Comparison of induced and spontaneous emission

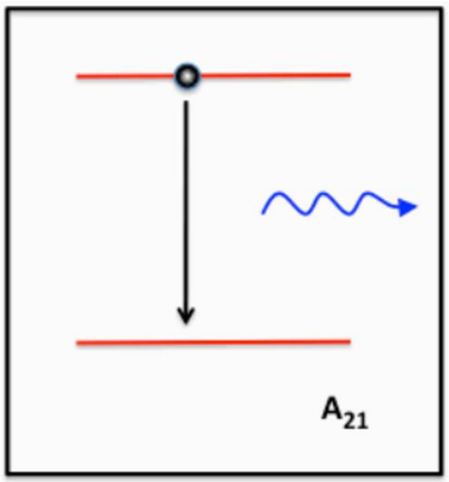
There was a home work:

- When (at what temperatures, wavelengths) is spontaneous or induced emission stronger?

  Assume LTE (blackbody)

Spontaneous & Stimulated emission

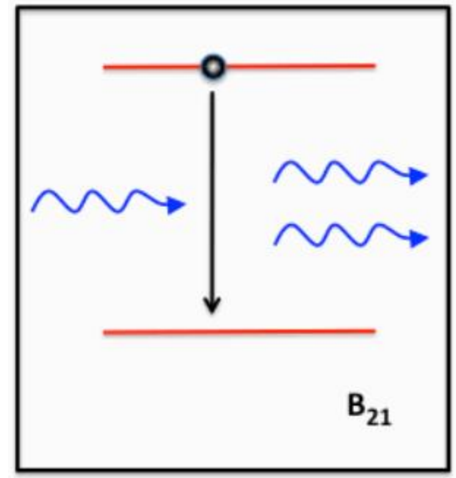
Spontaneous emission



Spontaneous emission

- The system goes from an upper level *u* to a lower level *l* spontaneously.
- Occurs independently of the radiation field.
- Emits **isotropically**.

Stimulated emission



Stimulated emission

- The system goes from an upper level *u* to a lower level *l* stimulated by the presence of a radiation field ($h\nu$ corresponding to the energy difference between levels *u* and *l*).
- Stimulated emission occurs into the **same** state (frequency, direction, polarization) as the photon that stimulated the emission.

Relation between Einstein coefficients

$$\frac{A_{21}}{B_{21}} = \frac{2h\nu_{21}^3}{c^2} \rightarrow A_{21} = B_{21}\frac{2h\nu_{21}^3}{c^2}$$

$$\frac{g_1 B_{12}}{g_2 B_{21}} = 1 \rightarrow g_1 B_{12} = g_2 B_{21}$$

Einstein's coefficients concern the probability that a particle spontaneously emits a photon, the probability to absorb a photon, and the probability to emit a photon under the influence of another incoming photon.
Einstein's coefficients are valid for all radiation fields.

Induced and Spontaneous emission

When is spontaneous emission stronger?

Total amount of emitted photons per unit time at a given frequency is
Spontaneous emission: $\eta_{sp}=n_2 A_{21}$
Stimulated emission: $\eta_{st}=n_2 B_{21} J_\nu$

$$A_{21} = B_{21}\frac{2h\nu_{21}^3}{c^2}$$

$$g_1 B_{12} = g_2 B_{21}$$

$$\frac{\eta_{sp}}{\eta_{st}} = \frac{n_2 A_{21}}{n_2 B_{21} J} = \frac{2h\nu_{21}^3}{c^2 J}$$

TE: blackbody, J=$B_\nu(T)$

$$B_\nu(T) = \frac{2h\nu_{21}^3}{c^2}\left(e^{\frac{h\nu_{21}}{kT}} - 1\right)^{-1}$$

$$\frac{\eta_{sp}}{\eta_{st}} = e^{\frac{h\nu_{21}}{kT}} - 1$$

$$e^{\frac{h\nu_{21}}{kT}} \geq 2 \quad \Rightarrow \quad h\nu_{21} \geq kT\ln 2 \quad \Rightarrow \quad \lambda_* \leq \frac{hc}{kT\ln 2} = \frac{2.076\times 10^8}{T}\text{Å}$$

At wavelengths shorter than λ_* spontaneous emission is dominant

T=5777K → λ_*≈41000 Å　　λ_* =6563 Å → T≈31600K　　λ_* =4340Å → T≈48000K

Spectral line formation

EINSTEIN COEFFICIENTS
LINE PROFILES: NATURAL BROADENING
BROADENING OF SPECTRAL LINES
NATURAL LINE BROADENING:
THERMAL (DOPPLER) BROADENING
CONVOLUTION OF DIFFERENT BROADENING PROCESSES
PRESSURE BROADENING
INGIS-TELLER RELATION
ROTATIONAL AND INSTRUMENTAL BROADENING

Natural and Thermal Broadenings

From above:

- Natural Line Broadening: $$\varphi_\nu = \frac{\Gamma/4\pi}{(\nu-\nu_0)^2 + (\Gamma/4\pi)^2} \qquad \Gamma = \sum_{i<j} A_{ji}$$

Lorentzian profile with FWHM $$\Delta\lambda_{1/2} = \frac{\lambda_0^2}{c}\Delta\nu_{1/2} = \frac{\lambda_0^2}{c}\frac{\Gamma}{2\pi} \approx f_{ij} \times 7 \times 10^{-4}\,\text{Å}$$

- Doppler broadening $$\varphi(\nu) = \frac{1}{\sqrt{\pi}\Delta\nu_D} e^{-(\nu-\nu_0)^2/\Delta\nu_D^2} \qquad \Delta\nu_D = \frac{\nu_0}{c}u_{th} = \frac{\nu_0}{c}\sqrt{\frac{2kT}{m}}$$

Gaussian line profile with FWHM $$\Delta\lambda_{1/2} = \frac{\lambda_0^2}{c}\Delta\nu_{1/2} = 7.1 \times 10^{-7}\lambda_0(\text{Å})\sqrt{(T/\mu)}\ \text{Å}$$

$$\Delta\nu_{1/2} = 1.67\Delta\nu_D$$

Comparing broadenings

- Thermal (Doppler):
  - $\Delta\lambda_{th}$=0.02 Å (at λ_0=5000 Å, T=6000K, Fe)
  - $\Delta\lambda_{th}$=0.5 Å (at λ_0=5000 Å, T=50000K, H)
- Radiation damping:
  - $\Delta\lambda_{FWHM}$=a few × 10^{-4} Å

- **But:** decline of Gauss profile in wings is much steeper than for Lorentz profile:

$$\text{Gauss } (10\Delta\lambda_{th}) \quad : \quad e^{-10^2} \approx 10^{-43}$$

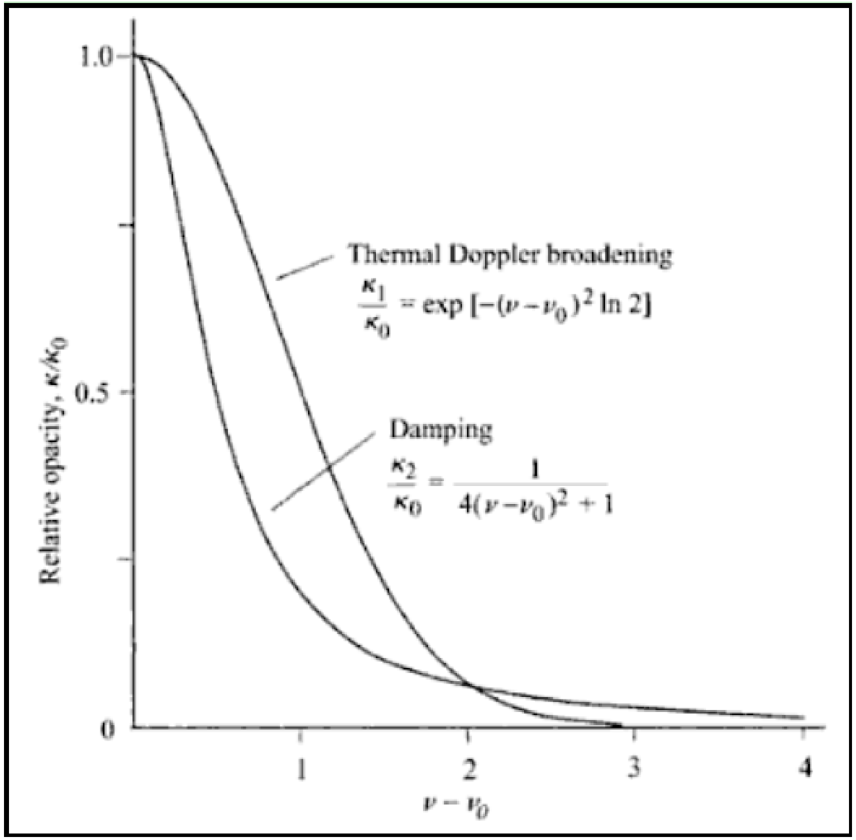
$$\approx$$

$$\text{Lorentz } (1000\Delta\lambda_{rad}) : \ 1/1000^2 \approx 10^{-6}$$

- In the line **wings** the **Lorentz** profile is **dominant**

Broadening mechanisms profiles

- Different broadening mechanisms have the form of
  - A Lorentzian function (natural profile and broadening, some pressure brodenings)
  - A Gaussian function (thermal broadening, instrumental broadening, etc.)
  - Other functions are possible (e.g., Linear Stark broadening)
- Generally, we have to consider both (all) types of profiles. For example, the pressure damping profile is negligible in the line core, but the Doppler profile decreases very steeply in the wings, whilst the damping profile decreases only as $1/\Delta\lambda^2$
- The Gaussian dominates the line core (or is confined to it), while the Lorentzian profile dominates in the line wings out to several times the FWHM.



Joint effect of different mechanisms

Mathematically: **convolution**

$$(f_A * f_B)(x) = \int_{-\infty}^{\infty} f_A(y) f_B(x-y) dy$$

Properties:

- commutative: $$f_A * f_B = f_B * f_A$$
- Fourier transformation: $$F(f_A * f_B) = normfactor \cdot F(f_A) \cdot F(f_B)$$
  where F denotes the Fourier transform of f.

i.e., in Fourier space the convolution is a multiplication

Application to profile functions

Convolution of two Gaussian profiles

$$G_A(x) = \frac{1}{A\sqrt{\pi}}\ e^{-\frac{x^2}{A^2}} \qquad G_B(x) = \frac{1}{B\sqrt{\pi}}\ e^{-\frac{x^2}{B^2}}$$

$$G_C(x) = G_A(x) * G_B(x) = \frac{1}{C\sqrt{\pi}}\ e^{-\frac{x^2}{C^2}} \quad \text{with} \quad C^2 = A^2 + B^2$$

Result: Gauss profile with **quadratic summation** of half-widths.

Convolution of two Lorentzian profiles (e.g., radiation + collisional damping)

$$L_A(x) = \frac{A/\pi}{x^2 + A^2} \quad L_B(x) = \frac{B/\pi}{x^2 + B^2}$$

$$L_C(x) = L_A(x) * L_B(x) = \frac{C/\pi}{x^2 + C^2} \quad \text{with} \quad C = A + B$$

Result: Lorentz profile with **sum** of half-widths

Voigt profile

Convolving Gauss and Lorentz profile (e.g. thermal + natural broadening)

$$G(\nu) = \frac{1}{\Delta\nu_D\sqrt{\pi}}\, e^{-\frac{(\nu-\nu_0)^2}{\Delta\nu_D{}^2}} \qquad L(\nu) = \frac{\gamma/4\pi^2}{(\nu-\nu_0)^2 + (\gamma/4\pi)^2}$$

$V = G * L$ depends on $\nu, \Delta\nu, \gamma, \Delta\nu_D$: $V(\nu) = \int_{-\infty}^{\infty} G(\nu')\, L(\nu - \nu')\, d\nu'$

Transformation: $\mathrm{v} := \frac{(\nu - \nu_0)}{\Delta\nu_D}$ $\quad a := \gamma/(4\pi\Delta\nu_D)$ $\quad y := \frac{(\nu' - \nu_0)}{\Delta\nu_D}$

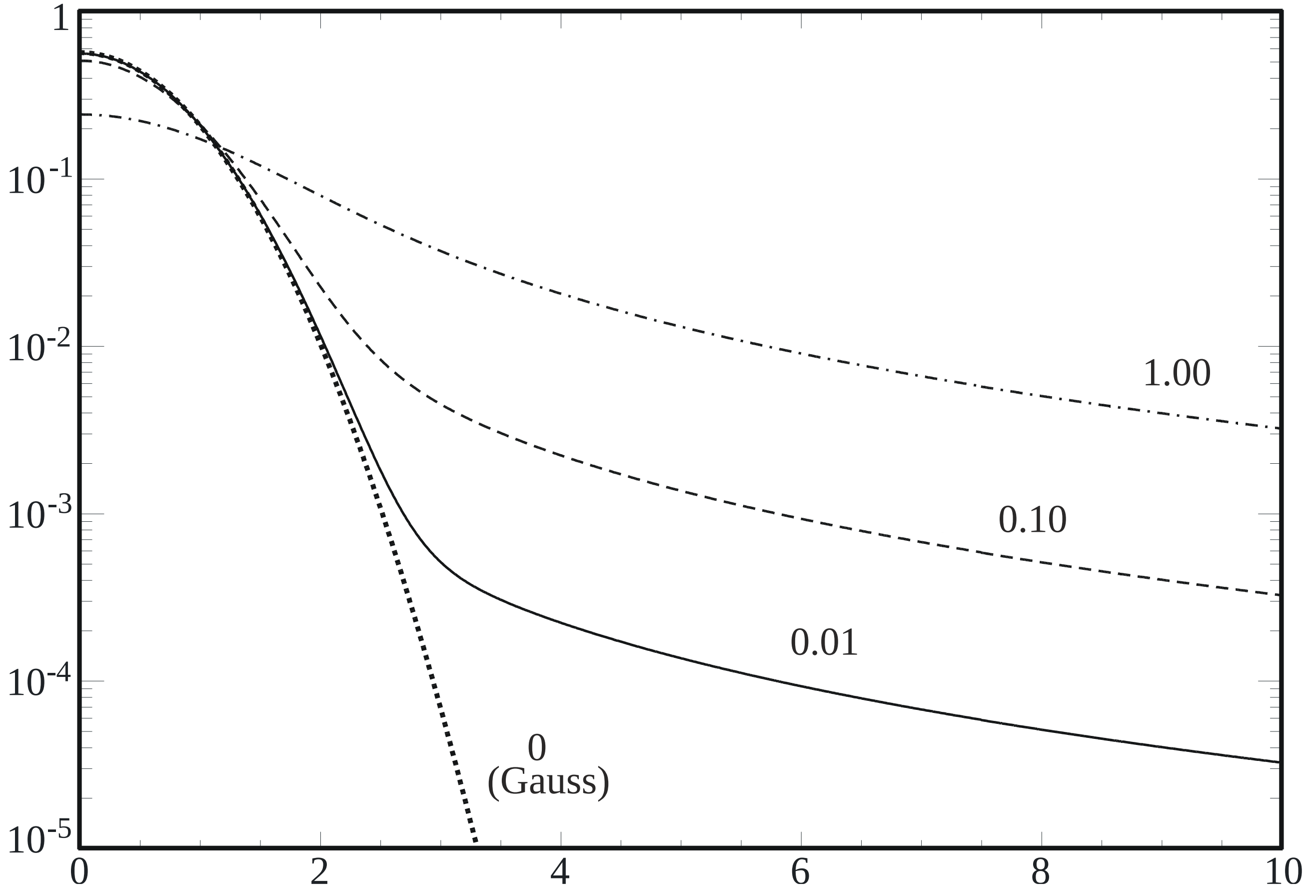
$$G(y) = \frac{1}{\Delta\nu_D\sqrt{\pi}}\, e^{-y^2} \qquad L(y) = \frac{a/\Delta\nu_D\pi}{y^2 + a^2} \qquad V = \frac{1}{\Delta\nu_D\sqrt{\pi}}\frac{a}{\pi}\int_{-\infty}^{\infty}\frac{e^{-y^2}}{(\mathrm{v}-y)^2 + a^2}\, dy$$

Def: $V = \frac{1}{\Delta\nu_D\sqrt{\pi}} H(a, \mathrm{v})$ with $H(a,\mathrm{v}) = \frac{a}{\pi}\int_{-\infty}^{\infty}\frac{e^{-y^2}}{(\mathrm{v}-y)^2 + a^2}\, dy$

Voigt function, no analytical representation possible.
(approximate formulae or numerical evaluation)

Normalization: $\int_{-\infty}^{\infty} H(a,\mathrm{v})\, d\mathrm{v} = \sqrt{\pi}$

The Voigt func for various a (1)

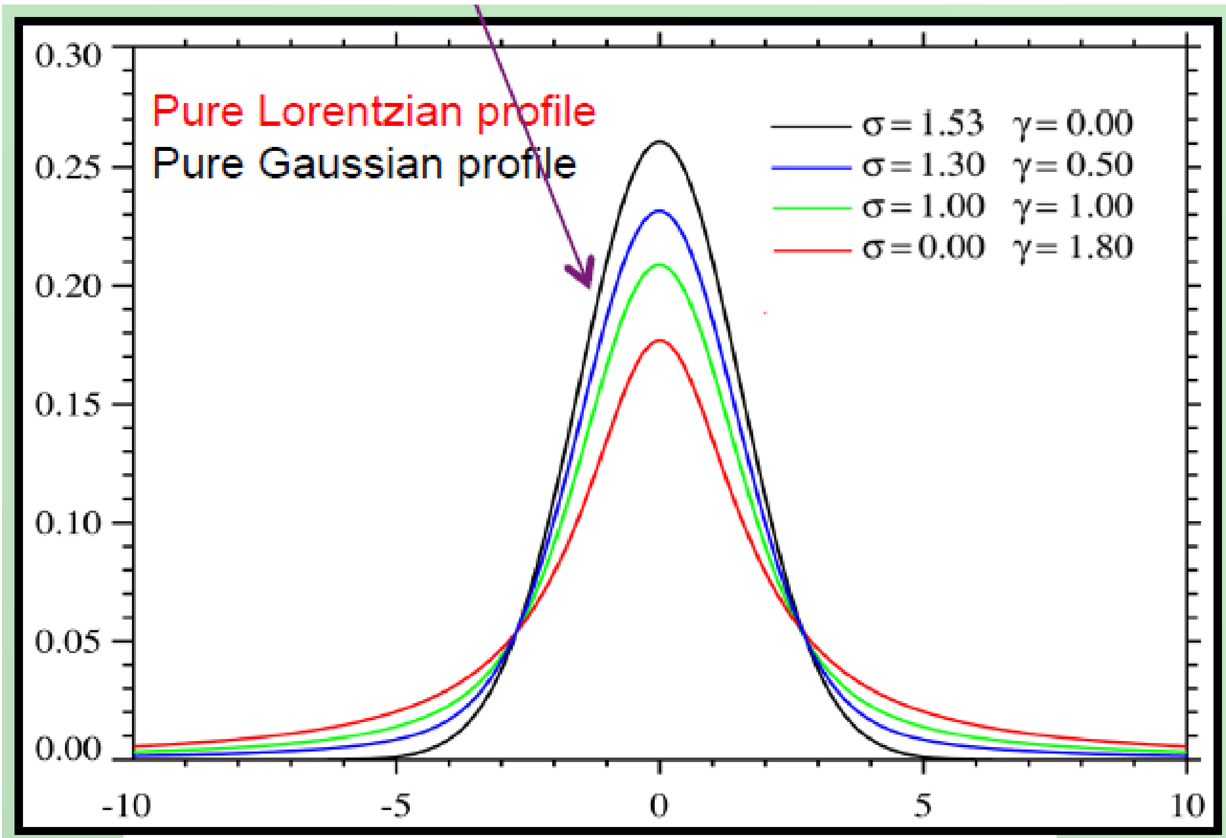


The final form of the combined **Voigt** profile depends on $\alpha = 2\pi a = \gamma/2\Delta\nu_D$, the ratio of the damping widths $\gamma/2$ to the Doppler width $\Delta\nu_D$

As **a rule of thumb**, the damping wings start to contribute a distance $-(\log \alpha)\Delta\lambda_D$ from the line centre

The Voigt func for various *a* (2)

Voigt profiles



The final form of the combined **Voigt** profile depends on $\alpha = 2\pi a = \gamma/2\Delta\nu_D$, the ratio of the damping widths $\gamma/2$ to the Doppler width $\Delta\nu_D$

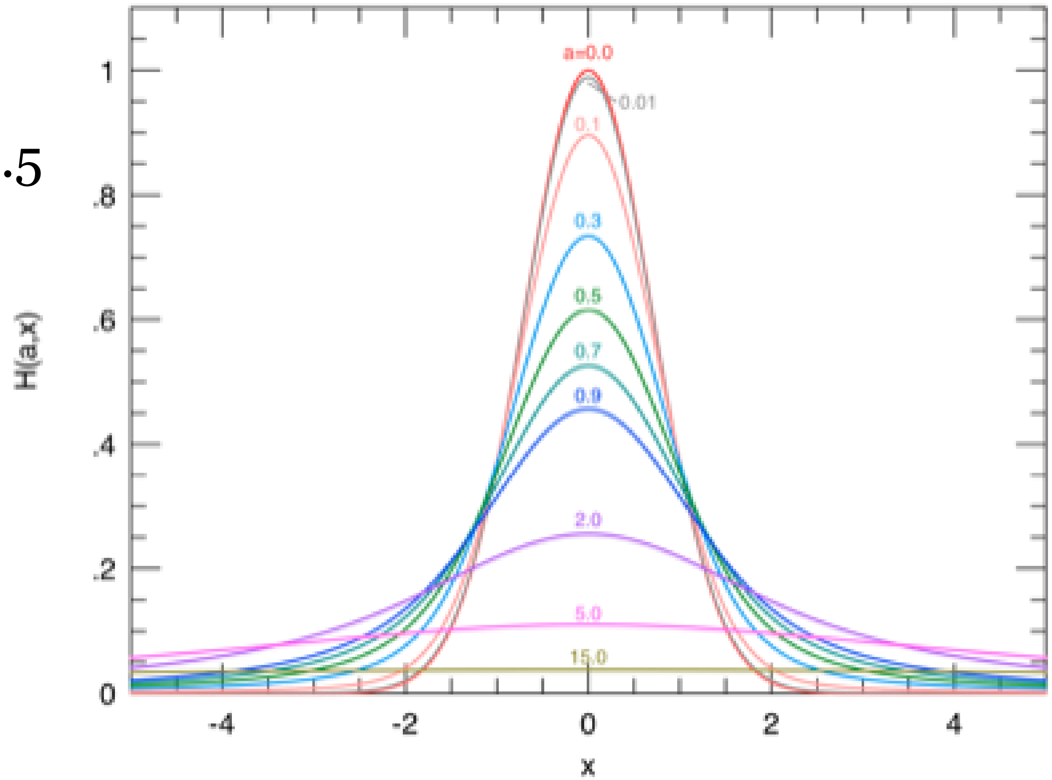
As **a rule of thumb**, the damping wings start to contribute a distance $-(\log \alpha)\Delta\lambda_D$ from the line centre

Calculation of a Voigt profile

No analytical representation is possible, but…

- IDL:
  IDL> u=findgen(201)/40.-2.5
  IDL> v=voigt(0.5,u)
  IDL> plot,u,v

- Python



Spectral line formation

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Other broadening mechanisms

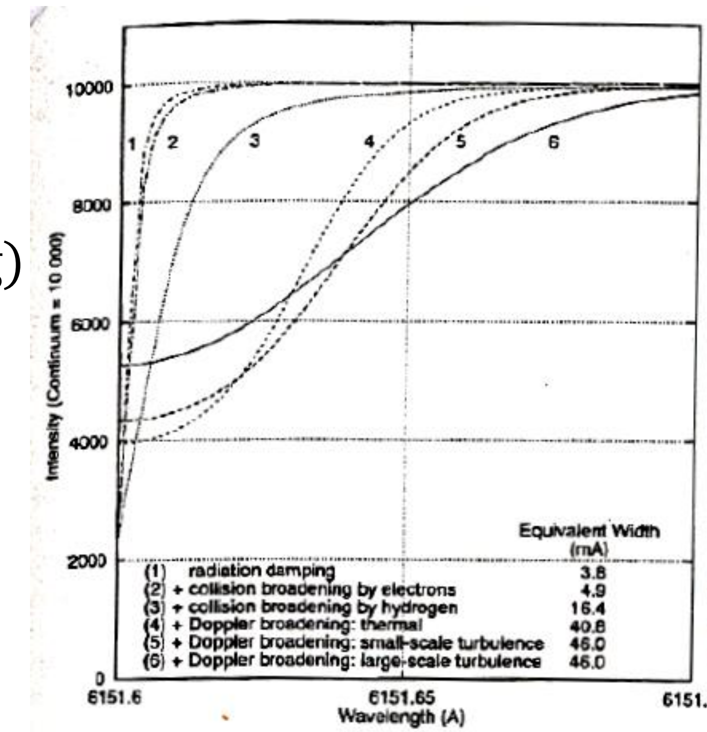
There are numerous broadening mechanisms which influence the apparent shape of spectral lines:

microscopic

1. Natural broadening √
2. Thermal broadening √
3. Microturbulence √
   (treated like extra thermal broadening)
4. Collisions (important for strong lines)
5. Isotopic shift, *hfs*, Zeeman effect

macro

6. Macroturbulence
7. Rotation
8. Instrumental broadening



Holweger 1996

Collisional and Pressure broadening

- The orbitals of an atom can be perturbed in a collision with a neutral atom (collisional broadening) or encounter with the electric field of an ion (pressure broadening).

Direct collisions?

- Collisions in the gas de-excite atoms before they naturally decay, shortening its lifetime.
- The resulting line profile is Lorenzian (as with natural broadening) with a width of $\nu_{1/2}=1/(\pi t)$ where t is the time between collisions.
- The number of collisions (per second) is the number of perturbers in the volume swept out by the atom, i.e. $N\sigma v$. Since $\frac{1}{2}mv^2=3/2\ kT$, the time between collisions is

$$t \approx 1/(N\sigma\sqrt{3kT/m})$$

- So, the FWHM in terms of pressure ($P=NkT$) is:

$$\Delta\nu_{1/2}(\mathrm{Hz}) = P\sigma/\pi\sqrt{3/kTm} = 3.6\times10^{19}P\sigma/\sqrt{mT/m_H}$$

- For the Sun (T=5800K, P=10^5 dyne/cm^2),
H atom *direct* collisions ($\sigma=\pi a_0^2$=8x10^{-17} cm^2) cause $\Delta\nu_{1/2} = 4\ \mathrm{MHz}$
i.e. **less than the natural width**

$$\Delta\lambda_{1/2} = \frac{\lambda_0^2}{c}\Delta\nu_{1/2} = 5\times10^{-5}\ \text{Å}$$

Impact broadening

- Nevertheless, the impact approximation can be used for some broadening mechanisms, which are important since atoms can interact without direct collision.

- The change in energy induced by the collision is a function of the separation $\boldsymbol{r}$ between the absorber and perturbing particle, and can be approximated by a power law of the form $\Delta E \sim \text{Constant} \times r^{-n}$ where n is an integer, such that the change in frequency is $\Delta \nu = \Delta E/h = C_n r^{-n}$

  Constants C_n are determined by laboratory measurements, or calculations.

Pressure broadening (1)

$$\Delta\nu = \frac{C_n}{r^n}$$

n =	name	interaction of
2	linear Stark effect	hydrogen-like ions + p, e
3	resonance broadening	neutral atoms with each other, H+H
4	quadratic Stark effect	ions + e, p
6	van der Waals broadening	metals + H

Two approximations exist – impact broadening for $n>2$ (n=3 resonance, n=4 quadratic Stark effect, n=6 van der Waals) and a quasi-static approximation (i.e. surrounding particles are nearly at rest; for linear Stark broadening, n=2).

Pressure broadenings...

The orbitals of an atom can be perturbed in a collision with a neutral atom or encounter with the electric field of an ion.

n =	name	interaction of
2	linear Stark effect	hydrogen-like ions + p, e
3	resonance broadening	neutral atoms with each other, H+H
4	quadratic Stark effect	ions + e, p
6	van der Waals broadening	metals + H

resonance broadening (n=3)
quadratic Stark effect (n=4)
van der Waals broadening (n=6)

} impact broadening approximations

Lorentz profile

$$\Delta\nu = \frac{C_n}{r^n}$$

(Ansatz): constants C_n are determined by laboratory measurements, or calculations → Let's discuss in a bit more detail

Collisional Broadening

- Frequency of collisions = $1/T_0$
- Suppose collisions occur if particles pass within distance = impact parameter ρ_0

$$\frac{1}{T_0} = N\pi\rho_0^2 v$$

N = #perturbers/cm^3, v = relative velocity cm/s

- Then damping parameter is

$$\Gamma = 2N\underbrace{\pi\rho_0^2}v$$

We used $\sigma=\pi a_0^2$ for direct collisions

Weisskopf approximation (1)

- perturber is a classical particle
- path is a straight line
- no transitions caused in atom
- interaction creates a phase shift or frequency shift given by

$$\Delta\omega = \frac{C_p}{r^p}$$

- p exponents of astronomical interest: 3,4,6

Weisskopf approximation (2)

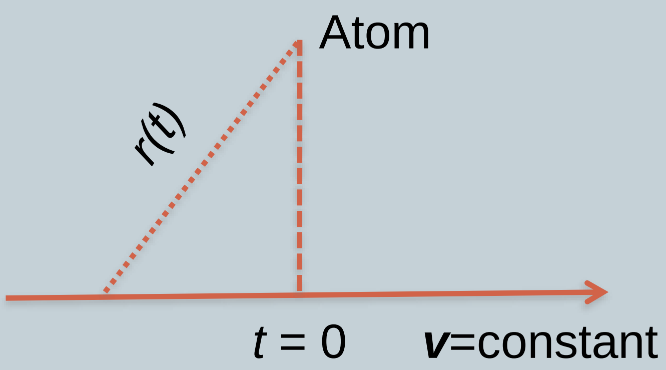
Total phase shift

$$\eta(\rho) = C_p \int_{-\infty}^{+\infty} \frac{dt}{r^p} = C_p \int_{-\infty}^{+\infty} \frac{dt}{[\rho^2 + v^2 t^2]^{p/2}} = \frac{C_p}{v \rho^{p-1}} \psi_p$$

$$\psi_p = \sqrt{\pi} \frac{\Gamma[(p-1)/2]}{\Gamma[p/2]}$$

p	$\boldsymbol{\psi_p}$
2	π
3	2
4	π/2
6	3π/8

$$r(t) = [\rho_0^2 + v^2 t^2]^{1/2}$$



perturber path

Weisskopf approximation (3)

- Assume that only collisions that produce a phase shift > η_0 are effective in broadening:
  then impact parameter is

$$\rho_0 = \left(\frac{C_p \psi_p}{\eta_0 v} \right)^{\frac{1}{p-1}}$$

- Weisskopf assumed $\eta_0 = 1$, yields damping

$$\Gamma_W = 2\pi N v \left(\frac{C_p \psi_p}{v} \right)^{\frac{2}{p-1}}$$

  depends on ρ, T

- Ignores weak collisions $\eta < \eta_0$

Better Impact Model: Lindholm-Foley

- Includes effects of multiple weak collisions, which introduce a phase shift $\Delta\omega_o$; $\Gamma_{LF} > \Gamma_W$

$$I(\omega) = \frac{\Gamma/(2\pi)}{(\omega - \omega_0 - \Delta\omega_0)^2 + (\Gamma/2)^2}$$

p	3	4	6
Γ	$2\pi^2 C_3 N$	$11.37\ C_4^{2/3}\ v^{1/3}\ N$	$8.08\ C_6^{2/5}\ v^{3/5}\ N$
$\Delta\omega_0$	0	$9.85\ C_4^{2/3}\ v^{1/3}\ N$	$2.94\ C_6^{2/5}\ v^{3/5}\ N$

- Impact theory fails for small *ρ*

Impact broadenings (n=3,4,6)

- **Resonance Broadening** (n=3) occurs between identical species, restricted to upper/lower level having an electric dipole transition to ground state (resonance line):

$$\Delta\lambda_{1/2} = 8.6\text{x}10^{-30}(g_i / g_k)^{1/2}\lambda^2\lambda_{res} f_{res} N_i$$

- **Quadratic Stark broadening** (n=4): Interaction of electron or proton with a system without dipole moment. The frequency shift depends on the square of the local electric field generated by passing electrons. With C_4 a constant obtained from laboratory data,

$$\log\gamma_4 = 19 + \frac{2}{3}\log C_4 + \log P_e - \frac{5}{6}\log T$$

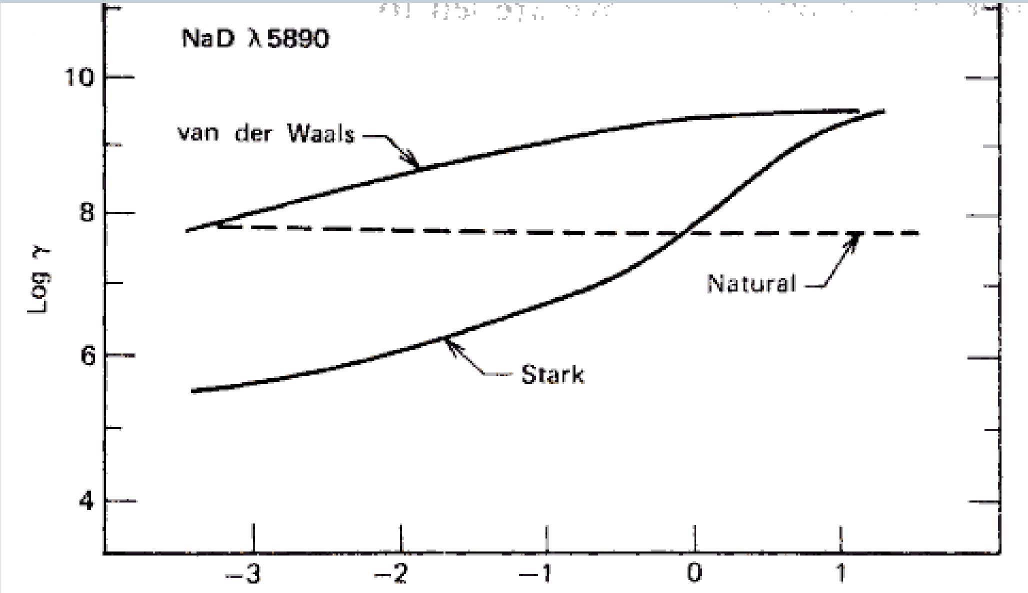
- **Van der Waals broadening** (n=6): A momentary dipole on one neutral atom induces a change in lifetime, by inducing a dipole on the other. Because of its overwhelming abundance, **neutral H** acts as a perturber. For C_6 a constant (excitation and ionization dependent),

$$\log\gamma_6 = 19.6 + \frac{2}{5}\log C_6 + \log P_g - \frac{7}{10}\log T$$

Example

A comparison of quadratic Stark and van der Waals broadening for the Na I 5890 line at various optical depths in the Sun.

The latter dominates here, and greatly exceeds the natural width by a factor of about 30.



In general,

- **Quadratic Stark broadening** (n=4) affects most lines in **hot stars** since **electron** pressure approaches gas pressure.
- **Van der Waals broadening** (n=6) affects most lines in **cool stars** since this involves interactions between **neutral atoms**

Linear Stark broadening (n=2)

- Atoms do **not** generally have permanent electric dipole moments. If there were such a moment, the Stark effect would be *linear*. Such a moment can occur only for two or more levels of the same energy (they are degenerate) but different orbital quantum numbers. This happens only for single electron atoms (H, He^{+}, Li^{2+}, ...).

- The frequency shift depends on the the local electric field generated by passing electrons.

- Unfortunately, impact theory is no longer satisfactory and we have to consider the distribution of electric fields. In the star there is no a uniform field – there is an average field distribution felt by an average atom (**statistical** Stark effect). This distribution is called the Holtsmark distribution.

Holtsmark Statistical Theory

- Ensemble of perturbers instead of single
- more particles, more chances for strong field
- e- attracted to ions, reduce perturbation by Debye shielding
- in stellar atmospheres density is low, number of perturbers is large, and Holtsmark distribution is valid

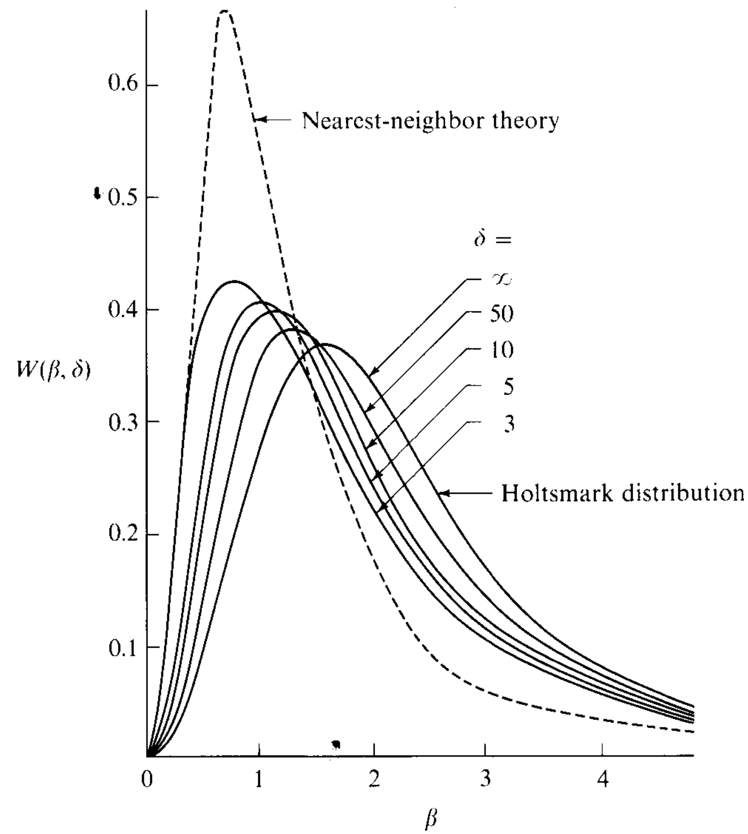


FIGURE 9-1
Probability distribution of field strength at a test point, including shielding effects; δ is the number of charged particles within the Debye sphere. From (**205**), by permission.

Hydrogen: Linear Stark Effect

- Each level degenerate with $2n^2$ sublevels.
- Perturbing field will separate sublevels.
- Observed profile is a superposition of components weighted by relative intensities and shifted by field probability function.
- $\Delta\lambda_{1/2} \approx 2.5\text{x}10^{-9}\alpha_{1/2}N_e^{2/3}$ where $\alpha_{1/2}$ is a half-width parameter widely used for plasma diagnostics (NIST).
- For Hα (n=2 to 3) in the Sun (P_e=20 dyne/cm^2, T=5800K), $\Delta\ \lambda_{\text{FWHM}}$=0.5 Å, i.e. a width 1000 times the natural width.
- Hot stars have very **high** electron pressures, so the Linear Stark effect greatly affects H I lines in hot stars (including white dwarfs), and is also relevant for hydrogenic ions (e.g. He II lines) in O stars.

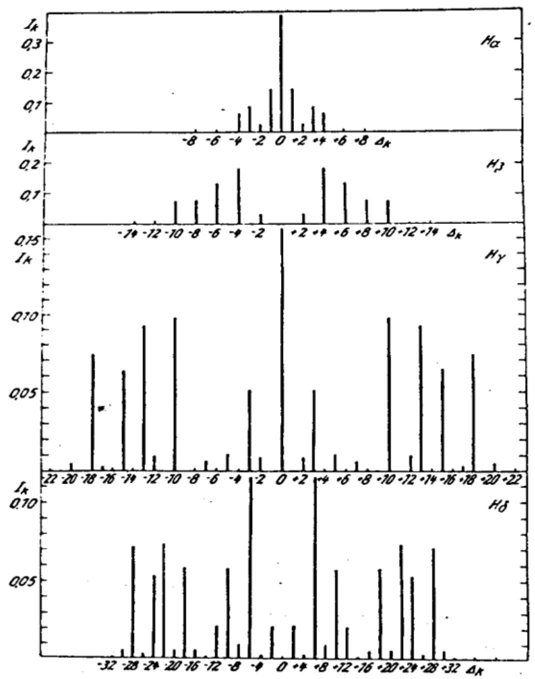
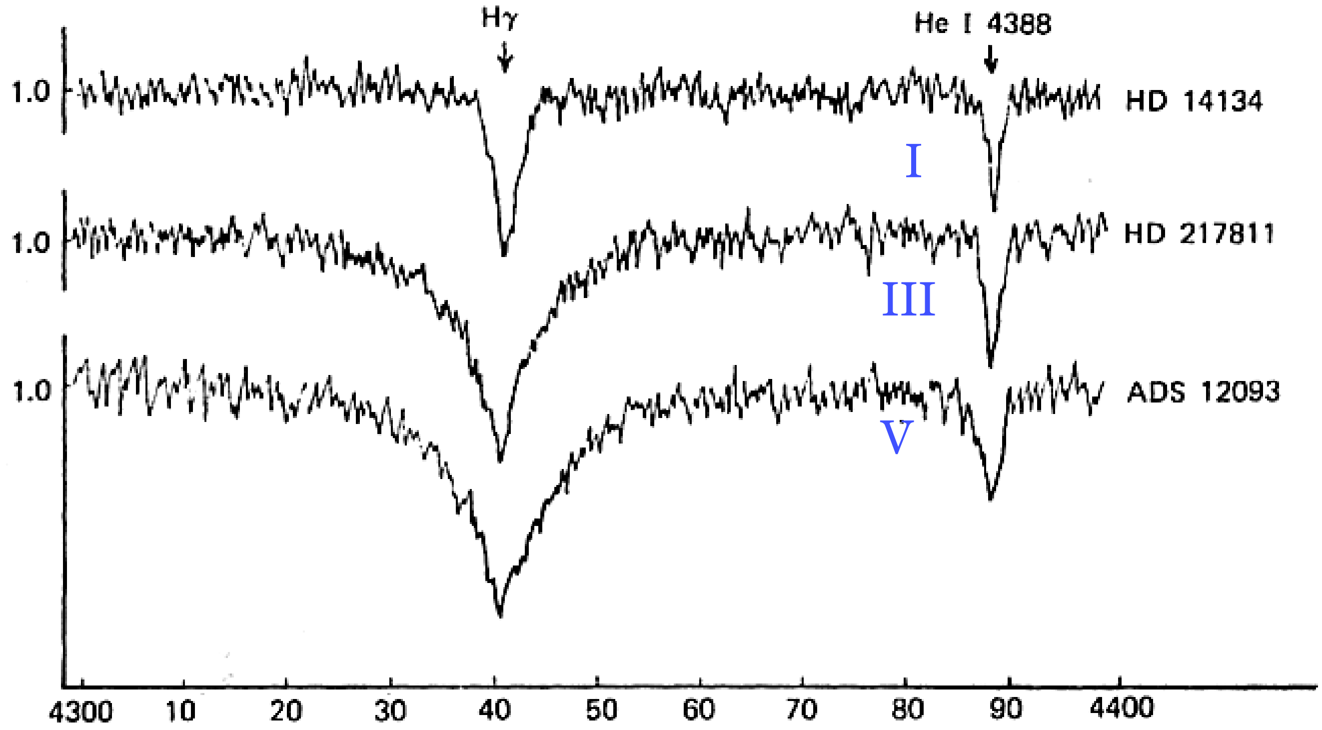


Fig. 11.1. The Stark effect splitting of the different Balmer lines (according to Unsöld, 1955, p. 320.)

Linear Stark broadening: examples (1)

Example of linear Stark broadening in early B stars – increased Hγ line width for increased pressure (this effect becomes significant for $T_{\rm eff}$>7500 K).



Linear Stark broadening: examples (2)

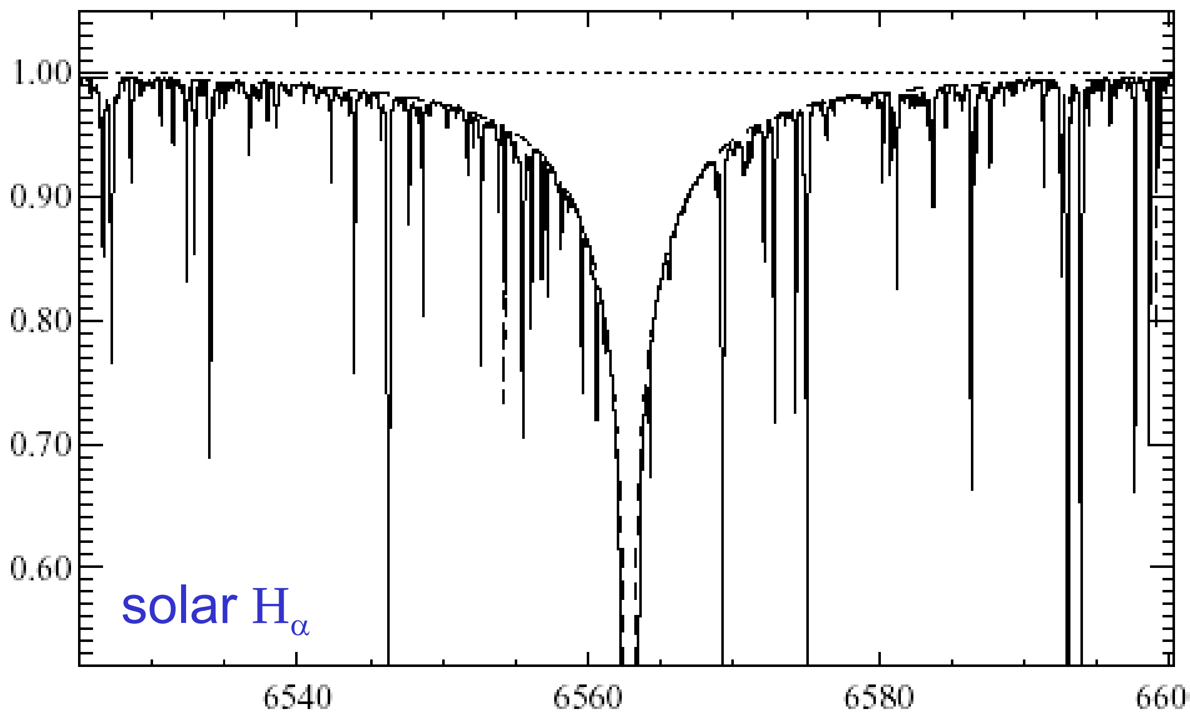
Vidal, Cooper & Smith (1973):

H I + p   quasistatic approach;

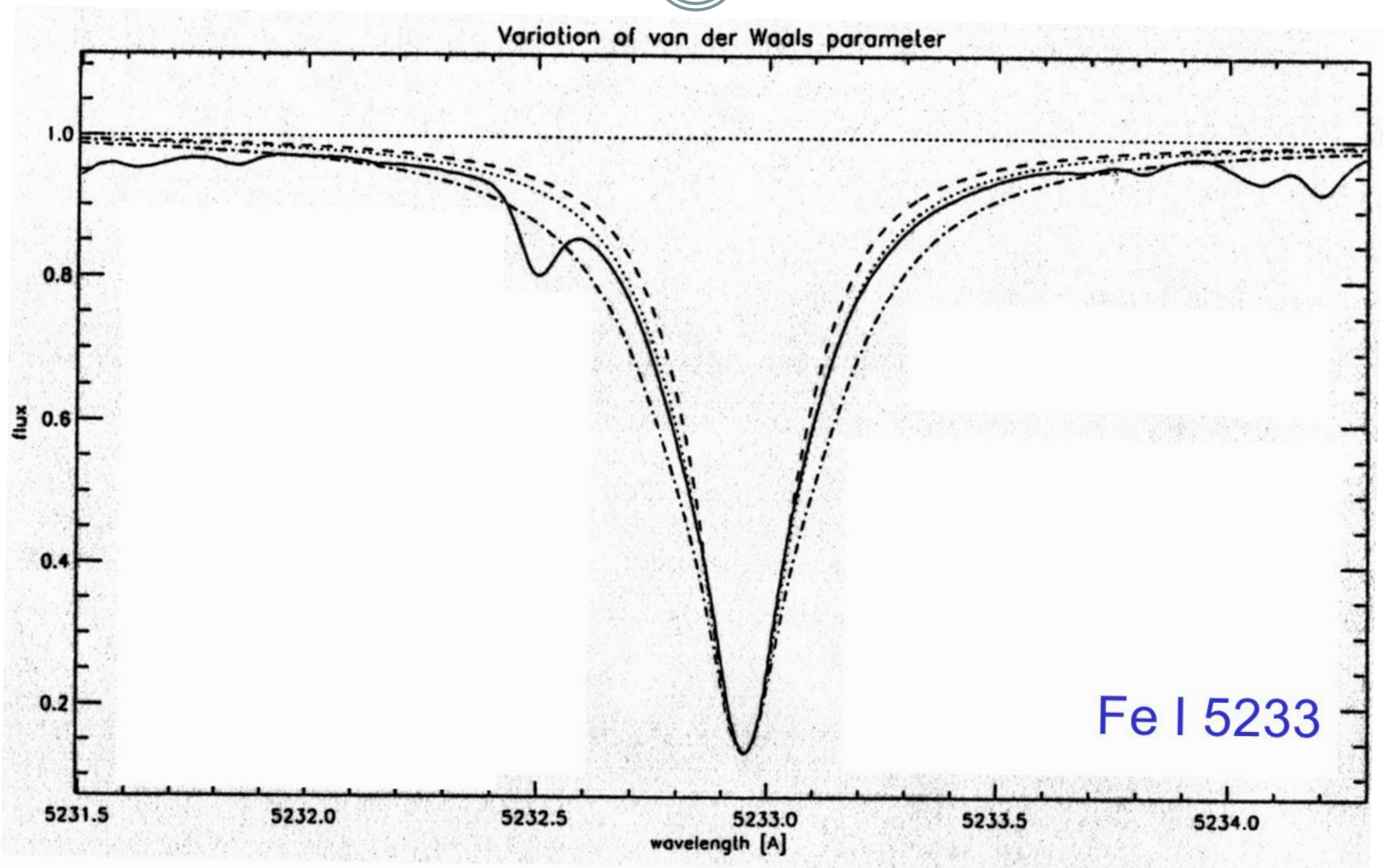
H I + e   collisional approximation – in a core

quasistatic approach   - in wings

Observations (solid line) and theory (dash line)



van der Waals broadening: example



Example: log C_6 varies from -31.40 (top), -31.10 (middle), to -30.50 (bottom)

New Developments in the Theory of Pressure-Broadening

♦ Linear Stark broadening

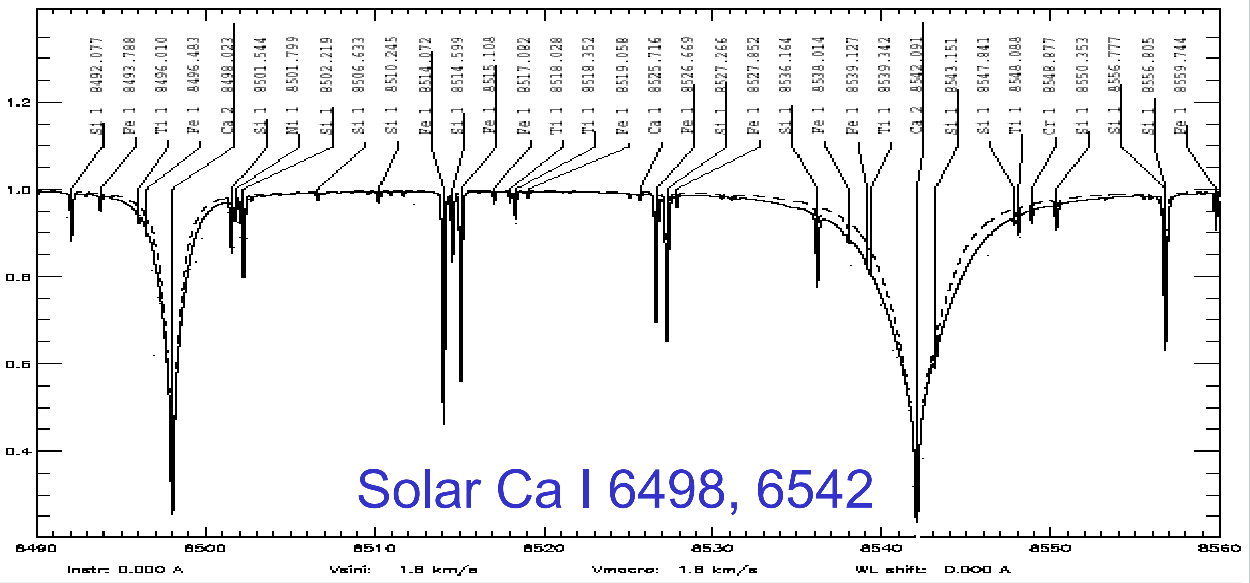
Stehle & Hutcheon (1999, A&AS, 140, 93) – tables of Stark profiles

♦ van der Waals broadening

Anstee & O'Mara (1995, MNRAS, 276, 859) and following papers

$$\gamma_6 / 4\pi = N_H (4/\pi)^{\alpha/2}\Gamma\big((4-\alpha)/2\big) \mathrm{v}\, \sigma_0 (\mathrm{v}/\mathrm{v}_0)^{-\alpha}$$

σ_0, α - tabulated parameters



Observations and Theory of *Anstee&O'Mara* are consistent!

Dash line – approximation of *Unsold* (1955)

Additional material for self-study

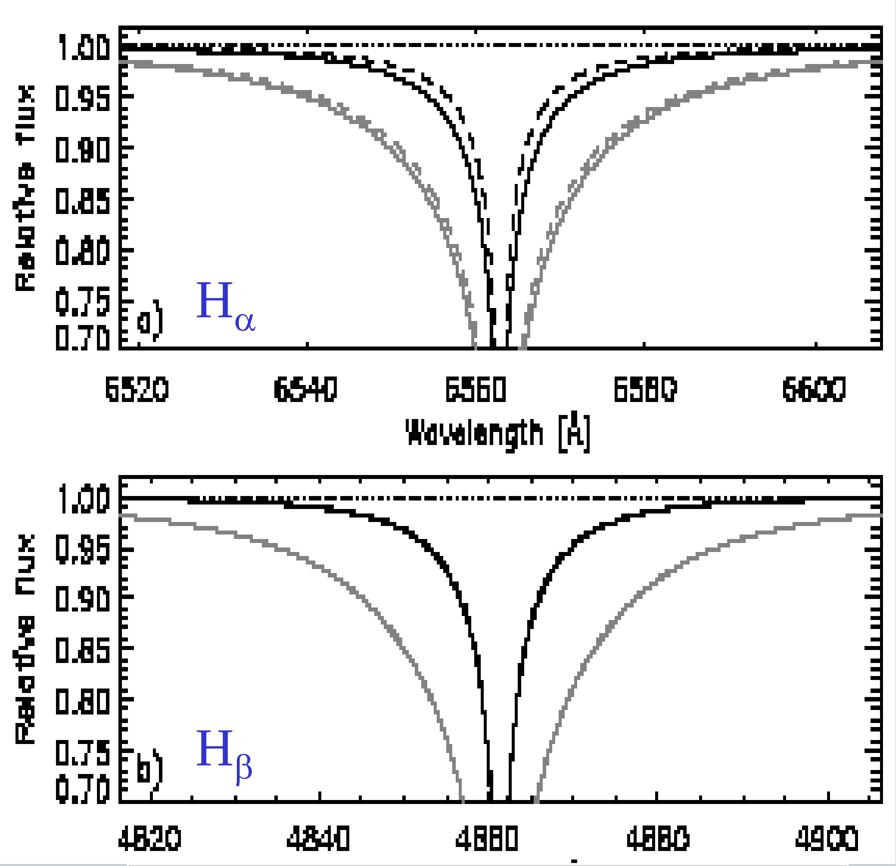
♦ Resonance Broadening

Barklem et al. (2000, A&A, 363, 1091)

Influence of resonance broadening on the line profiles of H_α and H_β

♦ Quadratic Stark broadening

Papers by *Dimitrijevic et al.*



$T_{\rm eff} = 5780$ K, log g = 4.44
$T_{\rm eff} = 7000$ K (grey)
Dash line:
Without resonance broadening

Additional material for self-study

Broadening of spectral lines

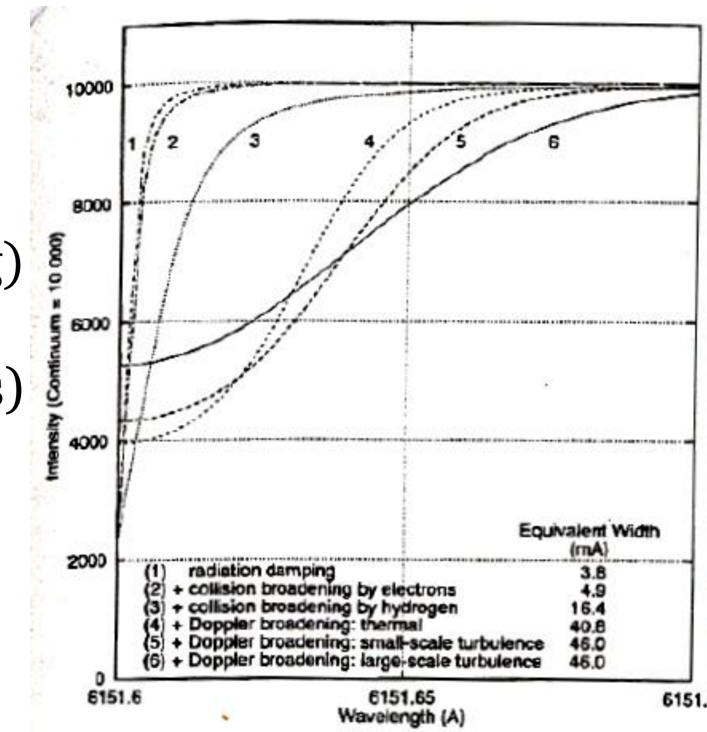
There are numerous broadening mechanisms which influence the apparent shape of spectral lines:

microscopic

1. Natural broadening
2. Thermal broadening
3. Microturbulence
   (treated like extra thermal broadening)
4. Collisions (important for strong lines)
5. Isotopic shift, hyperfine splitting (hfs)
   Zeeman effect

macro

6. Macroturbulence
7. Rotation
8. Instrumental broadening



Holweger 1996

Other broadening mechanisms

- **Turbulent Broadening:** In addition to microscopic (thermal) and macroscopic (rotation) motions, there are other motions in stellar atmospheres which are introduced, operating on microscopic (**microturbulence**) and macroscopic (**macroturbulence**) scales, via convolutions with Gaussian velocity distribution

- **Isotope splitting:** Different isotopes have different nuclear mass and so slightly different term energies – the effect is greatest for hydrogen (e.g. deuterium vs hydrogen).

- **Zeeman splitting:** Magnetic fields split magnetically sensitive lines – at optical wavelengths the splitting is seen as line broadening, towards the IR the splitting becomes more noticeable since it increases as λ^2 versus λ for Doppler broadening.

Turbulent broadening

- Added to thermal broadening in quadrature. The Gaussian line profile (normalized to unity) remains.
Recall the convolution of two Gaussian profiles!

$$\phi(\nu) = \frac{1}{\sqrt{\pi}\sqrt{\Delta\nu_D^2 + \xi_t^2}} \exp[-(\nu - \nu_0)^2 / (\Delta\nu_D^2 + \xi_t^2)]$$

where ξ_t is a microturbulence velocity.

- Note that the broadening because of microturbulence does **not** depend on the mass of an atom!

Spectral line formation

EINSTEIN COEFFICIENTS

LINE PROFILES: NATURAL BROADENING

BROADENING OF SPECTRAL LINES

NATURAL LINE BROADENING:

THERMAL (DOPPLER) BROADENING

CONVOLUTION OF DIFFERENT BROADENING PROCESSES

PRESSURE BROADENING

INGIS-TELLER RELATION

ROTATIONAL AND INSTRUMENTAL BROADENING

Inglis-Teller relation

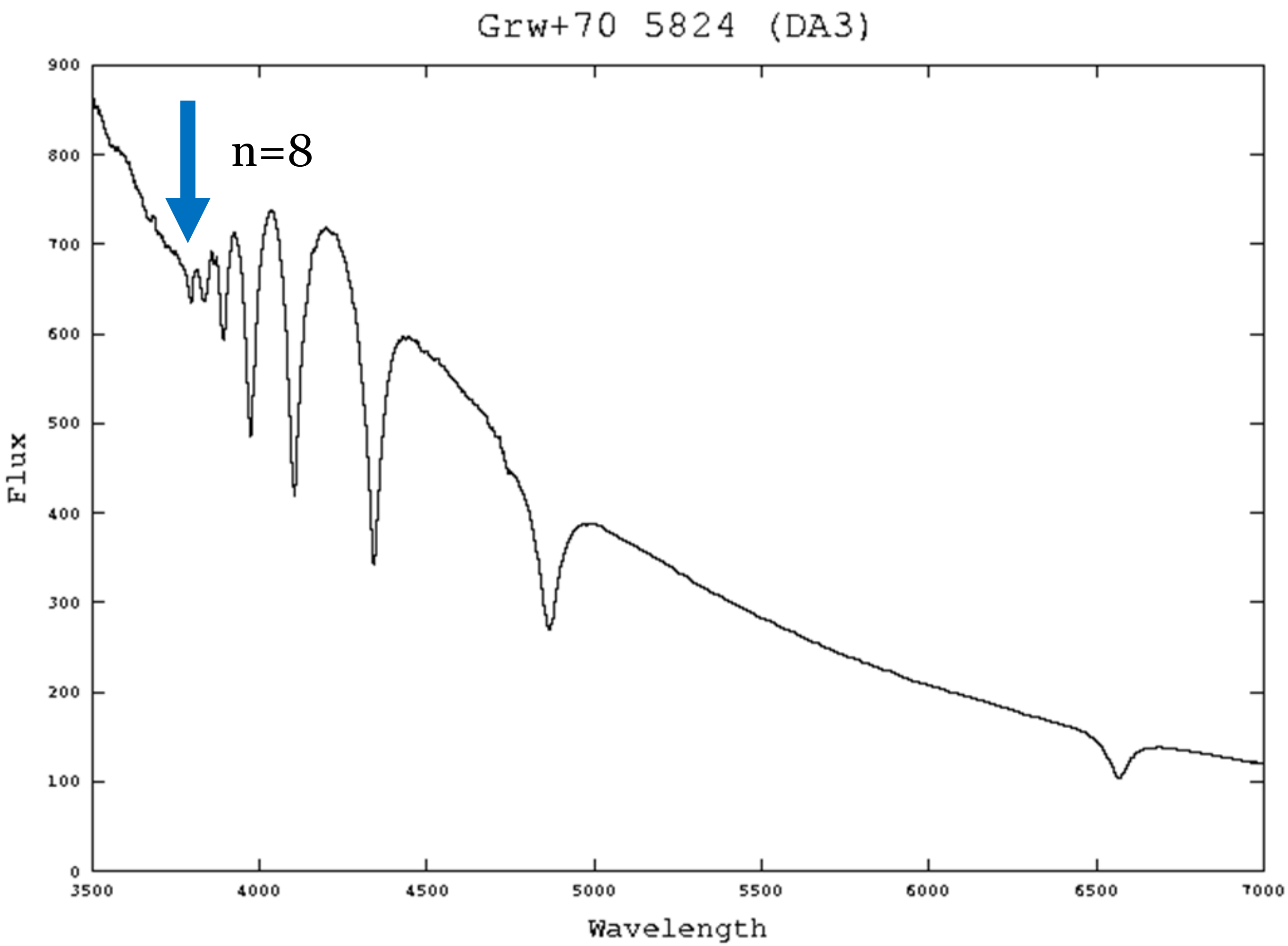
- Balmer lines, due to linear Stark broadening, overlap with each other close to the series limit, merging into a quasi-continuum at frequencies well below the nominal threshold.
- **If** linear Stark broadening is the dominant mechanism, one can estimate the N_e from the highest frequency Balmer line n_{max} that is still visible – the Inglis & Teller (1939) relation:

$$\log N_e = 23.26 - 7.5 \log n_{max}^{Balmer}$$

Star	SpT	n_{max}	Log N_e
α Cyg	A2I	29	12.2
Sirius	A2V	18	13.8
τ Sco	B0V	14	14.6
White dwarf	DA	8	16.4

From Mihalas (1970)

Inglis-Teller in White Dwarfs



Spectral line formation

EINSTEIN COEFFICIENTS
LINE PROFILES: NATURAL BROADENING
BROADENING OF SPECTRAL LINES
NATURAL LINE BROADENING:
THERMAL (DOPPLER) BROADENING
CONVOLUTION OF DIFFERENT BROADENING PROCESSES
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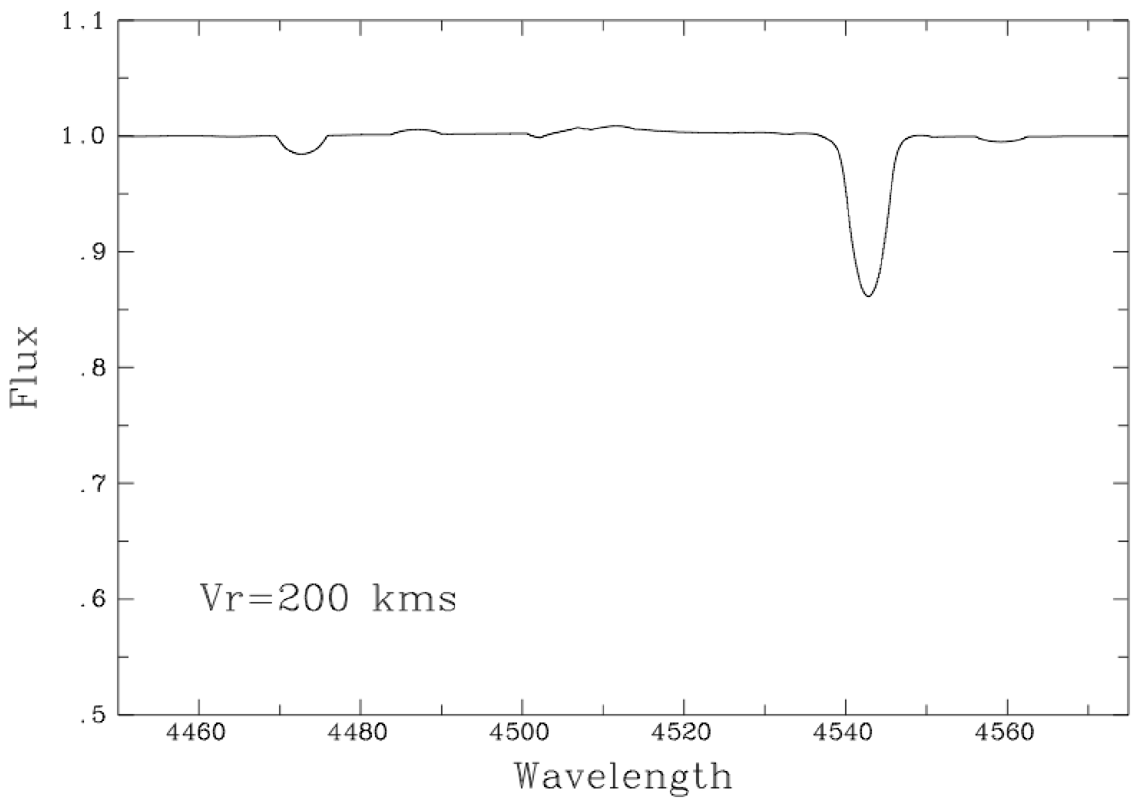
INGIS-TELLER RELATION
ROTATIONAL AND INSTRUMENTAL BROADENING

Rotational broadening

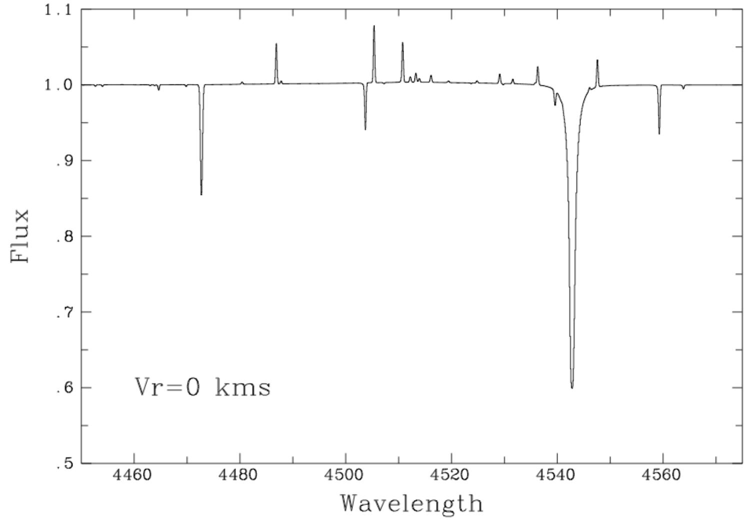
Thermal Doppler broadening describes the microscopic motion of individual particles in the atmosphere. The other scale extreme is **macroscopic** broadening of the lines caused by the **rotation** of the whole star. The maximum (critical) rotation velocity $V_c = \sqrt{(GM/R_e)}$ where R_e is the equatorial radius.

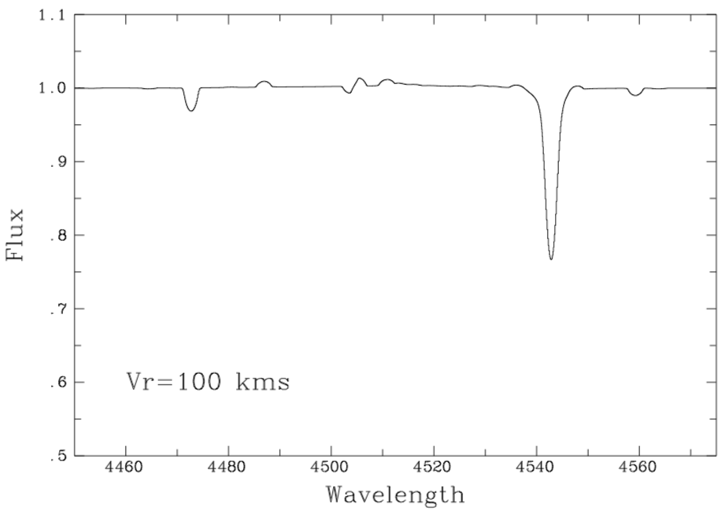
Successive synthetic models allowing for Doppler and Stark broadening are shown here for $V_{rot} \sin i = 0, 100, 200$ km/s.

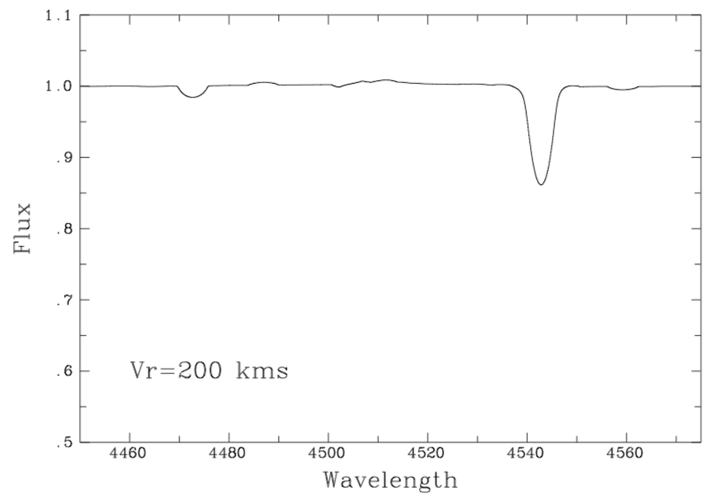


Rotational broadening

Successive synthetic models allowing for Doppler and Stark broadening are shown here for $V_{\mathrm{rot}} \sin i = 0, 100, 200$ km/s.







V_{rot} sin *i* ?

Many early-type OB stars are observed to be rotating rapidly (Be stars close to critical rotation), so this is the major broadening mechanism in these stars. Why sin(*i*)? **Inclination is rarely known**, except for eclipsing binaries.

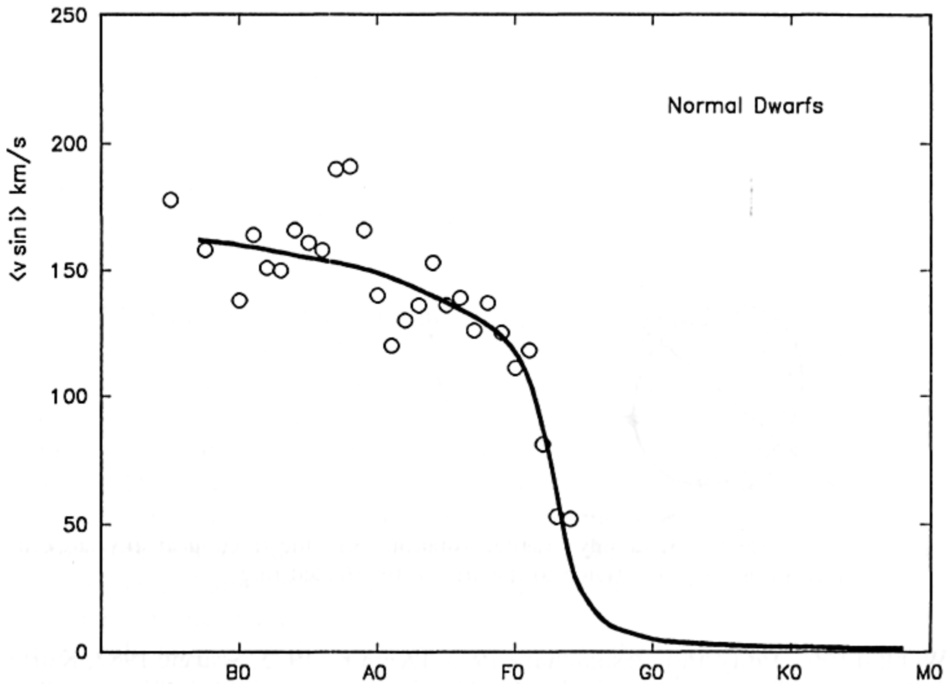


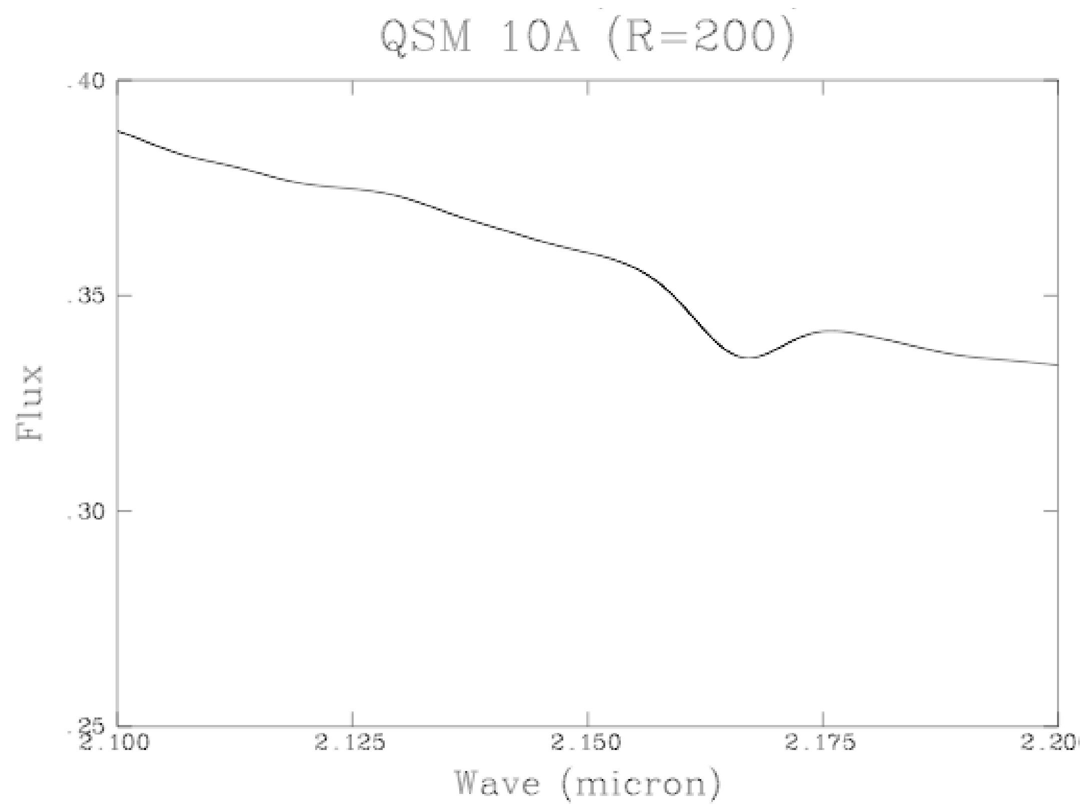
Fig. 17.16. The average rotation rates are shown for spectral intervals as a function of spectral type. (Data are from Uesugi and Fukuda (1982), Soderblom (1983), and Gray (1982b, 1984b).)

Instrumental Broadening

Any spectrograph used to observe a star has a finite resolution ($R=\lambda/\Delta\lambda$), regardless of the sharpness of the spectral line. For low resolution data (necessary when observing faint objects), this may affect the observed line profile more than everything else.

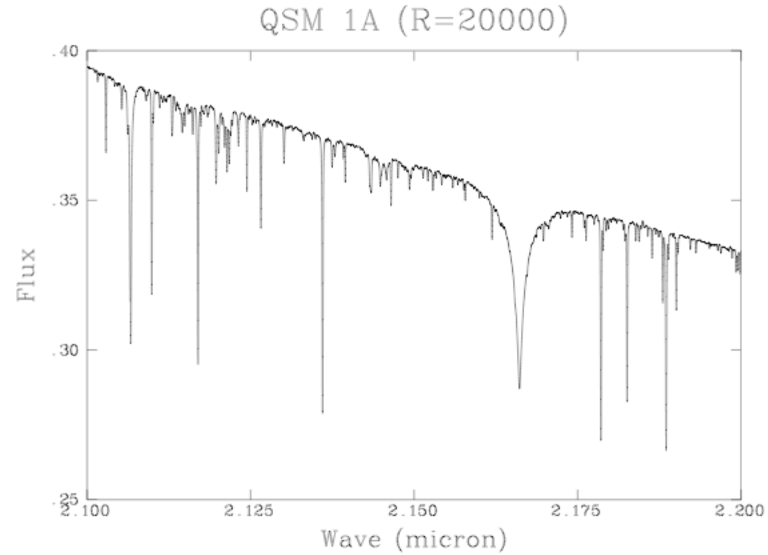
High (R=20,000),medium (R=2,000) and low (R=200) resolution Solar spectra at 2microns.

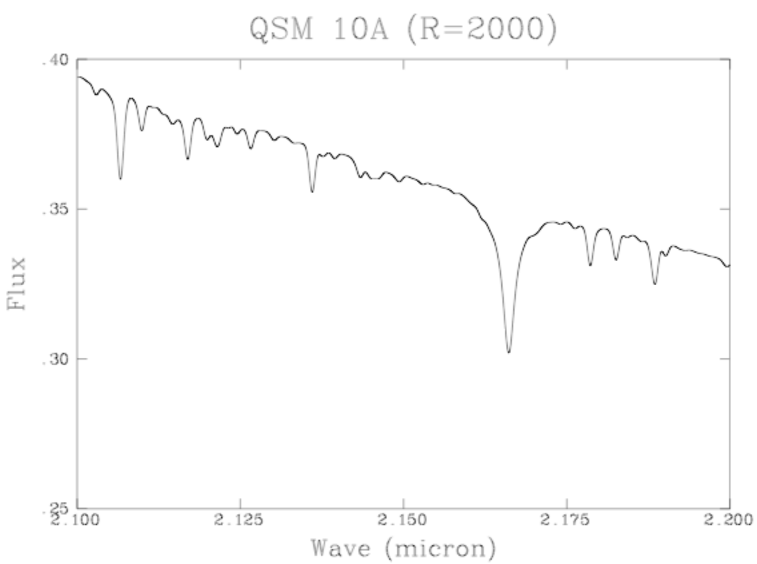
Faint stars with intrinsically narrow lines are generally broadened the most by the spectrograph!

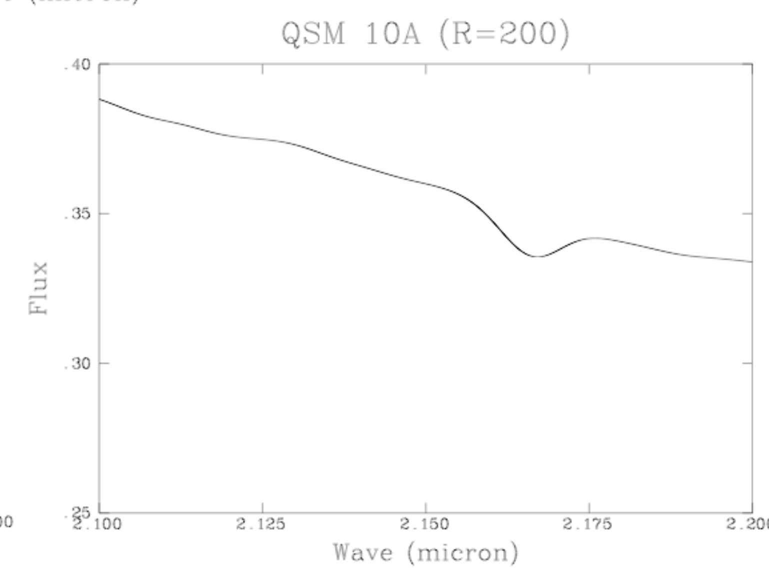


Instrumental Broadening

High (R=20,000), medium (R=2,000), and low (R=200) resolution Solar spectra at 2microns.







Summary

- Final profile is a convolution of all the key broadening processes.
- Convolution of Lorentzian profiles: $\Gamma_{total}=\Sigma\Gamma_i$
- Convolution of Lorentzian and Doppler broadening yields a Voigt profile.
- Pressure/collisional broadening via linear Stark broadening (only for hydrogenic ions), quadratic Stark broadening (interaction with electrons – hot stars) or Van der Waals broadening (interaction between neutral atoms – cool stars).
- Inglis-Teller relation allows estimate of N_e from overlapping Balmer lines in hot stars.
- Non-pressure broadening mechanisms include microscopic (thermal Doppler), macroscopic (rotational Doppler), turbulent, Zeeman, instrumental.
- Line profiles typically have characteristic Voigt profiles – Gaussian (thermal) cores and Lorenzian (pressure) wings.