



OBSERVATIONAL ASTRONOMY

AUTUMN 2024

Lecture 9

Vitaly Neustroev

Physical limitations on the precision of photometric measurements (6)

352

- In dark sky at a dark site (no moon, no reflected street light), the magnitude of a 1 arcsecond patch of sky in the V band is approximately $V_{\text{sky}} = 21.5$ mag.

Thus, every square arcsecond of sky gives:

$$S_{\text{sky}} = 2.5 \times 10^{-3} \text{ photons / (cm}^2 \text{ arcsec}^2 \text{ second)}$$

Physical limitations on the precision of photometric measurements (7)

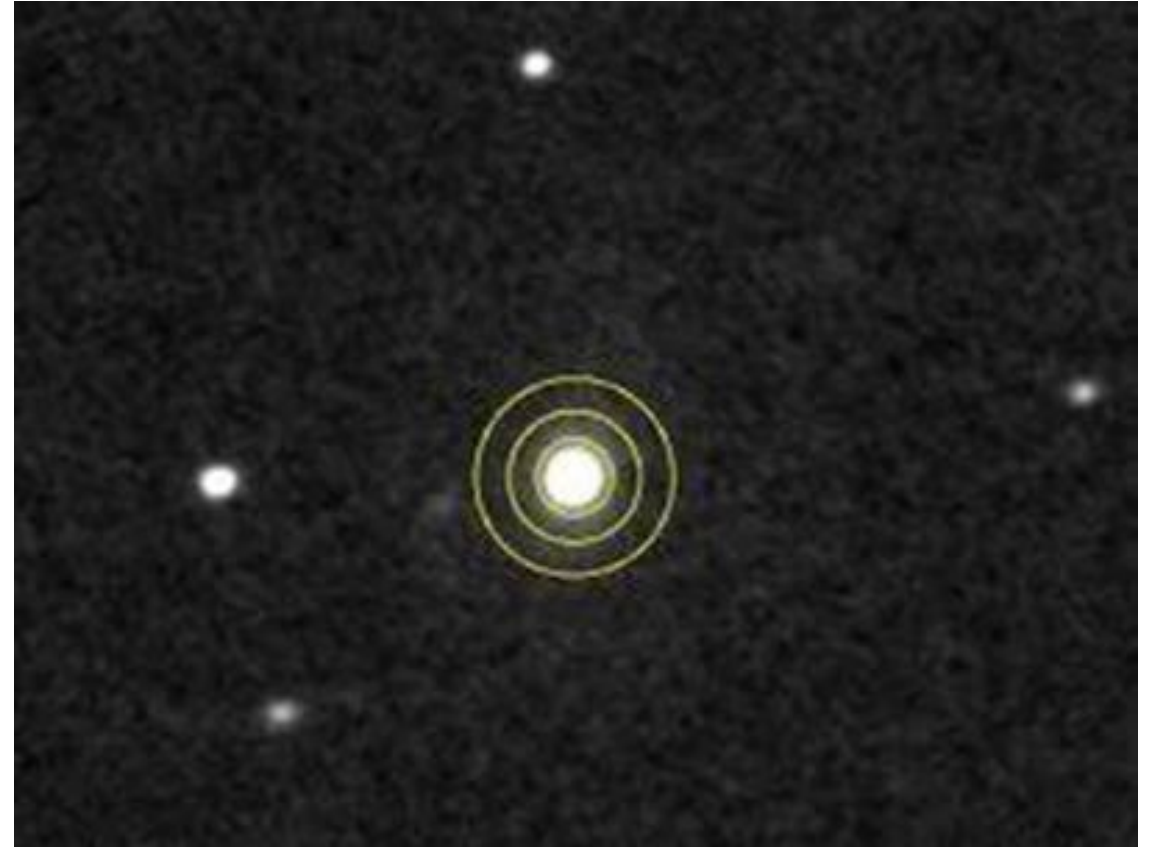
353

- To calculate the **Output Signal-To-Noise Ratio** of an observation we need to know the signal, and all sources of noise. These are:
 - Photon noise (shot noise) from the signal;
 - Photon noise from the sky background under the signal;
 - Photon noise from the sky background measurement to be subtracted off;
 - Readout noise from all sources;
 - **Fixed pattern noise;**
 - **Bias noise;**
 - **Dark current noise.**

A case study of simple aperture photometry (1)

354

- We observe a star on a CCD detector, and process the data in the simplest way possible.
- An area centred on the star is defined to be **the object area** and is large enough to contain all the photons from that star.
- An equal area some distance away, which is found to be free of stars, is defined as **the sky background area (sky aperture)**, and the sky background is measured from that.



A case study of simple aperture photometry (2)

355

- We will make some assumptions:
 - ▣ We have eliminated fixed pattern noise by dividing the image by a normalised long exposure of a uniform light source, this is called *a flat field*.
 - ▣ Bias noise and dark current noise are negligible, as this is a cryogenically cooled, buried channel CCD.

Aperture photometry

356

- There are a number of parameters we need to take into account to calculate the signal which reaches the detector:
 - t – exposure time
 - β – angular size of a source (defined by the seeing)
 - D – diameter of the telescope
 - S_{sky} [photons / (cm² arcsec² second)] – brightness of the sky
 - η – quantum efficiency of a detector (QE)
 - f_* [photons / (cm² second)] – the source flux to be measured

Signal calculation (1)

357

- We start from the number of photons incident from this **star**, from the **sky**, and from **the star + the sky**:
 - $A \sim D^2$ is the telescope collecting area [cm²]
 - $B \sim \beta^2$ is the source area on the sky [arcsec²]
- $n_* \approx \eta D^2 t f_*$ – an average number of photons from the source
- $n_{\text{sky}} \approx \eta D^2 t \beta^2 S$ – an average number of photons from the sky
- $n_{*+\text{sky}} \approx \eta D^2 t (f_* + \beta^2 S)$ – an average number of photons from the source and the sky

Signal calculation (2)

358

- That is without the Readout noise and other detector noises. If we want to take them into account – we must add $N_d = n_d t$ to the right side of equations.
- There is also a geometric efficiency factor as part of the aperture of the telescope is blocked by the secondary mirror.

For a while, we will not take these factors into account.

Noise on the measurements

359

- Noise on the measurements is given by the square root of the number of photons:

$$\sigma_{*+sky} = \sqrt{n_{*+sky}}$$

$$n_* \approx n_{*+sky} - n_{sky}$$

$$\sigma_* = \sqrt{n_{*+sky} + n_{sky}} = \sqrt{n_* + 2n_{sky}}$$

(If x and y have independent random errors σ_x and σ_y , then the error in $z = x \pm y$ is $\sigma_z^2 = \sigma_x^2 + \sigma_y^2$)

Signal to Noise ratio (1)

360

$$S/N = \frac{n_*}{\sigma_*} = \frac{n_*}{\sqrt{n_* + 2n_{\text{sky}}}} = \frac{\eta D t f_*}{\sqrt{\eta t (f_* + 2\beta^2 S)}}$$

$n_* \approx \eta D^2 t f_*$

$n_{\text{sky}} \approx \eta D^2 t \beta^2 S$

Let's now consider two special cases:

- If the Source dominates over the Sky: $n_* \gg n_{\text{sky}}$
- If the Sky noise dominates: $n_{\text{sky}} \gg n_*$

Signal to Noise ratio (2)

361

$$S/N = \frac{n_*}{\sqrt{n_* + 2n_{\text{sky}}}} = \frac{\eta D t f_*}{\sqrt{\eta t (f_* + 2\beta^2 S)}}$$

If the Source dominates over the Sky: $n_* \gg n_{\text{sky}}$

$$S/N \cong \frac{n_*}{\sqrt{n_*}} = \sqrt{n_*} = D \sqrt{\eta t f_*}$$

$f_{\min} \sim 1 / (D^2 t)$ for the given S/N

the telescope aperture is most important!

Signal to Noise ratio (3)

362

If the Sky noise dominates: $n_{\text{sky}} \gg n_*$

$$S/N = \frac{n_*}{\sqrt{n_* + 2n_{\text{sky}}}} = \frac{\eta D t f_*}{\sqrt{\eta t (f_* + 2\beta^2 S)}}$$

$$S/N \cong \frac{n_*}{\sqrt{2n_{\text{sky}}}} = \frac{\eta D t f_*}{\sqrt{2\eta t \beta^2 S}} = \frac{D f_*}{\beta} \sqrt{\frac{\eta t}{2S}}$$

$$f_{\text{min}} \sim \frac{\beta}{D} \sqrt{\frac{S}{t}} \text{ for the given } S/N$$

the seeing (angular size of a source) is most important!

Signal to Noise ratio (4)

363

Source dominates over the Sky

$$S/N \cong D \sqrt{\eta t f_*}$$

$$f_{\min} \sim \frac{1}{D^2 t}$$

for the given S/N

the telescope aperture

most important is

Sky noise dominates

$$S/N \cong \frac{D f_*}{\beta} \sqrt{\frac{\eta t}{2 S}}$$

$$f_{\min} \sim \frac{\beta}{D} \sqrt{\frac{S}{t}}$$

for the given S/N

the seeing

Photometry

The technique that measures the relative amounts of light in different wavelength ranges.

Stellar magnitudes

365

□ Magnitudes:

□ Apparent magnitudes $m_1 - m_2 = -2.5 \log \frac{E_1}{E_2}$

□ Absolute magnitudes

$$M - m = -2.5 \log \left(\frac{D}{10} \right)^2$$

D is the object's distance in parsecs

$$M = m + 5 - 5 \log D$$

$$M = m + 5 - 5 \log D - A \cdot D$$

A is the interstellar absorption in magnitudes per parsec. Within the galactic plane A is $\sim 0.002 \text{ mag pc}^{-1}$.

□ Sometimes M may be estimated by some independent method. Then:

$$D = 10^{[(m-M+5)/5]} \text{ pc}$$

Filters and photometric systems

366

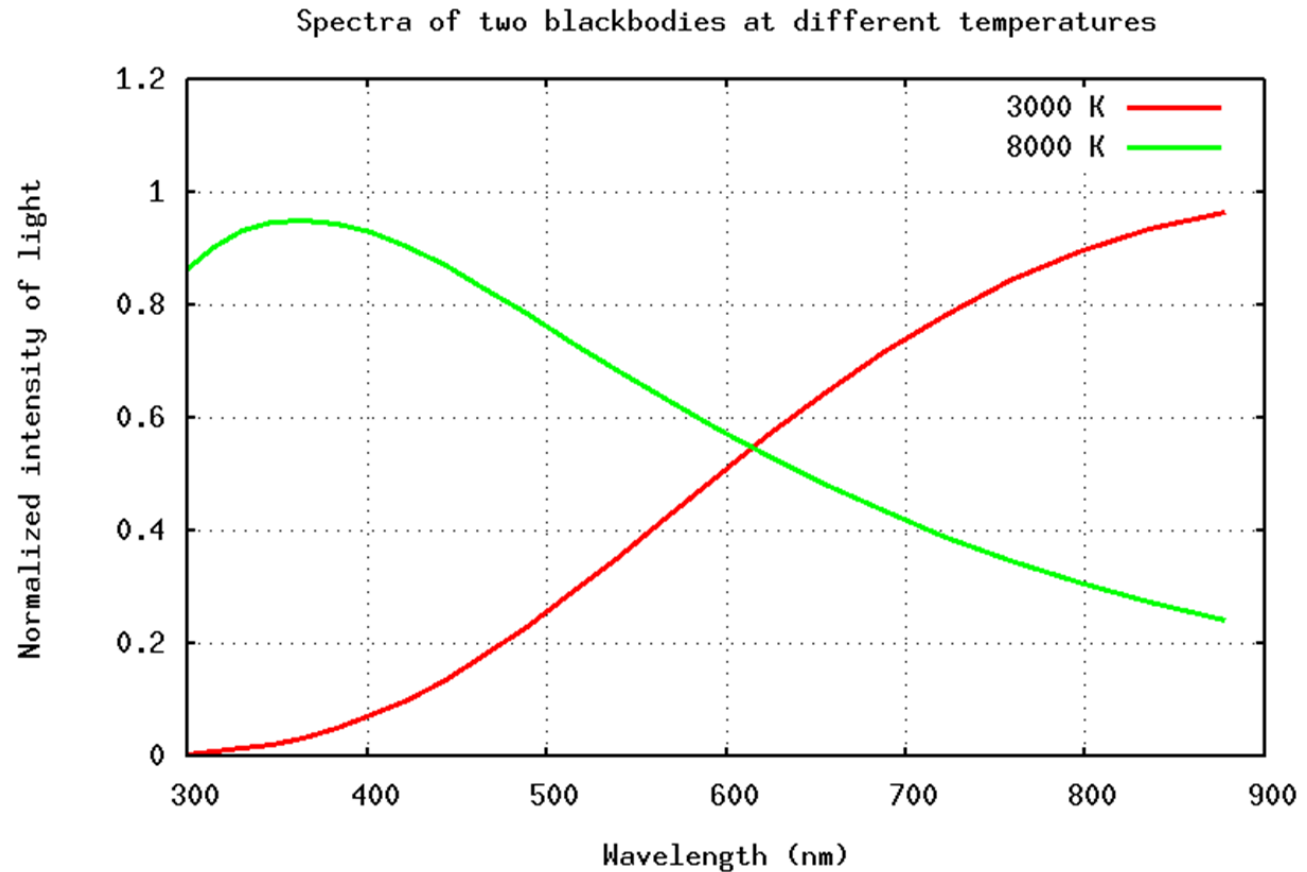
- Filter (photometric) systems:
 - ▣ Filters are used to restrict the wavelengths of electromagnetic radiation that hit the detector.
- Why may we want to do that?
 - ▣ Because stars have different colours that means they have different temperatures.

Observing through filters (1)

367

Hot objects emit most of their light at short wavelengths

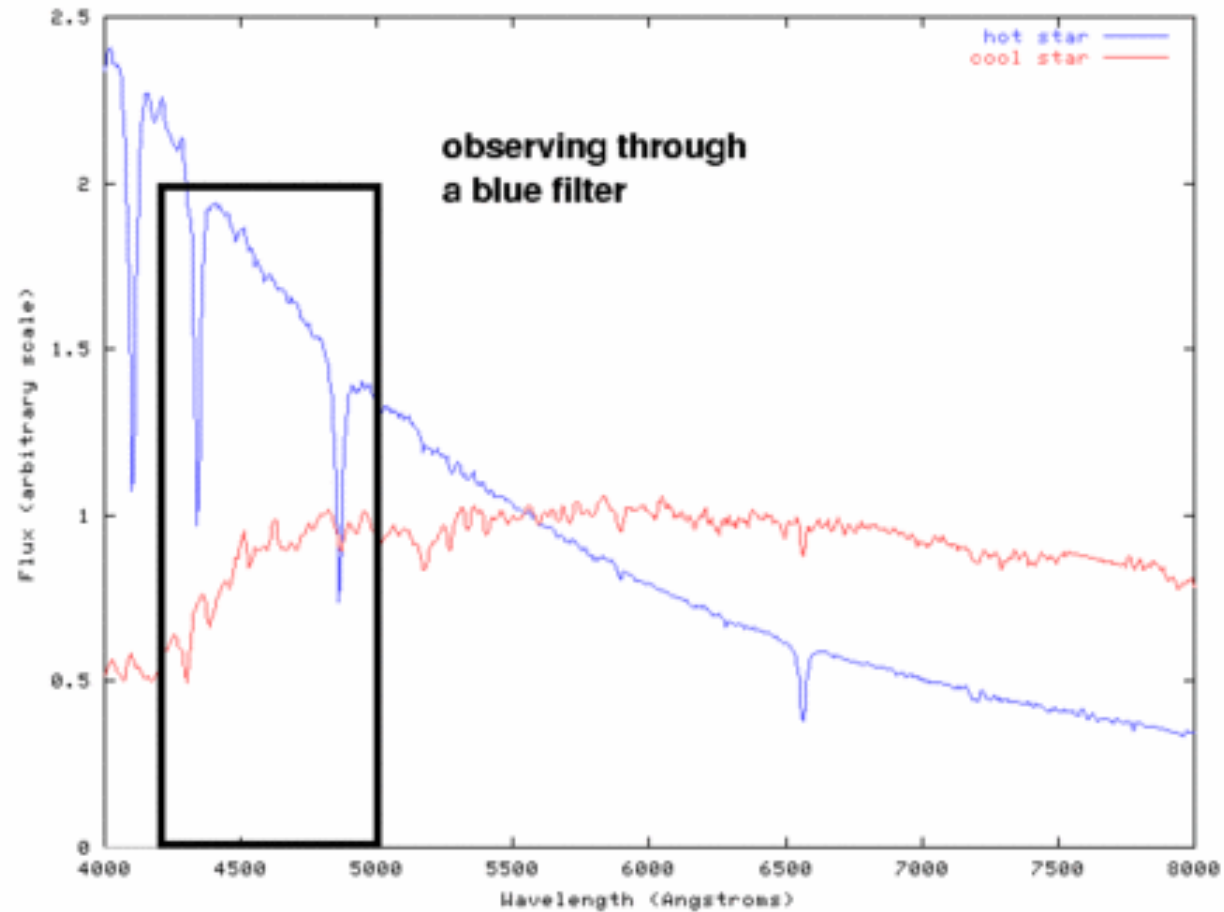
Cool objects emit most of their light at long wavelengths



Observing through filters (2)

368

Observing through filters allows us to estimate temperatures.



Observing through filters (3)

369

Which are the three brightest stars?

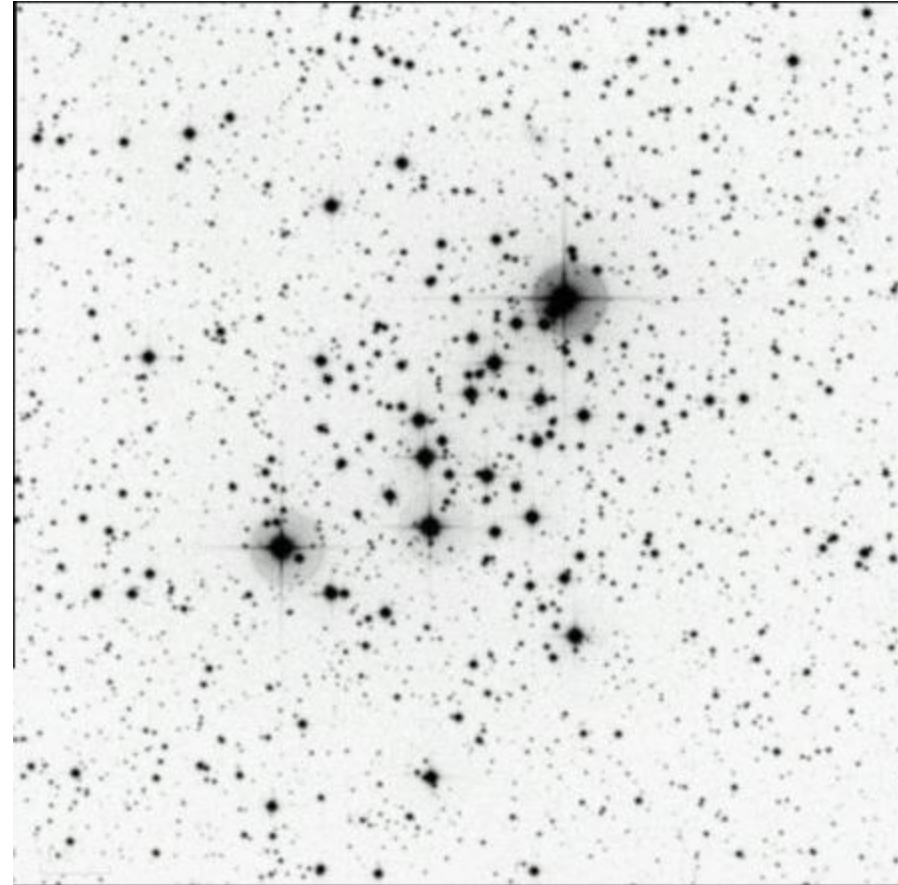


Observing through filters (4)

370

Which are the brightest stars?

It depends on
the bandpass through which
one observes them.



Photometric systems

371

- There is a number of different **photometric systems**, each one based on a particular passband (i.e. a particular combination of filter and detector and telescope).
- They may be grouped into wide, intermediate, and narrowband systems according to the bandwidth of their transmission curves. In the visible region:
 - ▣ Wide (broadband) filters have bandwidths of $\sim 1000 \text{ \AA}$
 - ▣ Intermediate: $100\text{-}500 \text{ \AA}$
 - ▣ Narrowband filters range from 0.5 to 100 \AA .
- One should **always remember** to specify the system when quoting the magnitude of a star.

Johnson-Cousins photometric system (UBVRI)

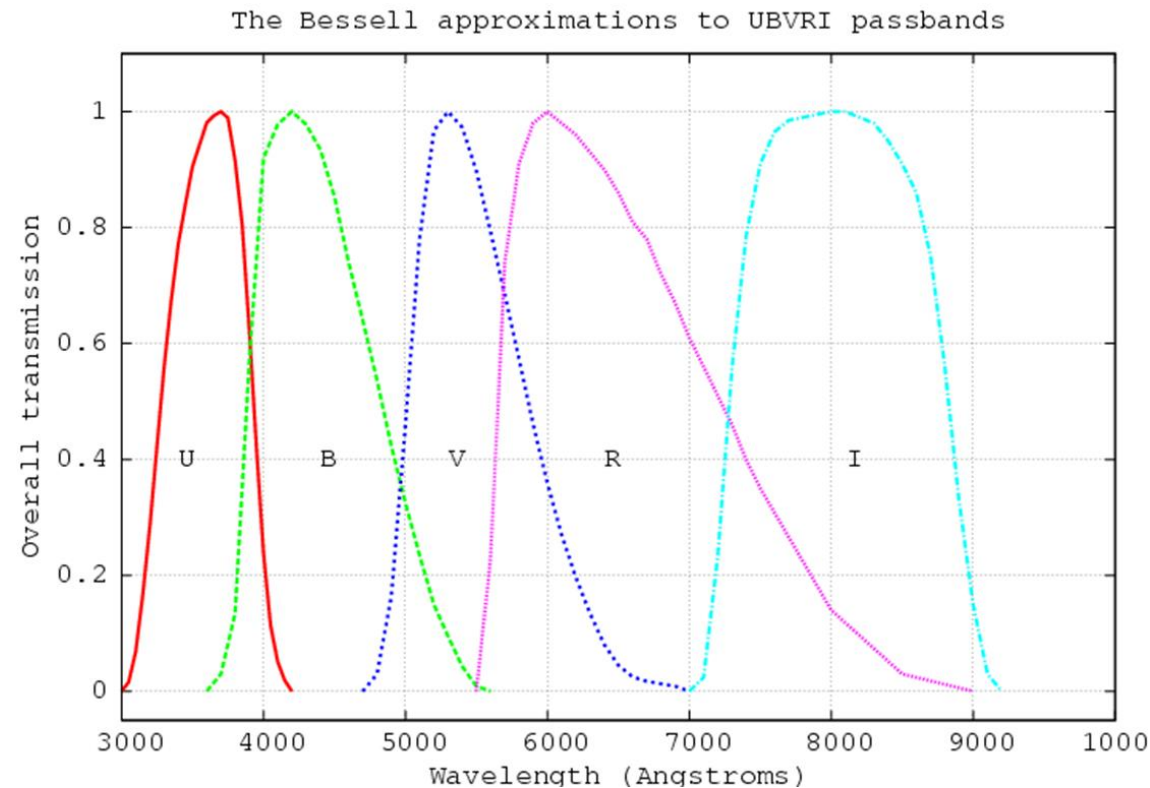
372

- Most astronomers working in the optical use **the Johnson-Cousins UBVRI** photometric systems:
 - Johnson and Morgan defines the **UBV** system with stars visible in the northern hemisphere
 - Cousins defines the redder **R** and **I** passbands.
- The systems are defined by particular combinations of glass filters and photomultiplier tubes (they were created many years ago before CCDs existed). Since photomultipliers and CCDs have very different spectral sensitivities, it is difficult to make the effective passband of a CCD-based instrument match that of a photomultiplier-based instrument.
- In 1990, Michael **Bessell** came up with a recipe for making filters out of common colored glasses which would reproduce pretty closely the official Johnson-Cousins **UBVRI** passbands → **Bessell filters**.

Johnson-Cousins photometric system

373

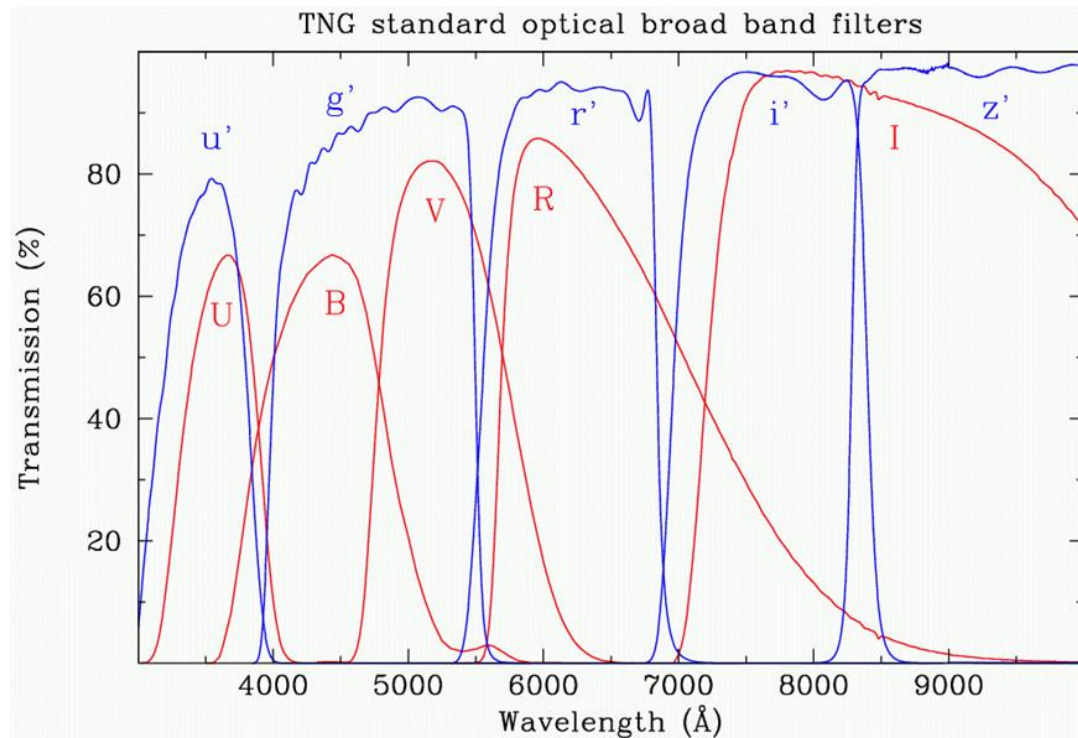
- The spectral resolution of the broadband UBVRI passbands is small:
 $R = \lambda/\Delta\lambda \approx 5$



SDSS (ugriz) photometric system

374

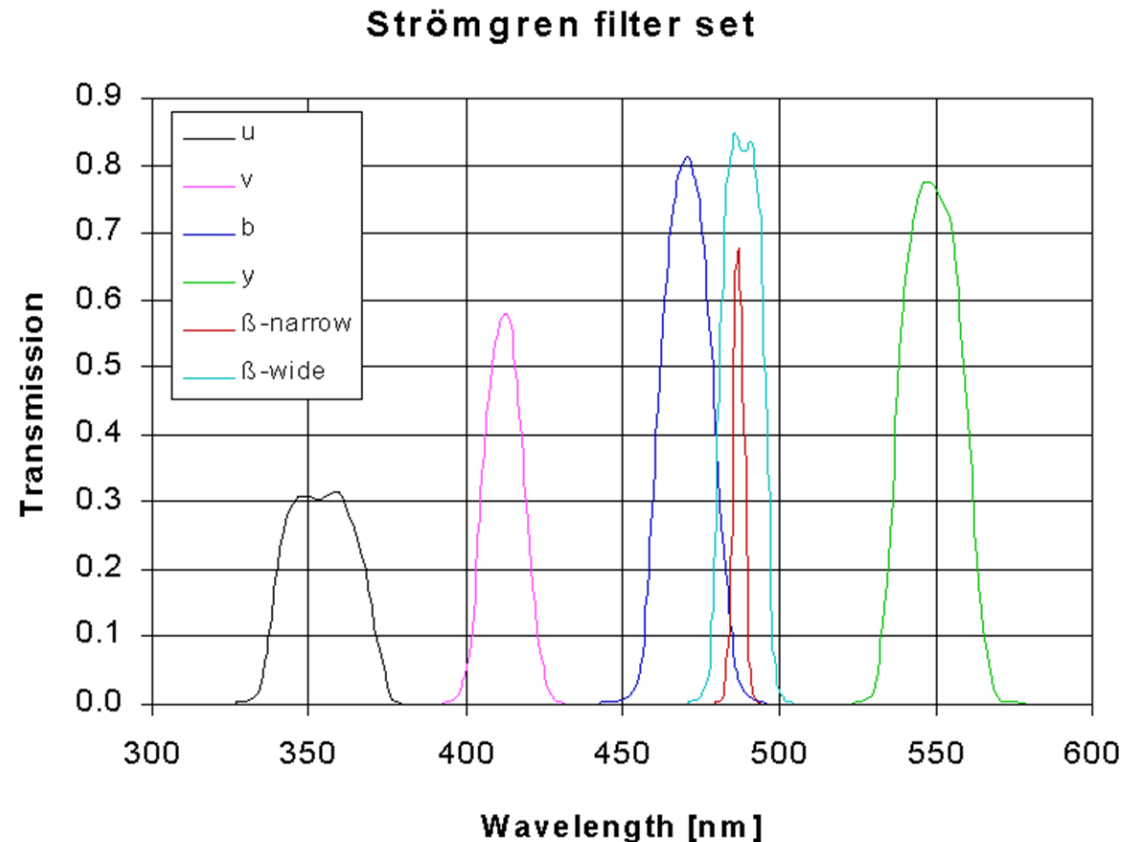
- Although **UBVRI** is the best known optical system, there are a number of others. Some were specifically designed to solve a particular astrophysical problem, others to mesh with particular detectors. One important system is the **u'g'r'i'z'** that is being used by the Sloan Digital Sky Survey (**SDSS**) and the Panoramic Survey Telescope and Rapid Response System (Pan-STARRS). It has become very popular recently.



Strömgren photometric system

375

- **Strömgren photometric system** (uvby) is four-colour intermediate-band photometric system (plus H β filters) for stellar classification. It was pioneered by the Danish astronomer Bengt Strömgren in 1956.



Narrowband photometric systems

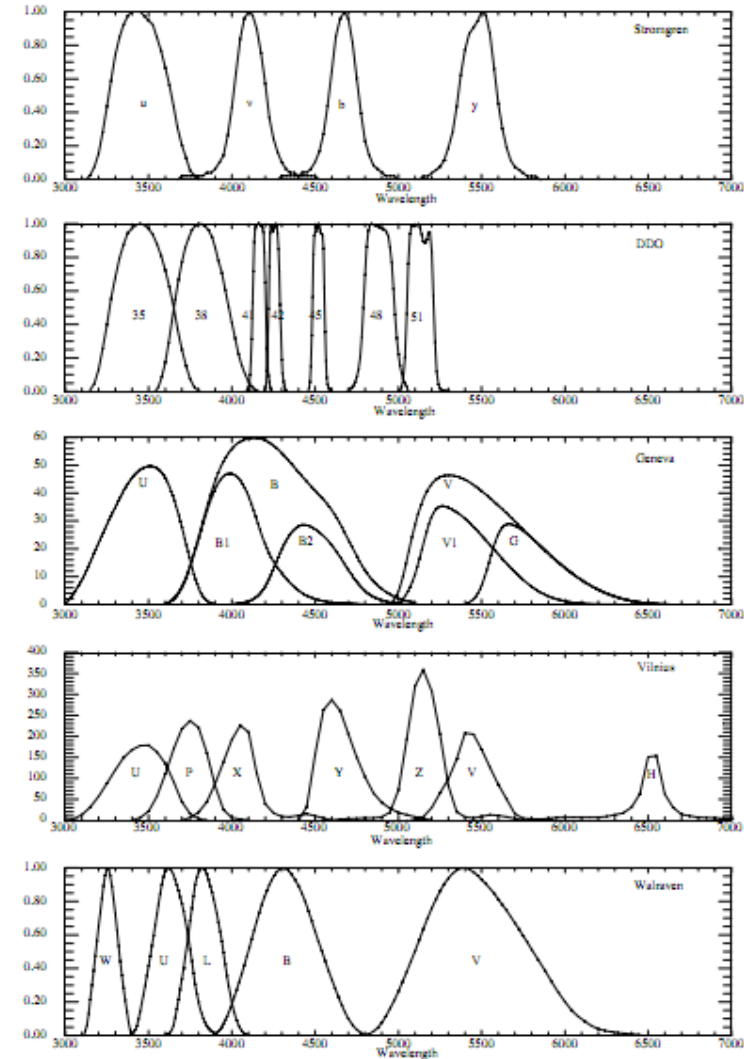
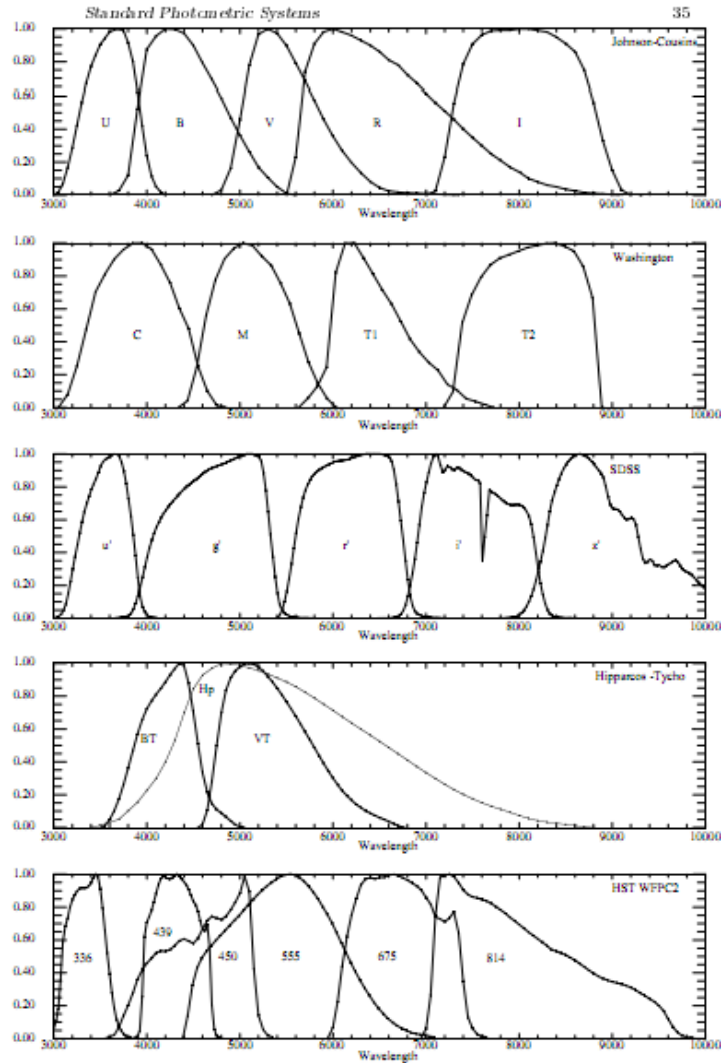
376

- For some applications, astronomers use **narrowband** filters; a common filter used to measure light emitted by hydrogen atoms is centered at 6563 Angstroms and roughly 20 Angstroms wide: $R = \lambda/\Delta\lambda \approx 330$
- A narrowband filter like this requires much longer exposure times to build up the same signal as a broadband filter. Since telescope time is so precious, astronomers tend to use **broadband** systems.

That's one reason for the popularity of the UBVRI or SDSS systems.

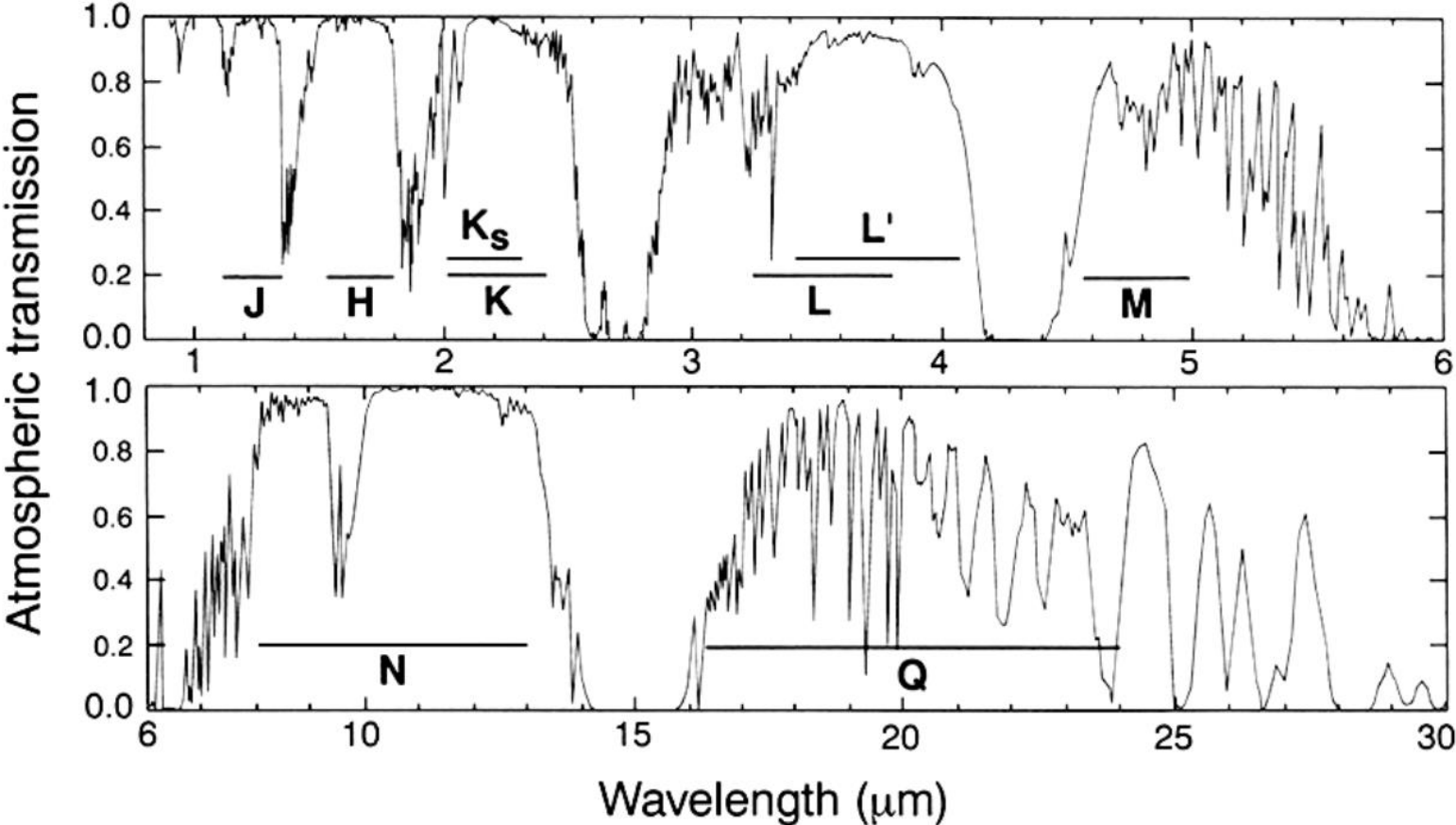
Photometric systems (optical)

377



The Infrared Photometric Bands: JHK+others

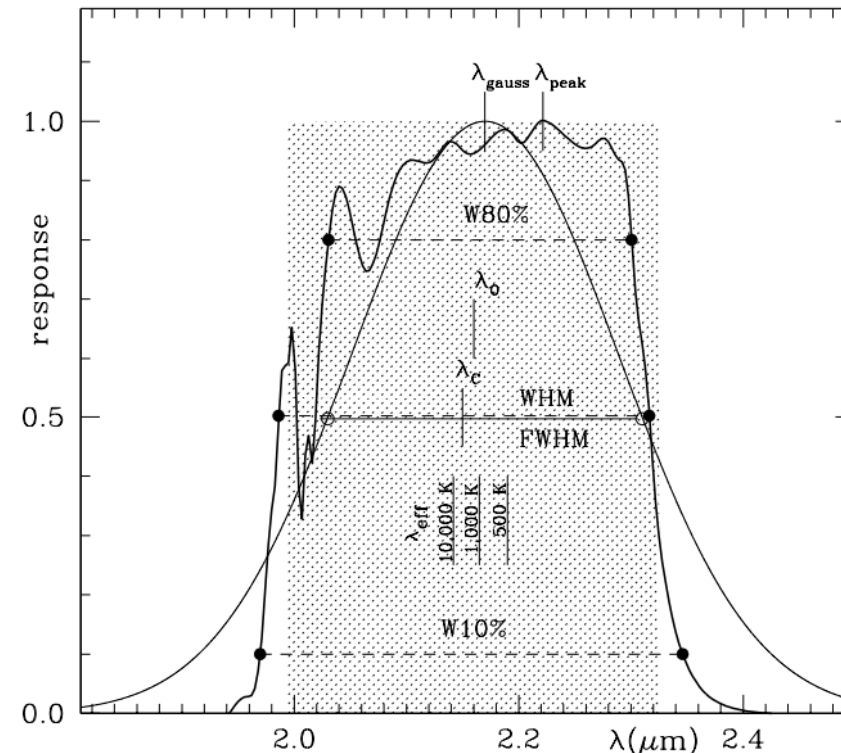
... where the atmospheric transmission windows are



Filter transmission curves (1)

379

- Typical broad-band transmission curves are **not** rectangular, and even **not** symmetric.
- Different quantities can be used to describe a filter, e.g.:
 - λ_c is the wavelength halfway between the points, where the band transmission profile reaches half of the maximum value.
 - WHM is the the full wavelength span between the points, where the band transmission profile reaches half of the maximum value.
 - λ_{peak} is the wavelength at which the band transmission profile reaches its maximum.



From Fiorucci and Munari, 2003, A&A, 401, 781

Filter transmission curves (2)

380

- Some important parameters depend on the source spectrum. For example,
 - λ_0 is the mean wavelength of the band, the property of just a band:

$$\lambda_0 = \frac{\int \lambda F(\lambda) d\lambda}{\int F(\lambda) d\lambda}.$$

- whereas the effective wavelength λ_{eff} is

$$\lambda_{\text{eff}} = \frac{\int \lambda F(\lambda) S(\lambda) d\lambda}{\int F(\lambda) S(\lambda) d\lambda}.$$

where

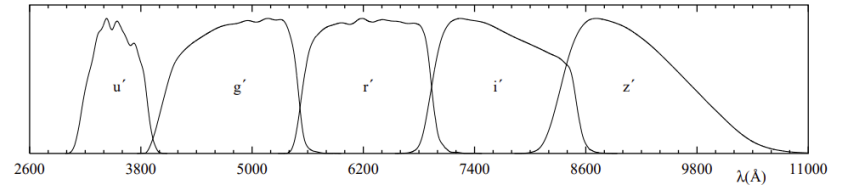
$F(\lambda)$ is the transmission profile of the band, and

$S(\lambda)$ the energy distribution of a source spectrum.

Filter transmission curves (3)

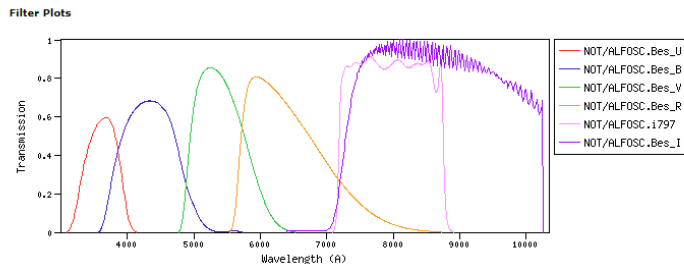
Good sources of info:

- *The Asiago Database on Photometric Systems* (218 systems; checked on 2023-09-20)
 - <http://ulisse.pd.astro.it/Astro/ADPS>
 - Fiorucci and Munari, 2003, A&A, 401, 781
- *Filter Profile Service* (10625 filters available on 2023-09-20)
 - <http://svo2.cab.inta-csic.es/theory/fps/>



Filter	λ_c	λ_0	λ_{peak}	λ_{gauss}	$B3$	Vega	Sun	K2	M2	Carbon
u'	3530	3521	3431	3519	3504	3551	3538	3593	3636	3525
WHM = 642	W10% = 831 W80% = 437 FWHM = 555				[599]	[602]	[565]	[517]	[448]	[498]
$W_c = 590$	$\frac{A(\lambda)}{A(V)}$ _{5.0} = 1.36	$\frac{A(\lambda)}{A(V)}$ _{3.1} = 1.61	$\frac{A(\lambda)}{A(V)}$ _{2.1} = 1.95	$\frac{A(\lambda)}{E(B-V)}$ = 2.038	$a = 0.934$	$b = 1.972$	<i>B3</i>			
$\mu = 201$	$a = 0.942$				$b = 1.985$				<i>Sun</i>	
$I_{asym} = 0.01$	$(4.946, 0.067)_{B3}^{r=0.99}$				$(5.271, 0.091)_{Sun}^{r=1.00}$		$(5.888, 0.075)_{M2}^{r=1.00}$			
$I_{kurt} = -0.88$	$\lambda_{eff} = 3521.5 + 41.9 \times E(B-V)$				$W_{eff} = 600.1 - 38.8 \times E(B-V)$				$r = -0.98$	
	$\lambda_{eff}(T) = 3472 + 145 \times \theta + 77 \times \theta^2 - 55 \times \theta^3$				$W_{eff}(T) = 556 + 263 \times \theta - 562 \times \theta^2 + 193 \times \theta^3$					
g'	4788	4803	5173	4820	4683	4708	4817	4903	5015	5100
WHM = 1411	W10% = 1641 W80% = 1111 FWHM = 1245				[1238]	[1271]	[1318]	[1230]	[1057]	[1004]
$W_c = 1325$	$\frac{A(\lambda)}{A(V)}$ _{5.0} = 1.12	$\frac{A(\lambda)}{A(V)}$ _{3.1} = 1.19	$\frac{A(\lambda)}{A(V)}$ _{2.1} = 1.28	$\frac{A(\lambda)}{E(B-V)}$ = 0.884	$a = 1.011$	$b = 0.656$	<i>B3</i>			
$\mu = 419$	$a = 1.015$				$b = 0.507$				<i>Sun</i>	
$I_{asym} = -0.12$	$(3.804, -0.006)_{B3}^{r=-0.78}$				$(3.949, 0.009)_{Sun}^{r=0.95}$		$(4.281, -0.000)_{M2}^{r=-0.08}$			
$I_{kurt} = -1.04$	$\lambda_{eff} = 4807.4 + 142.3 \times E(B-V)$				$W_{eff} = 1371.8 - 208.0 \times E(B-V)$				$r = -0.99$	
	$\lambda_{eff}(T) = 4647 + 312 \times \theta + 241 \times \theta^2 - 173 \times \theta^3$				$W_{eff}(T) = 1156 + 909 \times \theta - 1424 \times \theta^2 + 387 \times \theta^3$					
r'	6242	6253	6191	6247	6160	6168	6220	6256	6307	6365
WHM = 1387	W10% = 1565 W80% = 1248 FWHM = 1262				[1282]	[1294]	[1335]	[1341]	[1315]	[1251]
$W_c = 1343$	$\frac{A(\lambda)}{A(V)}$ _{5.0} = 0.88	$\frac{A(\lambda)}{A(V)}$ _{3.1} = 0.83	$\frac{A(\lambda)}{A(V)}$ _{2.1} = 0.77	$\frac{A(\lambda)}{E(B-V)}$ = 0.88	$a = 0.947$	$b = -0.205$	<i>B3</i>			
$\mu = 407$	$a = 0.933$				$b = -0.235$				<i>Sun</i>	
$I_{asym} = -0.07$	$(2.615, 0.020)_{B3}^{r=0.99}$				$(2.770, 0.028)_{Sun}^{r=1.00}$		$(3.099, 0.013)_{M2}^{r=0.98}$			
$I_{kurt} = -1.09$	$\lambda_{eff} = 6253.4 + 91.0 \times E(B-V)$				$W_{eff} = 1370.2 - 106.1 \times E(B-V)$				$r = -0.98$	
	$\lambda_{eff}(T) = 6145 + 139 \times \theta + 156 \times \theta^2 - 80 \times \theta^3$				$W_{eff}(T) = 1255 + 289 \times \theta - 183 \times \theta^2 - 109 \times \theta^3$					
i'	7704	7667	7242	7635	7573	7584	7620	7649	7732	7653
WHM = 1532	W10% = 1756 W80% = 1005 FWHM = 1291				[1322]	[1335]	[1359]	[1369]	[1340]	[1453]
$W_c = 1374$	$\frac{A(\lambda)}{A(V)}$ _{5.0} = 0.66	$\frac{A(\lambda)}{A(V)}$ _{3.1} = 0.61	$\frac{A(\lambda)}{A(V)}$ _{2.1} = 0.54	$\frac{A(\lambda)}{E(B-V)}$ = 0.66	$a = 0.818$	$b = -0.487$	<i>B3</i>			
$\mu = 453$	$a = 0.814$				$b = -0.497$				<i>Sun</i>	
$I_{asym} = 0.14$	$(1.906, 0.020)_{B3}^{r=0.99}$				$(2.028, 0.026)_{Sun}^{r=1.00}$		$(2.260, 0.016)_{M2}^{r=0.99}$			
$I_{kurt} = -1.05$	$\lambda_{eff} = 7666.9 + 78.9 \times E(B-V)$				$W_{eff} = 1391.2 - 65.7 \times E(B-V)$				$r = -0.97$	
	$\lambda_{eff}(T) = 7562 + 101 \times \theta + 123 \times \theta^2 - 52 \times \theta^3$				$W_{eff}(T) = 1310 + 144 \times \theta - 9 \times \theta^2 - 104 \times \theta^3$					
z'	9038	9115	8717	9018	9022	9046	9057	9086	9136	9076
WHM = 1408	W10% = 2212 W80% = 845 FWHM = 1326				[1379]	[1390]	[1400]	[1412]	[1410]	[1562]
$W_c = 1411$	$\frac{A(\lambda)}{A(V)}$ _{5.0} = 0.48	$\frac{A(\lambda)}{A(V)}$ _{3.1} = 0.45	$\frac{A(\lambda)}{A(V)}$ _{2.1} = 0.42	$\frac{A(\lambda)}{E(B-V)}$ = 0.48	$a = 0.677$	$b = -0.622$	<i>B3</i>			
$\mu = 536$	$a = 0.673$				$b = -0.611$				<i>Sun</i>	
$I_{asym} = 0.48$	$(1.417, 0.018)_{B3}^{r=0.99}$				$(1.512, 0.023)_{Sun}^{r=1.00}$		$(1.702, 0.015)_{M2}^{r=0.99}$			
$I_{kurt} = -0.39$	$\lambda_{eff} = 9113.4 + 73.5 \times E(B-V)$				$W_{eff} = 1422.0 - 42.8 \times E(B-V)$				$r = -0.97$	
	$\lambda_{eff}(T) = 8997 + 88 \times \theta + 105 \times \theta^2 - 36 \times \theta^3$				$W_{eff}(T) = 1357 + 91 \times \theta + 24 \times \theta^2 - 76 \times \theta^3$					

Filter ID	λ_{ref}	λ_{mean}	λ_{eff}	λ_{min}	λ_{max}	W_{eff}	ZP _v	ZP _A	Obs. Facility	Instrument	Description
NOT/ALFOSC.Bes_U	3600.85	3617.41	3670.73	3102.79	4129.62	580.28	1758.31	4.07e-9	NOT	ALFOSC	Bessell U
NOT/ALFOSC.Bes_B	4306.12	4346.66	4319.73	3579.81	5682.69	1004.43	3923.93	6.34e-9	NOT	ALFOSC	Bessell B
NOT/ALFOSC.Bes_V	5389.63	5417.18	5365.72	4786.00	6447.52	885.24	3670.94	3.79e-9	NOT	ALFOSC	Bessell V
NOT/ALFOSC.Bes_R	6396.64	6464.12	6329.59	5551.69	8522.76	1279.53	3085.76	2.25e-9	NOT	ALFOSC	Bessell R
NOT/ALFOSC.I797	7927.59	7966.41	7886.27	7103.45	8872.45	1499.39	2435.41	1.16e-9	NOT	ALFOSC	interference i
NOT/ALFOSC.Bes_I	8559.60	8682.26	8466.07	6392.60	10246.30	2578.97	2338.38	9.57e-10	NOT	ALFOSC	Bessell I.



Magnitudes & Photometric systems

382

- When writing the magnitude of a star, astronomers use an abbreviation to denote the photometric system of the measurement:
 - $V = 1.03$ (or $1.03V$) means “magnitude of this star in the V system is 1.03”
 - $B = 0.46$ (or $0.46B$) means “magnitude of this star in the B system is 0.46”

But a magnitude system can be different!

Magnitude systems

383

$$m_1 - m_2 = -2.5 \log \frac{F}{F_0}$$

- The flux F_0 defines the reference or **zeropoint** of the magnitude scale. The choice is **arbitrary**.
- Standardizing magnitudes (magnitude systems):
 - **Vega** system
 - **AB** system
 - **ST** Magnitudes

A magnitude system is **not** a photometric (filter) system
(you can use a filter in any system)

Photometry: Vega system

384

- Astronomers have chosen to use the bright star **Vega** (α Lyr) as their starting point.
- In the UBVRI systems, the star Vega is **defined** to have a magnitude of zero in all bands (**actually, this is not quite true**):

$$U = 0.0; B = 0.0; V = 0.0; R = 0.0; I = 0.0$$

- This means also that **all** the colours of Vega are **zero**.
- The zero-point of this system depends on the flux of Vega (outside the atmosphere) and is **different** in different bands.

Photometry: AB system

385

- In the **AB system**, which is not based on Vega, it is assumed that the flux constant F_0 is the **same** for all wavelengths and passbands.
- That constant is per definition such that in the V filter: $m_V^{Vega} = m_V^{AB} = 0$
(or more accurately: $F_\lambda dv \equiv F_\lambda d\lambda$ when averaged over the V filter, or at the effective wavelength of the V filter, $\lambda_{\text{eff}} = 5480 \text{ \AA}$. Based on the work of Oke (1974), then

$$m_v = -2.5 \log F_v - (48.585 \pm 0.005)$$

where $F_v(\lambda)$ is the spectral flux density per unit frequency of a source at the top of the Earth's atmosphere in units of $\text{erg s}^{-1} \text{cm}^{-2} \text{Hz}^{-1}$.

- **Note that the AB magnitude system is expressed in c rather than F_λ !**

The flux density in F_v is related to the flux density in F_λ by:

$$F_v [\text{ergs s}^{-1} \text{cm}^{-2} \text{Hz}^{-1}] = 10^{-8} \frac{\lambda [\text{\AA}]^2}{c [\text{cm s}^{-1}]} F_\lambda [\text{ergs s}^{-1} \text{cm}^{-2} \text{\AA}^{-1}]$$

- One can easily convert between AB magnitudes and Janskys:
In AB magnitudes, mag 0 has a flux of **3631 Jy**.

AB and VEGA systems compared

386

- The difference between AB and VEGA magnitudes becomes very large at redder wavelengths!
- The spectrum of Vega is very complicated at IR wavelengths and often model atmospheres are used adding to uncertainties

