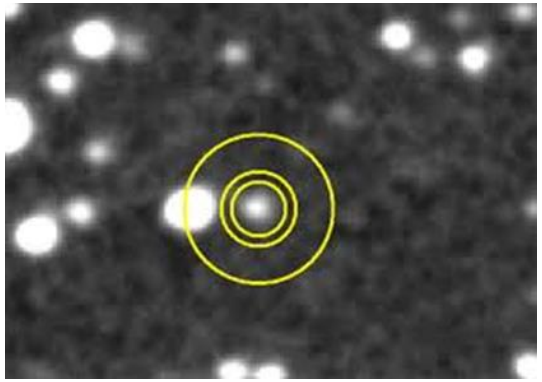
- Bias level/structure correction
- Flat-fielding errors
- Charge Transfer Efficiency (CTE)
  0.99999/pixel transfer
- Non-linearity when approaching full well
- Scale changes in focal plane
- A zillion other potential problems

Improving the Signal to Noise (1)

- Larger Sky Aperture – Increasing the sky aperture and scaling it to the size of the object aperture, or using several sky apertures and averaging them, reduces the noise to:

$$\sigma_*^2 = \zeta\, \beta\, N_{sky} + N_{star} + \zeta\, n_{pix*}\, \sigma_R^2$$

where $\zeta = (1 + n_{pix*} / n_{pix\_sky})$, and n_{pix*} and n_{pix_sky} are the number of pixels in the star and sky apertures respectively. In practice, the sky aperture is often an annulus around the star aperture. Must be careful that stars do not get in the sky aperture!

Improving the Signal to Noise (2)

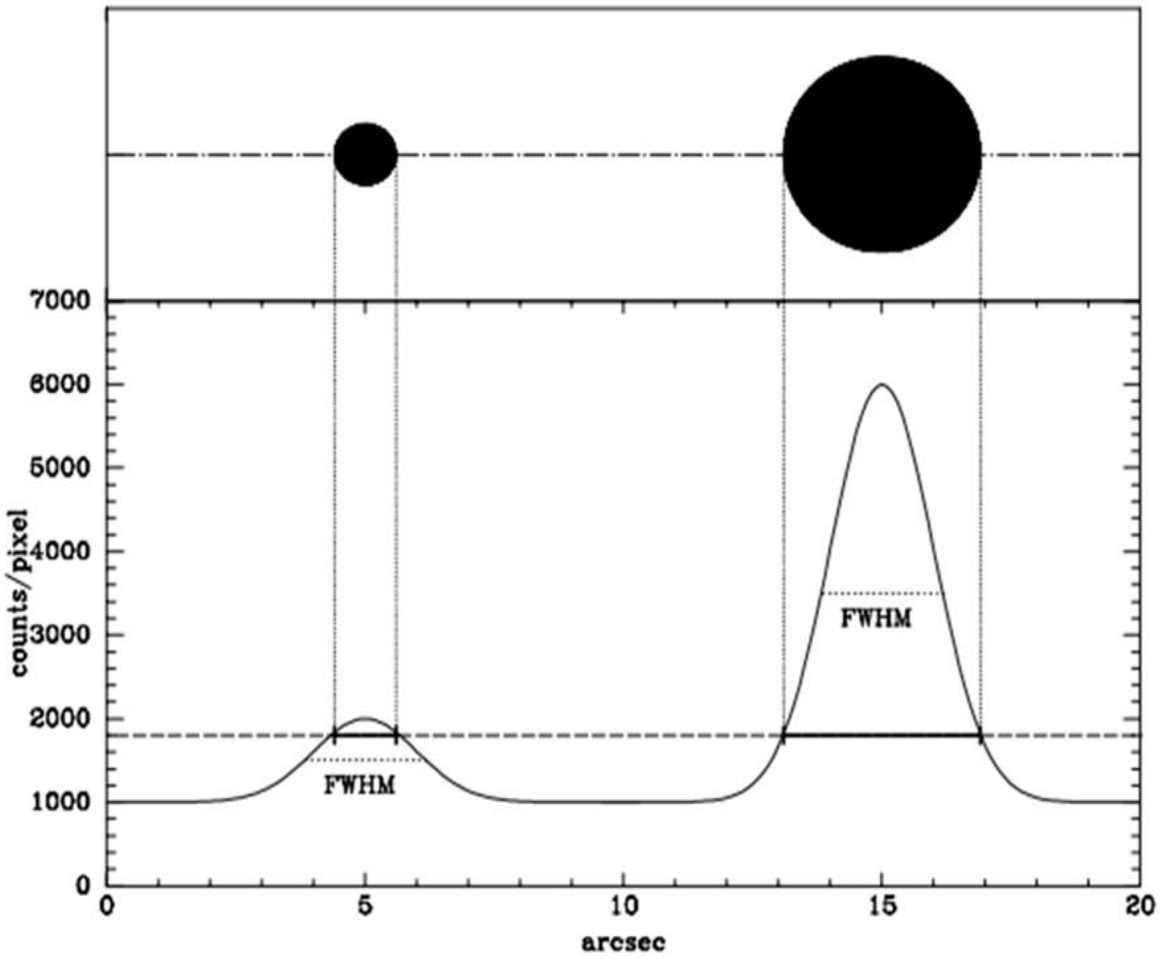
- Smaller object aperture – reducing the object aperture reduces both sky noise and readout noise. However, you lose signal. The problem is if you are comparing the signal in different images, and fluctuations in image size (seeing) cause the amount of signal you lose to vary, then this introduces systematic errors in the brightness measured (photometry).

Solution – Aperture Correction

Aperture Correction (1)

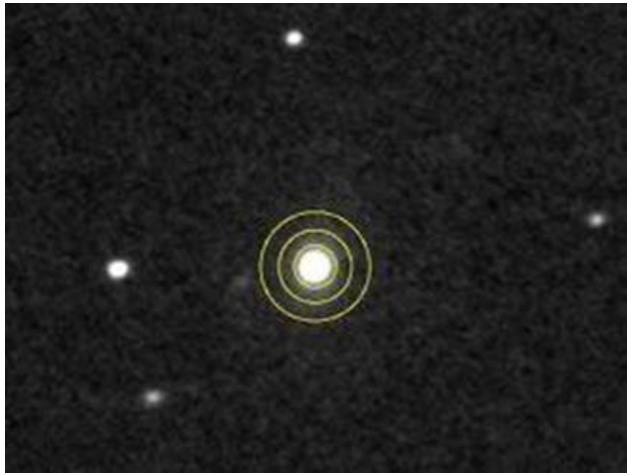
- The **point spread function** (PSF) is the shape of the CCD image of a point (unresolved) source of light.
- Since the PSF is the shape of a point of light on the CCD, and since all stars are points, then all stars have exactly the same shape and size on the CCD, if aberrations are not significant.
- The PSF does not have an edge. The intensity of the star fades smoothly to zero with increasing radius, but there is no place that we could call an "edge".

Aperture Correction (2)



Brighter stars may look bigger, but that is caused by the following effect:

the shape of the faint and bright star are exactly the same, we are simply looking at a larger diameter at a **given intensity** for a bright star than for a faint star.



Aperture Correction (3)

- If we want to measure all the light from a star,
  how far out in radius do we have to go?
  - One logical answer might be: as big as possible, to get “all” the light from the star.
    This is **not** a good answer.
  - Reducing the object aperture **reduces** both sky noise and readout noise.

- **But**, a small aperture will only encompass a **fraction** of the total light from the star!
  However, if the seeing were constant, any aperture would measure the same fraction of light for any star, and when comparing one star with another the effect would cancel out.

Aperture Correction (4)

- The problem is that seeing is not constant. A small aperture might measure 0.5 of the total light from a star on one CCD image, then, if the seeing worsens, the same size aperture might measure only 0.4 of the light from the star on the next CCD image.
- Seeing affects mostly the inner Gaussian core of the image. Using an aperture 4 to 10 times the diameter of the typical FWHM will get most of the light. In this size aperture, reasonable variations in the seeing will not result in measurable variations in measured counts.

Aperture Correction (5)

- However, for faint objects, an aperture 4 times the FWHM will contain a lot of sky signal. This will result in a low S/N ratio.
- Aperture Correction: If we measure the bright object in a small aperture (say radius = 1 FWHM) and also in a bigger aperture which gets "all" the light (say 4 FWHM) we can easily find the ratio of light in the small to large aperture (which we express as a magnitude difference).

Aperture Correction (6)

- The aperture correction is defined as:

$$\Delta = m_{inst}(4\ \text{FWHM}) - m_{inst}(1\ \text{FWHM})$$

(Δ is always a negative number)

- How do we use the aperture correction?

$$\text{total} = m_{inst}(1\ \text{FWHM}) + \Delta$$

- "Total" is our estimate of the total instrumental magnitude in the faint star, $m_{inst}(1\ \text{FWHM})$ is the measured magnitude in the small aperture for the faint star, and Δ is the aperture correction derived from a bright star in the same frame.

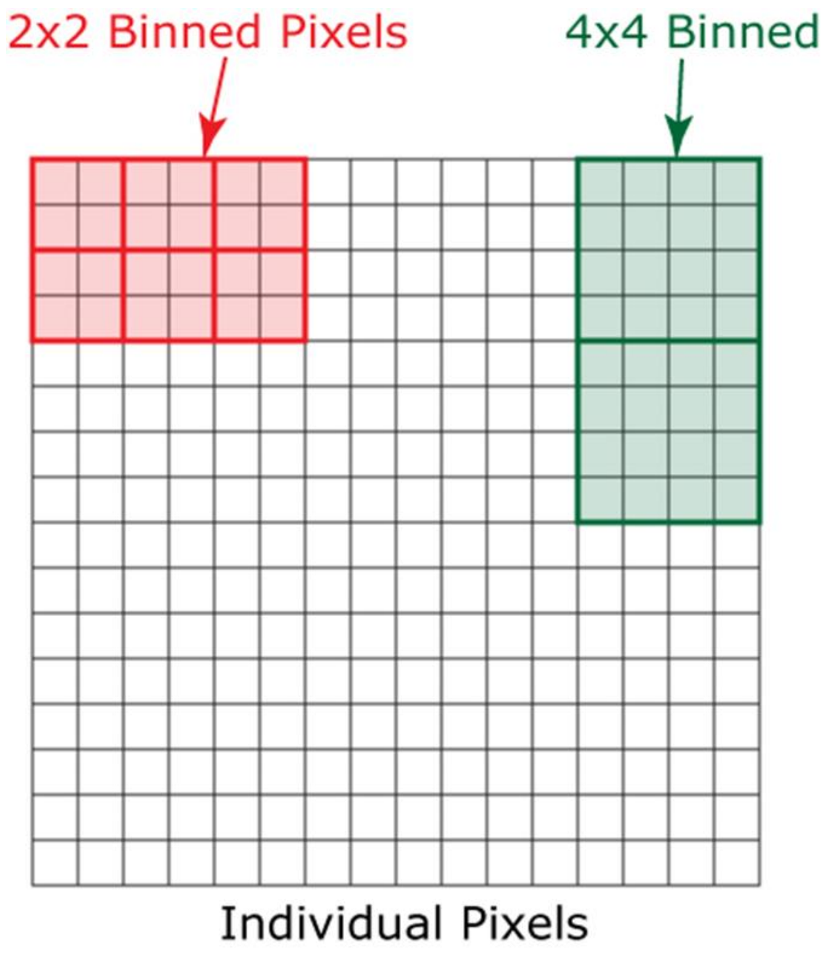
Aperture Correction (7)

- There must be an optimum aperture size that gives the maximum S/N.
- The optimum size of the small aperture has been studied by several authors.
- The optimum aperture seems to be achieved when the measurement aperture has a diameter about $1.4 \times$ FWHM of the PSF. At this aperture, the aperture correction is about -0.3 mag.
- However, the S/N does not appear to be too sensitive to the exact small aperture size.

Improving the Signal to Noise (3)

- On-chip binning – Most CCDs have the option of *binning*: combining a set of adjacent pixels into a single pixel produced as output. For example, a square of 4 pixels on the CCD chip might be reported as one pixel containing their combined value. But you only have to read the output capacitor out once and you only get one lot of readout noise.

On-chip binning



- This way you reduce readout noise at the expense of resolution. Resolution should always be smaller than the characteristic size of the star images.

Profile Fitting (1)

- Profile fitting is used most commonly in crowded fields, where it is difficult or impossible to define a sky aperture free of stars (or galaxies).
- It does however offer an advantage in precision even in sparse fields, because it weights the data more correctly.

Profile Fitting (2)

- Basic assumption is that the intensity profile (which is in principle a 2 dimensional function) is the same for all stars in a particular CCD image.
- Intensity profile is determined by seeing or by diffraction, or occasionally by aberrations.
- If it is determined by aberrations you need to be very careful, because the assumption that the profile is the same at all positions on the CCD may **not** be correct.

Profile Fitting (3)

- From a set of isolated, comparatively bright (but not saturated) stars in the frame, determine the image profile, this is called the **Point Spread Function (PSF).**
- For ground based data an empirical approximation to the PSF is the Moffat function:

$$f(r) = C_i \ (1 + r^2/R_0{}^2)^{-\beta} + B_i \quad (r < r_{max})$$

$$f(r) = B_i \ (r > r_{max})$$

R_0 is the characteristic radius of the star image,
r is the distance from the centre of the image,
β describes the overall shape of the PSF,
B_i is the background in the region of star i, and
C_i is the relative brightness of star i.

Profile Fitting (4)

$$\text{FWHM}=2\,R_0\sqrt{2^{1/\beta}-1}$$

- Fit this function for each of the stars in the image to the data, using a least squares or similar technique.
- For each star determine B_i and C_i.
  R_0 and β are constant within an image.

Profile Fitting (5)

- Then we have a set of scaling factors, which can be converted to a relative magnitude.
- We need aperture photometry of one star, either from this CCD frame or from another, this can be a bright isolated star with high S/N, this gives the magnitudes of all of the stars in the frame.
- The fit gives the correct weighting, rather than adding in lots of pixels with very little signal, S/N from profile fitting is usually at least a factor of 2 higher than from aperture photometry.
- Profile fitting can cope with fields in which stars are close or their images even overlap.

Profile Fitting (6)

- For ground-based data the PSF is determined by the seeing, and must be redetermined for each CCD image.
- For space based (e.g. Hubble Space Telescope) data the PSF is fixed, and is often available as part of the standard calibration data produced with the observations.
  It still depends upon the passband (filter).