Wavelength dependence of α(H)

- Consider the H absorption coefficient α (per atom) for T=5040K (Θ=5040/T=1). Let us compare the value of α in the Balmer (n=2) to Lyman (n=1) continua at 912Å:

$$\frac{\alpha(Balmer)}{\alpha(Lyman)} = \frac{\sigma_{i2}}{\sigma_{i1}}\frac{N_2}{N_1} = \frac{\sigma_{i2}g_2}{\sigma_{i1}g_1}e^{-(10.2eV/kT)} = \frac{\sigma_{i2}g_2}{\sigma_{i1}g_1}10^{-(10.2\times5040/T)}$$

- From above, $\sigma_n \propto n^{-5}$ and $g_n=2n^2$ so $$\frac{\alpha(Balmer)}{\alpha(Lyman)} = \frac{2^{-5}\times 8}{1\times 2}6.3\times10^{-11} \approx 8\times10^{-12}$$

- There is a huge difference in hydrogen absorption coefficient at 912Å (Lyman edge) at T=5040K.

- Similar calculations at T=25200K (Θ=5040/T=0.2) give

$$\frac{\alpha(Balmer)}{\alpha(Lyman)} = \frac{2^{-5}\times 8}{1\times 2}0.009 = 0.001$$

- Hydrogen absorption coefficient is very T sensitive!

Wavelength dependence of α(H)

- Consider the H absorption coefficient α (per atom) for T=5040K (Θ=5040/T=1). What about the value of α in the Paschen (n=3) to Balmer (n=2) continua at 3647Å?

$$\frac{\alpha(+)}{\alpha(-)} = \frac{\sigma_u}{\sigma_l}\frac{N_u}{N_l} = \frac{\sigma_u g_u}{\sigma_l g_l} e^{-(\chi_{ul}/kT)} = \frac{\sigma_u g_u}{\sigma_l g_l} 10^{-(\chi_{ul}\times 5040/T)}$$

Transition between levels u and l:

$$\chi_{ul} = C\left(\frac{1}{u^2} - \frac{1}{l^2}\right)$$

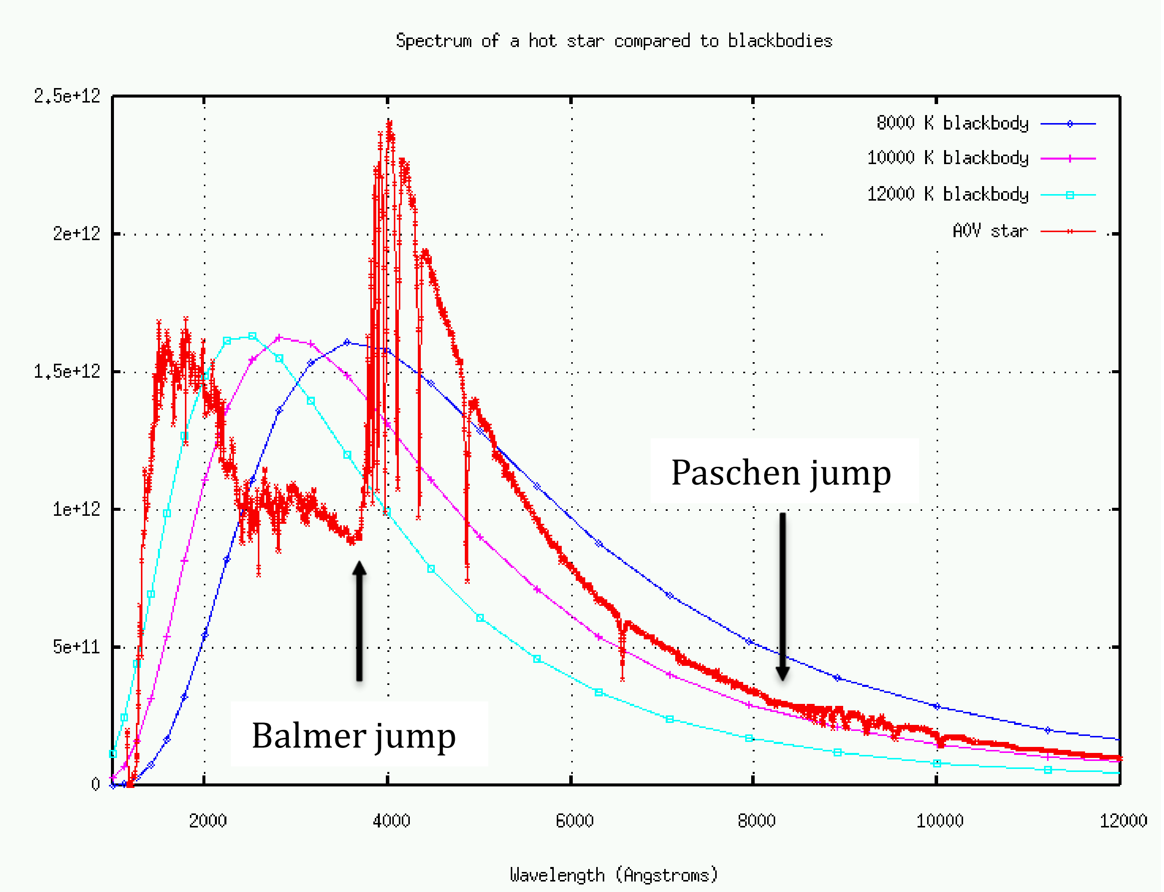
where $C=\chi_{\text{ion}}$=-13.6 eV

- From above, $\sigma_n \propto n^{-5}$ and $g_n=2n^2$ so $\frac{\alpha(Paschen)}{\alpha(Balmer)} \overset{?}{=} 0.004$

- There is a huge difference with Lyman edge (8×10^{-12}). Still, Balmer jump is notable.
- Obviously, all the following jumps will be less and less prominent.

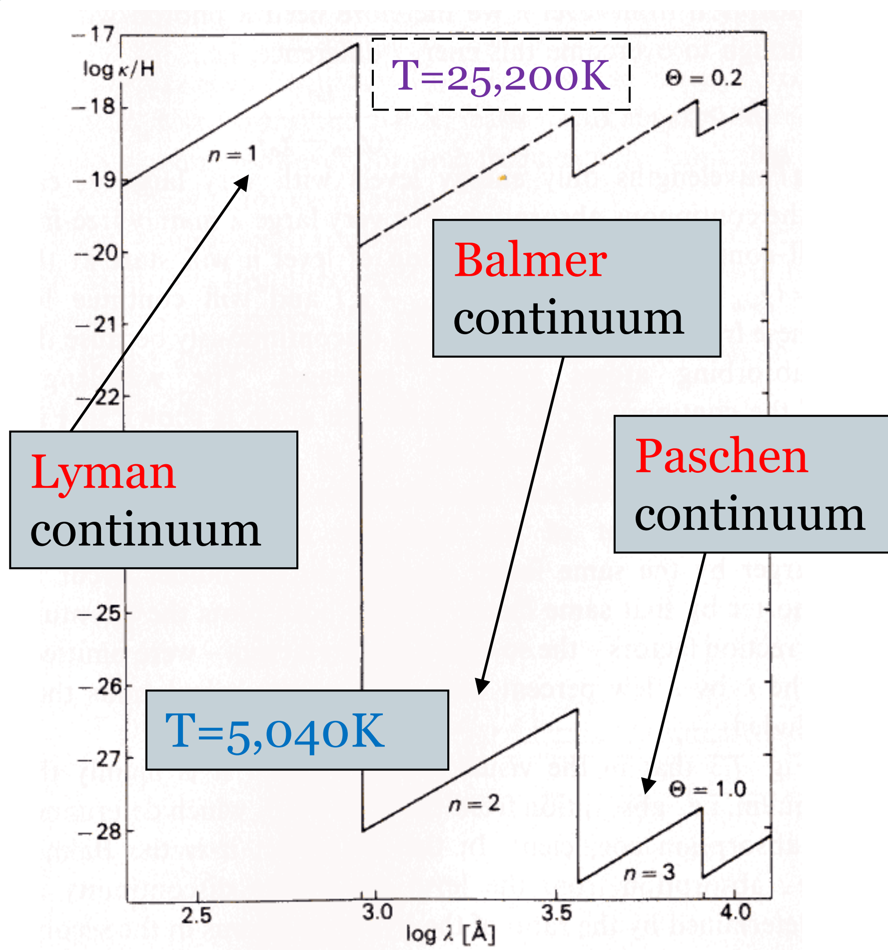
Wavelength dependence of $\alpha(H)$

- There is a huge difference between Lyman edge (8×10^{-12}) and Balmer jump (0.004). Still, Balmer jump is notable.
- Obviously, all the following jumps are less and less prominent.



Wavelength dependence of $\alpha(H)$

- Primarily, the Paschen continuum (absorption from n=3) determines the H absorption coefficient in the visual ($3647\text{Å} < \lambda < 8205\text{Å}$).
- For He^+, the ionization energy is larger by a factor of $Z^2=4$ than that of the H atom. All discontinuities occur at wavelengths shorter by a factor of 4, i.e. 228Å instead of 912Å for the He^+ Lyman continuum.



Negative hydrogen ion H^-

- The H atom is capable of holding a second electron in a bound state (binding energy 0.754eV). All photons with $\lambda < 1.64\mu m$ have sufficient energy to ionize the H^- ion back to neutral H atom plus a free electron. The extra electrons needed to form H^- come from ionized metals (such as Ca^+).
- For Solar-like stars, it turns out that H^- is the dominant continuum opacity source at optical wavelengths. In early-type stars H^- is too highly ionized to play a role, whilst in late-type stars there are too few free electrons (since no ionized metals).

Importance of H^- in the Sun (1)

We can use the Saha equation to derive the relative population of $N(H^-)$ in the Sun ($u^-=1$, T=5777K, χ_{ion}=0.754 eV),

$$\log\frac{N^+}{N^0} = \log\frac{u^+}{u^0} + \log 2 + \frac{5}{2}\log T - \chi_{ion}\Theta - \log P_e - 0.48$$

$$\log\frac{N(H^0)}{N(H^-)} = \log\frac{2}{1} + \log 2 + 9.40 - 0.66 - 1.18 - 0.48 = +7.68$$

So, only **2 out of 10^8** hydrogen atoms is in the form of H^-.

Why then the H^- absorption coefficient so important?
Recall, only H atoms in the 3rd quantum level (n=3, Paschen continuum) can contribute to the visual continuous opacity. From the Boltzmann formula

$$\log N(H_{n=3})/N(H_{n=1}) = \log 2(3)^2/2(1)^2 - 5040/5777 \times 12.1 = -9.6$$

i.e. $N_H(n=3)/N_H(n=1)$=2.4x10^{-10} for the Sun. We can now compare the number of H^- ions and H atoms in the Paschen continuum:

$$\log N(H_{n=3})/N(H^-) = 2.4 \times 10^{-10}/2.1 \times 10^{-8} = 0.01$$

Importance of H^- in the Sun (2)

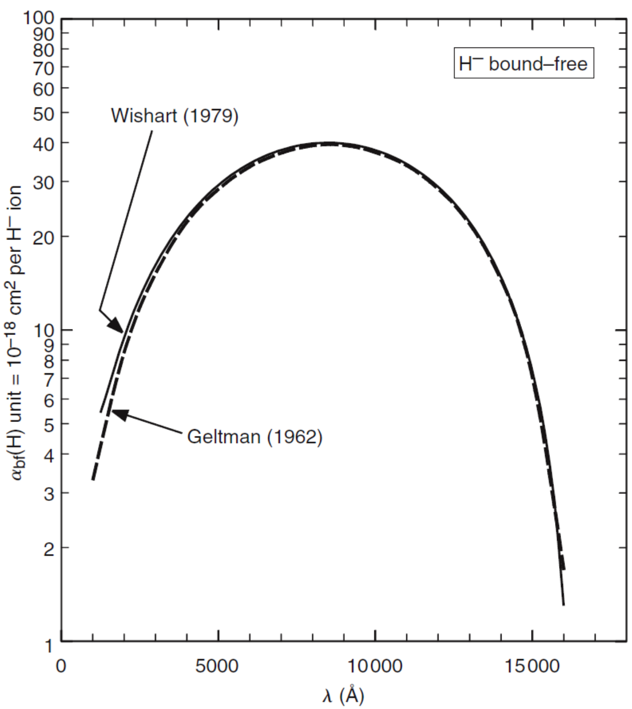
The atomic absorption coefficients per absorbing atom are comparable, so we expect H^- b-f absorption to be 100 times more important than the H Paschen continuum for the Sun.

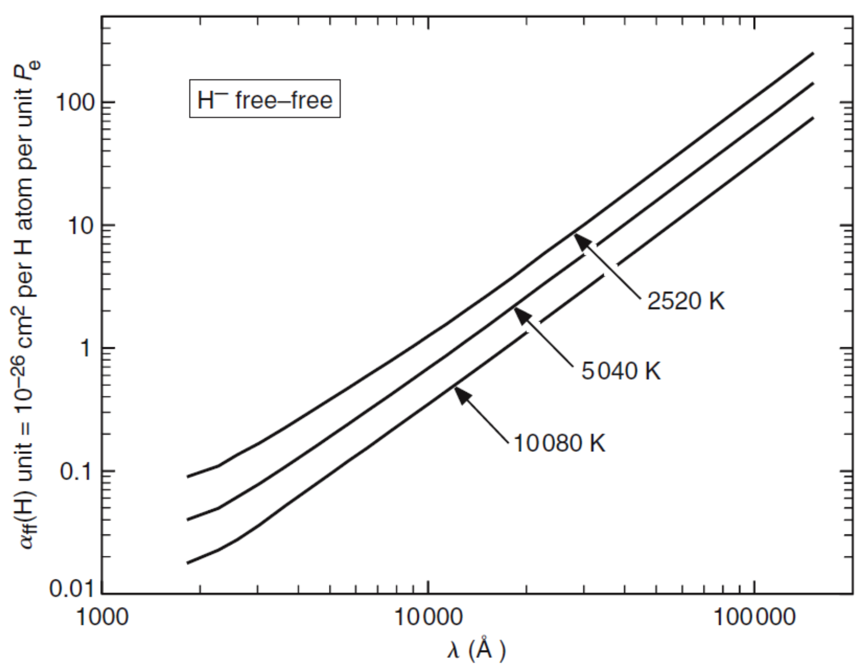
The Balmer continuum (n=2) cannot so easily be neglected and does contribute to the opacity at shorter wavelengths.

Note: For early type stars (A and earlier) we find $N_H(n=3)/N(H^-) \gg 1$ so absorption of neutral H is much more important than H^-. This is why such stars have very strong discontinuities in the Balmer & Paschen limits. We will discuss the importance of the Balmer jump shortly.

H⁻ continuous opacity

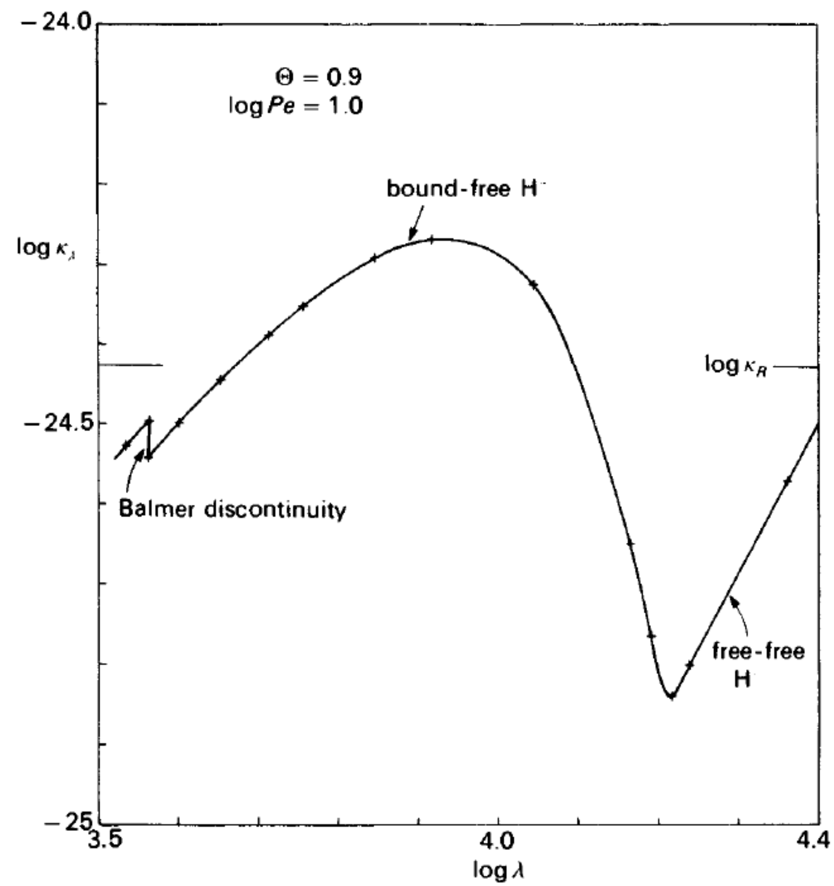
The bound-free H^- absorption can occur for $\lambda < 16500Å$, with a different behaviour from H, reaching a maximum at 8000Å, and decreasing towards the ultraviolet. At longer wavelengths, there is only free-free H^- absorption (with a $\nu^{-3} \propto \lambda^3$ dependence).





Hydrogen continuous opacity

- We have identified H^- (bound-free) in the visual and H^- (free-free) in the IR as principal sources of opacity in the Sun.
- The H Balmer continuum shortward of the 3647Å Balmer jump is an additional contributor.
- What observational evidence is there that this is true for the Sun, and what other forms of opacity play a role in other stars?

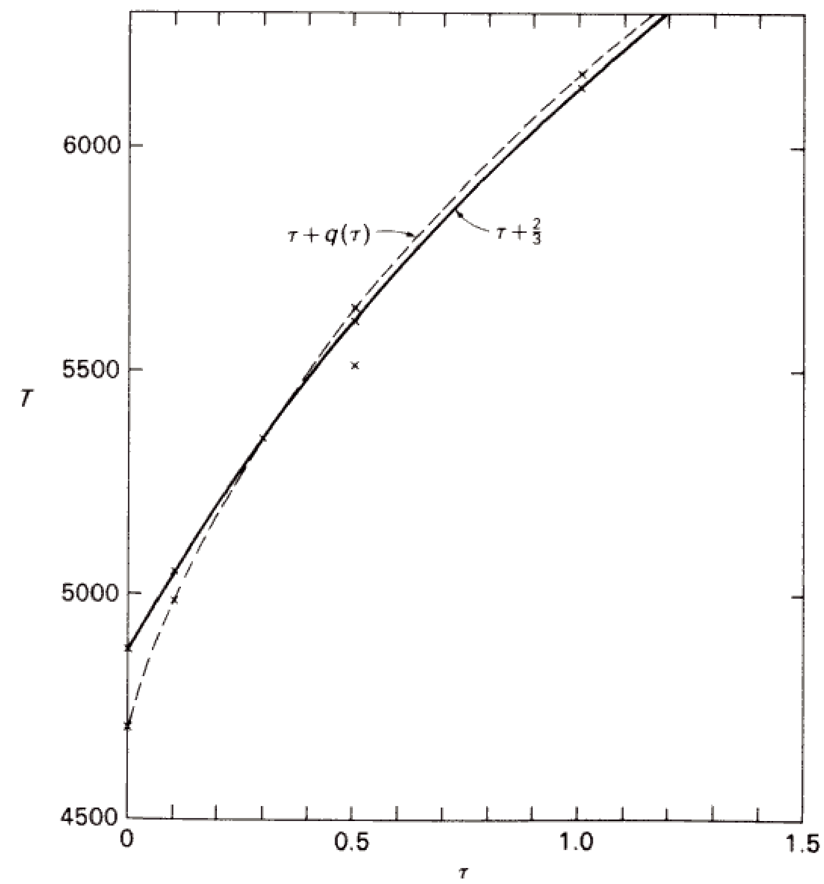


$T(\tau_\lambda)$ from Eddington approximation

We can use the observed limb darkening of the Sun at different λ to derive the depth dependence of the source function, $S_\lambda(\tau_\lambda)$.

Assuming LTE, $S_\lambda(\tau_\lambda)=B_\lambda[T(\tau_\lambda)]$ we can obtain the temperature as a function of τ_λ.

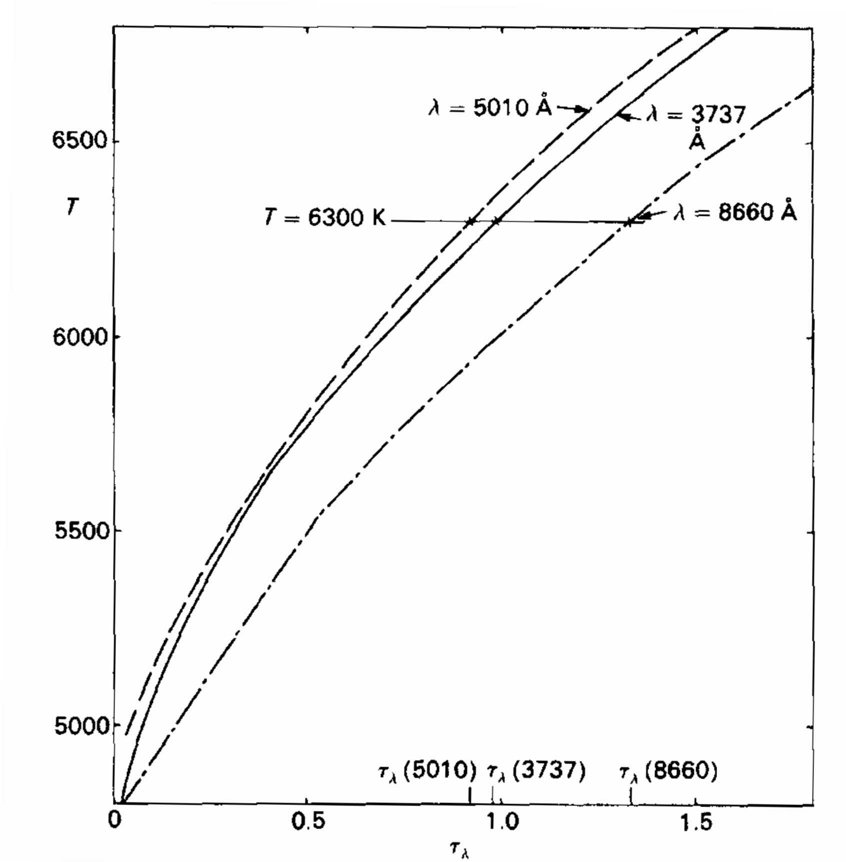
Recall from radiative equilibrium (assuming the Eddington approximation), $T(\tau_\lambda)$ can be obtained for a **grey** atmosphere.



$T(\tau_\lambda)$ from limb darkening

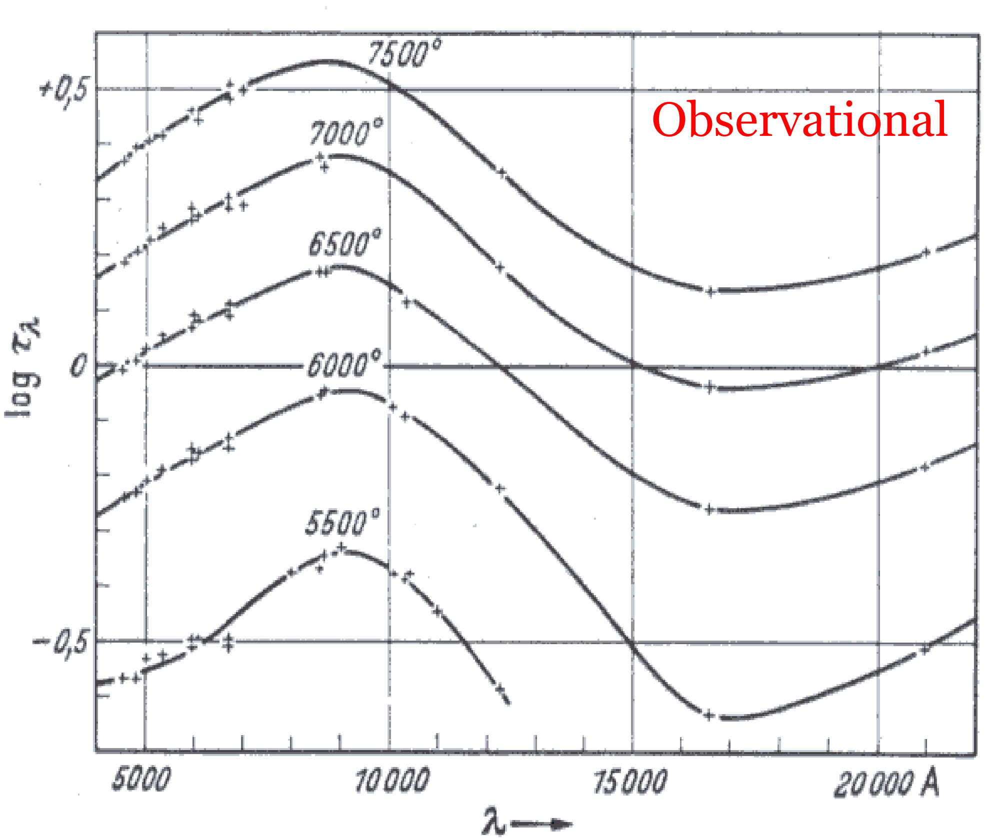
Limb darkening observations of the Sun at different wavelengths (via imaging using suitable filters) to derive $T(\tau_\lambda)$ at various wavelengths (e.g. 3737, 5010 & 8660 Å shown here).

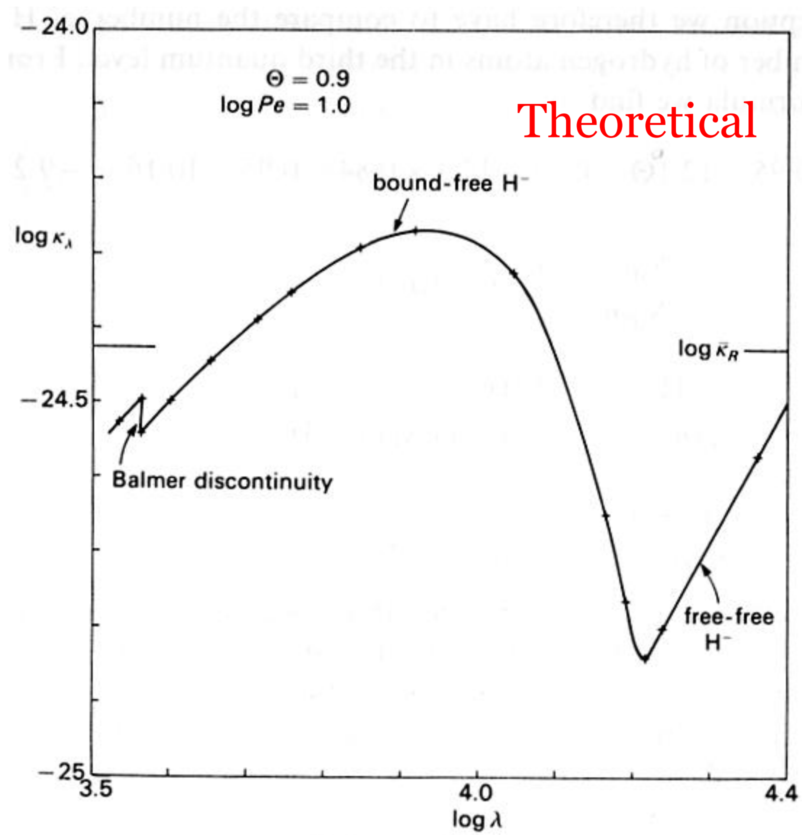
The horizontal line shown at T=6300 K connects points which correspond to the same ***geometrical*** depth, so it is possible to derive the wavelength dependence of τ_λ.



Confirmation of H⁻

The wavelength dependence of τ_λ (and hence κ_λ **or** α_λ) can be observationally derived for the Sun – the optical and IR dependence **agrees** remarkably well with the theoretical absorption coefficient for b-f and f-f H^-.





Other sources of opacity

He ABSORPTION
METALLIC ABSORPTION
SCATTERING
EFFECT OF NONGREYNESS OF THE TEMPERATURE STRUCTURE

Many physical processes contribute to opacity

- **Bound-Bound Transitions** – absorption or emission of radiation from electrons moving between bound energy levels.
- **Bound-Free Transitions** – the energy of the higher level electron state lies in the continuum or is unbound.
- **Free-Free Transitions** – change the motion of an electron from one free state to another.
- **Electron Scattering** – deflection of a photon from its original path by a particle, without changing its wavelength.
  - **Rayleigh scattering** – photons scatter off **bound** electrons (varies as λ^{-4}).
  - **Thomson scattering** –photons scatter off **free** electrons (independent of wavelength).
- **Photodissociation** may occur for molecules.

What can various particles do?

- Free electrons – Thomson scattering
- Atoms and Ions –
  - Bound-bound transitions
  - Bound-free transitions
  - Free-free transitions
- Molecules –
  - BB, BF, FF transitions
  - Photodissociation
- Most continuous opacity is due to hydrogen in one form or another

He opacity?

Helium is the next most abundant element after H, so
is it important for the continuous absorption in the Sun or other stars?

Ionization of He to He^+ requires an energy of 24.6eV ($\lambda<504$Å are needed). Indeed, even the first excited level lies 19.8eV above the ground state, which can contribute only below 600 Å where there is very little radiation coming from the Sun. From the Boltzmann formula ($g_1=1$, $g_2=3$):

$$\log(N_{He}(2s^3S) / N_{He}(1s^1S)) = 0.48-19.8(5040/5777) = -16.8$$

So, only 10^{-17} of the He atoms can contribute to the absorption, and since He is 10% as abundant as H, only one in 10^{-18} atoms are He atoms in the 1st excited state.

Consequently, He opacity plays a **negligible** role for the Sun. The bound-free absorption from He^- is generally negligible, whilst free-free He^- (with a form similar to free-free H^-) can be significant at long wavelengths in cool stars.
Photoionization (bound-free processes) from He only plays a significant role for the hottest, O-type, stars.

Metal (Iron) opacity

- If He only plays a role for very hot stars, do any metals contribute to the continuous opacity in cools stars?

- **Iron** ($Fe/H=10^{-4}$) is generally the dominant metal continuous opacity source in stellar atmospheres.

- In the Sun, let's consider absorption by atomic Fe in the ultraviolet (2000 Å) for which an excitation energy of ~1.7 eV is required. The fraction of excited Fe atoms is 4×10^{-2} relative to the ground-state (from Boltzmann formula), whilst the fraction of ionized to neutral Fe is approximately 6 (from Saha equation).
- Accounting for the abundance of Fe, we obtain the fraction of atomic Fe atoms absorbing at 2000 Å relative to the total number of H atoms to be $4 \times 10^{-2} \times 10^{-4} \times 1/6 = 6 \times 10^{-7}$,

- We previously obtained 2×10^{-8} for H^-, so metallic lines in the UV are much more important for the absorption than the H^- ion, or the neutral H atom. Even more important is the absorption by the metal atoms in the ground level, which is <1570 Å for Fe, <1520 Å for Si.

Molecular opacity

- CN^-, C_2^-, H_20^- , CH_3, TiO are important sources of opacity in late (K-type) & very late (M-type) stars.
- Molecular Hydrogen (H_2) is more common than atomic H in stars cooler than mid-M (brown dwarfs!)
- H_2 does not absorb in the visible spectrum, so only plays a role in the IR.
- H_2^+ does absorb in the visual but is less than 10% of H^-. H_2^+ is a significant absorber in the UV for such very cool stars.

Scattering

In the classical picture of an atom, we can consider the electron as being bound to the atom. Any force trying to remove it will be counteracted by an opposing force. If a force were to pull on the electron and then let go, it would oscillate with eigenfrequencies $\omega = 2\pi\nu$.

The scattering cross-section for a *classical oscillator* can be written as

$$\sigma_s = \frac{8\pi}{3}\frac{e^4}{m_e^2 c^4}\left[\frac{\nu^4}{(\nu^2 - \nu_0^2)^2 + \gamma^2\omega^2}\right] \qquad \omega = 2\pi\nu$$

where ν_0 is the eigenfrequency of an atom and γ is the damping constant.

Thomson & Rayleigh Scattering

Two cases are of interest:

1. **Thompson (electron) scattering** (ν_o=0, γ=0) (photons scatters off a free electron, no change in λ, just direction):

classical electron radius

$$\sigma_T = \frac{8\pi}{3}\frac{e^4}{m_e^2 c^4} = \frac{8\pi}{3} r_e^2 = 6.65 \times 10^{-25}\ \mathrm{cm}^2/\mathrm{electron}$$

2. **Rayleigh scattering** by atoms/molecules ($\nu \ll \nu_o$, $\gamma \ll \nu_o$)

$$\sigma_R(\nu) \propto \sigma_T \nu^4 = \sigma_T \lambda^{-4}$$

Electron scattering vs. f-f transition

- Electron scattering (Thomson scattering) – the path of the photon is altered, but not the energy.
- Free-Free transition – the electron emits or absorbs a photon. A free-free transition **can only occur in the presence of an associated nucleus**.
  An electron in free space cannot gain the energy of a photon.

Thompson Scattering

Since an electron is tiny it makes a poor target for an incident photon so the cross-section for Thomson scattering is very small (σ_T=6.65x10^{-25} cm^2) and has the same value for photons of all wavelengths: **As such electron scattering is the only grey opacity source.**

Although e$^-$ are very abundant in the Solar photosphere, the small cross-section makes it unimportant.

Electron scattering is most effective as a source of opacity at high temperatures. In atmospheres of OB stars where most of the gas is completely ionized, other sources of opacity involving bound electrons are excluded. In this regime, α_T **dominates** the continuum opacity.

Rayleigh Scattering

- Rayleigh scattering by H atoms in Solar-type is more relevant than e^- scattering since atoms are much more common (recall **$N(H) \gg N(H^+)$**).
- In M stars, H_2 becomes the dominant form for hydrogen, with strong electronic transitions in the UV, so Rayleigh scattering by molecular H_2 can be important.
- The cross-section for Rayleigh scattering is much smaller than σ_T, and is proportional to λ^{-4} so increases steeply towards the blue. (In the same way the sky appears blue, due to a steep increase in the scattering cross-section of sunlight scattered by molecules in our atmosphere).
- The cross-section is sufficiently small relative to metallic absorption coefficients that Rayleigh scattering only plays a dominant role in extended envelopes of supergiants.

Total extinction coefficient κ

- The total extinction coefficient is given by:

$$\kappa_\nu = (1 - e^{-h\nu/kT}) \sum_j x_j (\kappa_j^{bb} + \kappa_j^{bf} + \kappa_j^{ff}) + \kappa^s$$

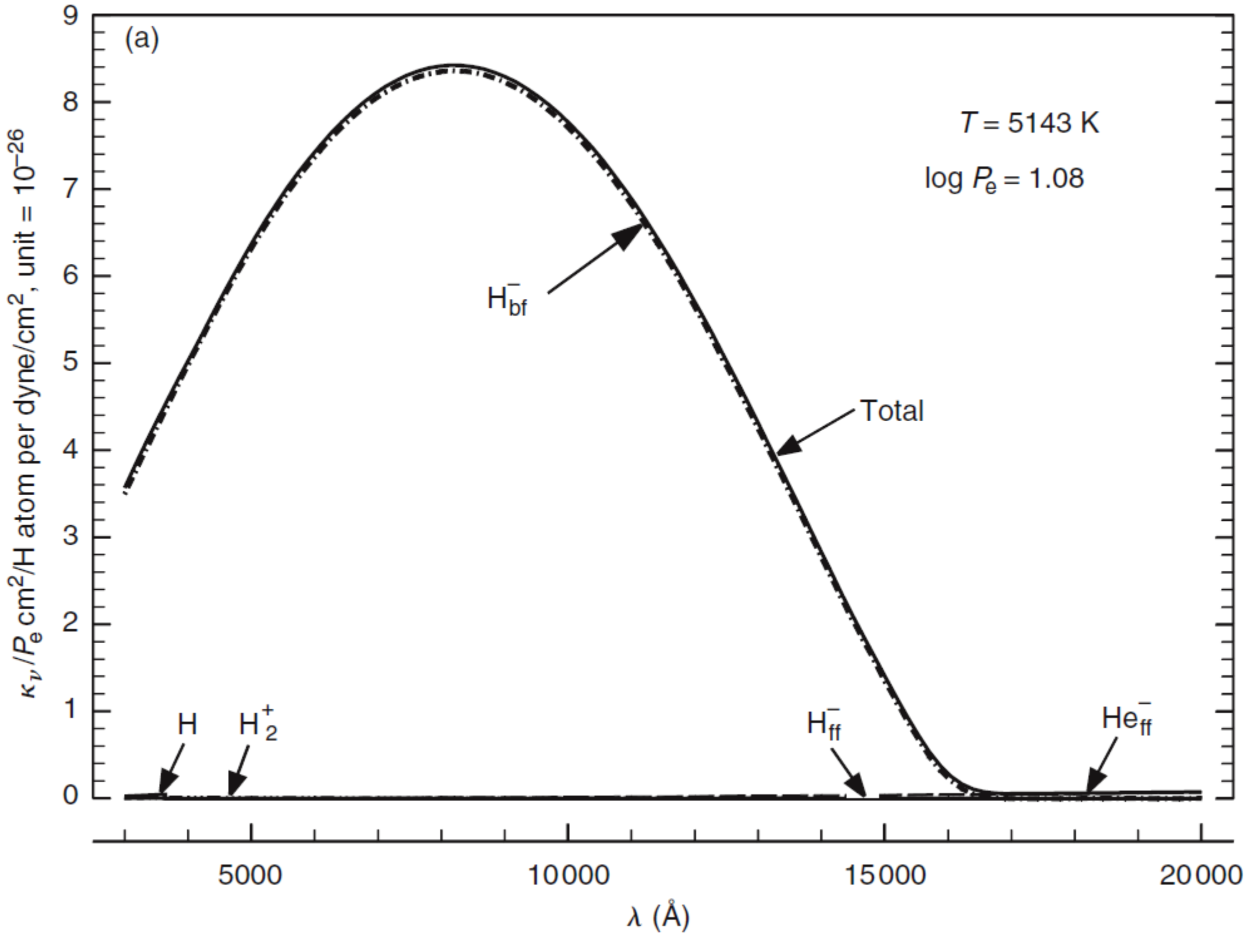
where the sum is over all elements j of number fraction x_j.

Here the $(e^{-\frac{h\nu}{kT}})$ term accounts for stimulated emission (incident photon stimulates electron to de-excite and emit photon with identical energy, as in a laser). We shall discuss it later.

- What is the total extinction coefficient for different types of star?

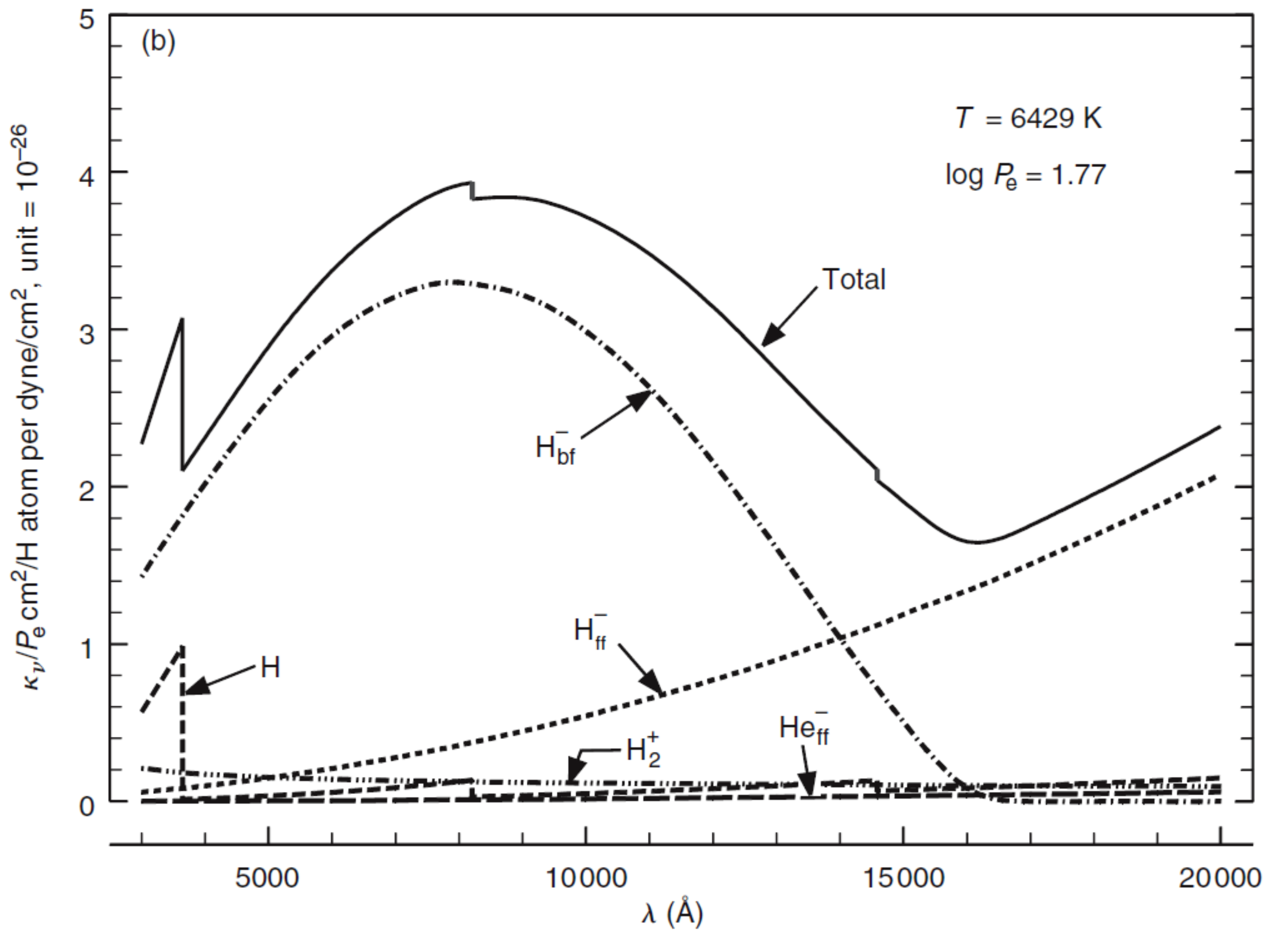
G-type (optical depth unity)

For G stars, the H⁻ ion (bound-free) dominates for optical.



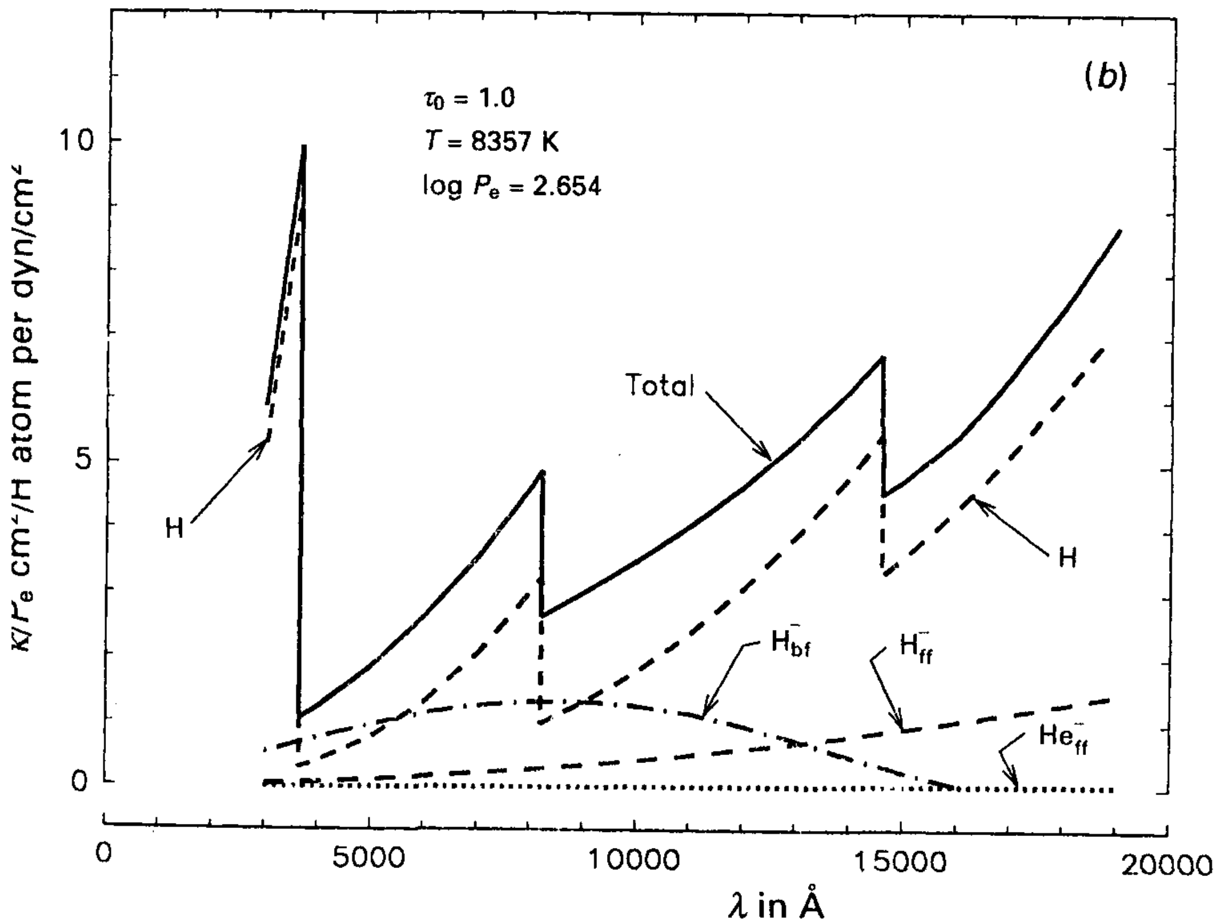
F-type (optical depth unity)

For F stars, the absorption is dominated by the two components of the H^- ion (bound-free) and (free-free), with a contribution from the Balmer continua below 3647Å.



A-type (optical depth unity)

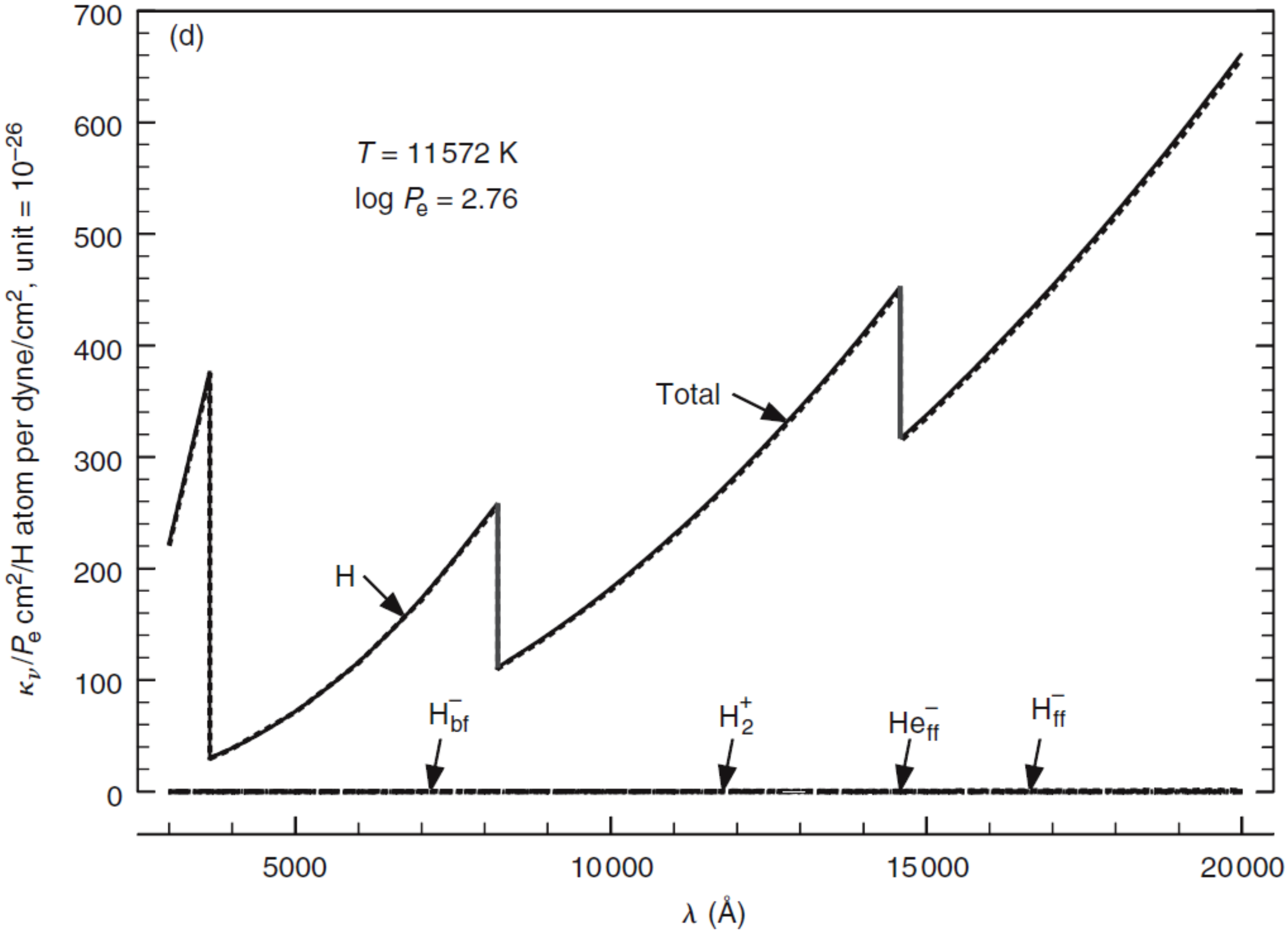
For a late A star, absorption from the H^- ion is dropping back compared to the cooler cases, while neutral hydrogen has grown with increasing temperature.
H (bound-free) Balmer, Paschen and Brackett continua start to dominate.



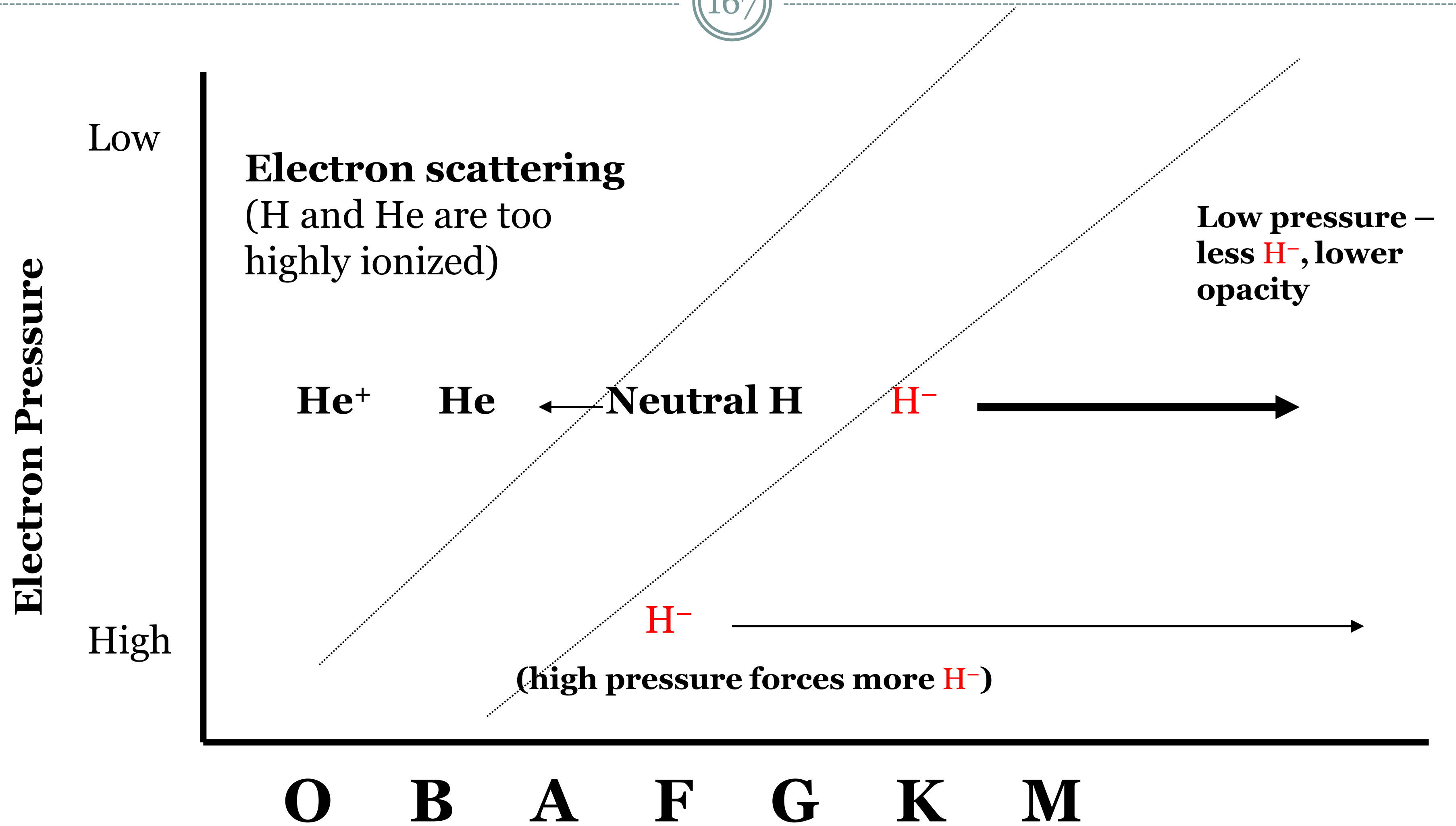
B-type (optical depth unity)

At late B, the H (bound-free) Balmer, Paschen & Brackett continua completely dominate.

For O stars electron scattering is the primary opacity source.



Dominant Opacity vs. Spectra Type

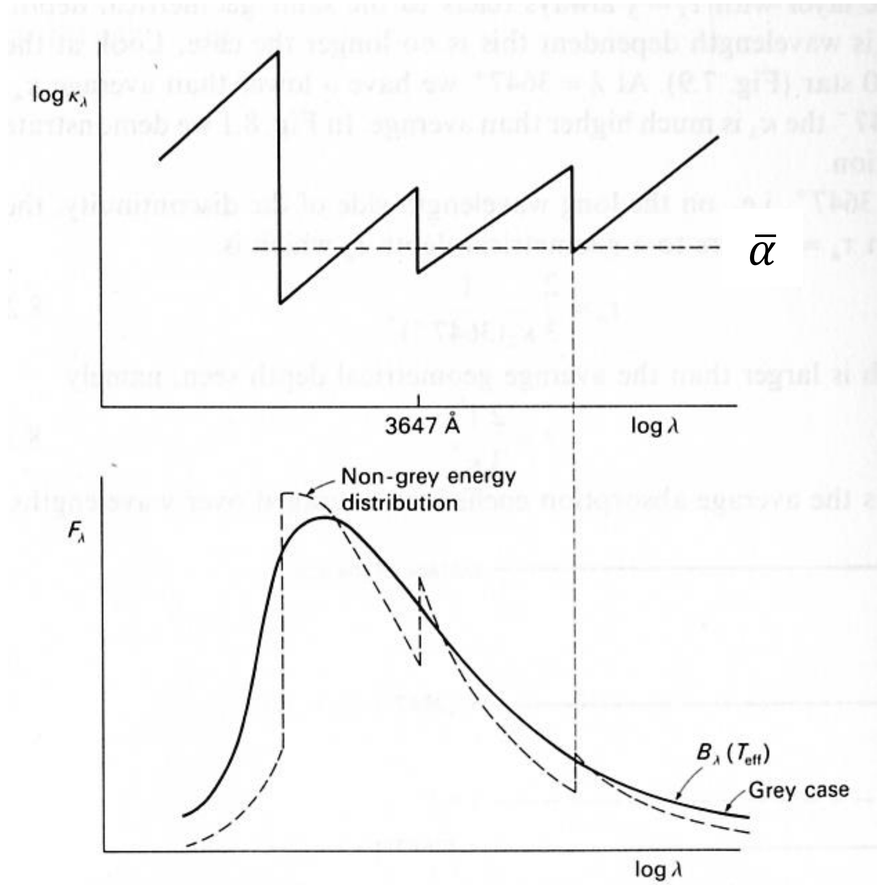


Continuum Energy Distribution

What is the effect of the λ dependence of α_λ on the emergent spectrum?

Consider the Balmer discontinuity at 3647Å. Immediately above the discontinuity (3647$^+$), the opacity α_λ is **lower** than average (shown as $\bar{\alpha}$), so we probe **deeper** than average into the atmosphere, where S_λ (and F_λ) is **higher** than the grey case, so F_λ exceeds the Planck function.

For 3647$^-$, the opacity is **higher** than average, so we probe **less deep** into the atmosphere (where T is smaller), and so receive a lower F_λ.



Balmer jump. Why is important?

- In hot stars, T>9000K, H^- negligible, only H contributes to opacity.

$$\frac{\alpha^+}{\alpha^-} = \frac{\sigma^+(\mathrm{H})}{\sigma^-(\mathrm{H})} \frac{N_H(n=3)}{N_H(n=2)}$$

Function of T only

"observed" known From Boltzmann law(T)

Thus, we can obtain the temperature.

In cooler stars (Solar-type)

$$\frac{\alpha^+}{\alpha^-} = \frac{\sigma(H^-)N(H^-) + \sigma^+(H)N_H(n=3)}{\sigma(H^-)N(H^-) + \sigma^-(H)N_H(n=2)}$$

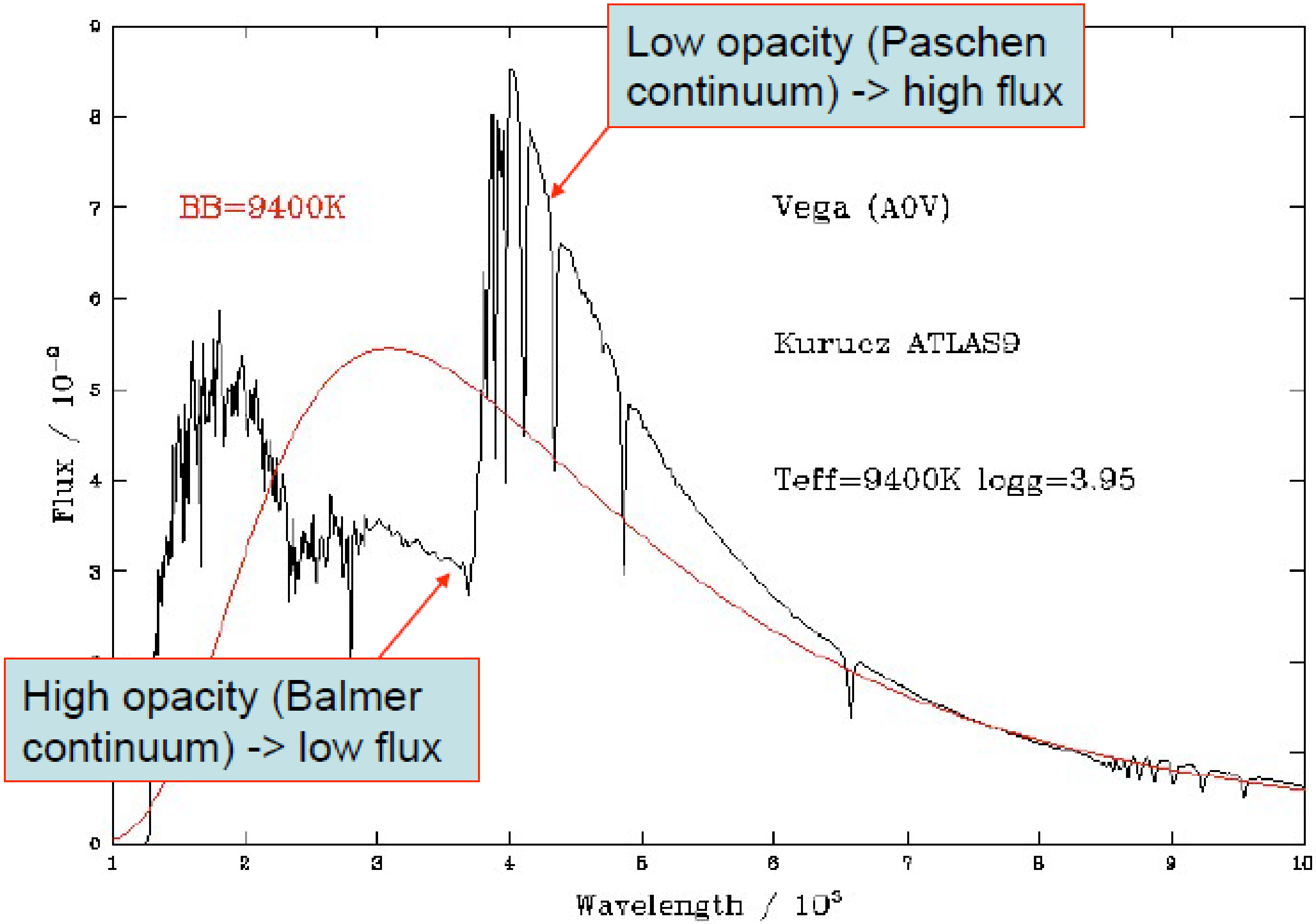
small

$$N(H^-) = N_H(n=1)n_e f(T) \quad \text{Saha eq} \Rightarrow \frac{\alpha^+}{\alpha^-}(n_e, T)$$

One of n_e or T can be determined

$$\text{if } n_e \uparrow \text{ then } \frac{\alpha^+}{\alpha^-} \to 1$$

Balmer jump in Vega



Summary

- Bound-bound transitions contribute to the line absorption. Bound-free and free-free transitions (plus scattering) contribute to the continuous absorption, mostly by H & He.
- Atomic H absorption coefficient highly *T* sensitive. For late-type stars in the optical and IR, bound-free and free-free transitions of the H^- ion dominate the continuous opacity, since the population of atomic H in n=3 (Paschen series) is so low.
- For early-type stars, atomic H dominates, producing strong jumps in the opacity at the Lyman, Balmer & Paschen edges.
- Negative H ion confirmed as dominant Solar optical & IR opacity source from limb darkening.
- He b-f opacity relevant only for very hot stars. Metal (Fe) opacity contributes to opacity in Solar-type stars in ultraviolet.
- Thompson (electron) scattering is grey & dominates continuum opacity in hot stars. Rayleigh scattering most important for late-type supergiants in UV
- Observed form of e.g. Balmer jump in A stars can be understood from the discontinuity in continuous H b-f opacity.
- Nongreyness changes the temperature structure.