[Theoretical] Astrophysics (765649S)

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Part 2

Theoretical Astrophysics

? Stellar Atmospheres

Interstellar Medium

Interacting Binary Stars

\+ Radiative processes

✓ Stellar Structure and Evolution

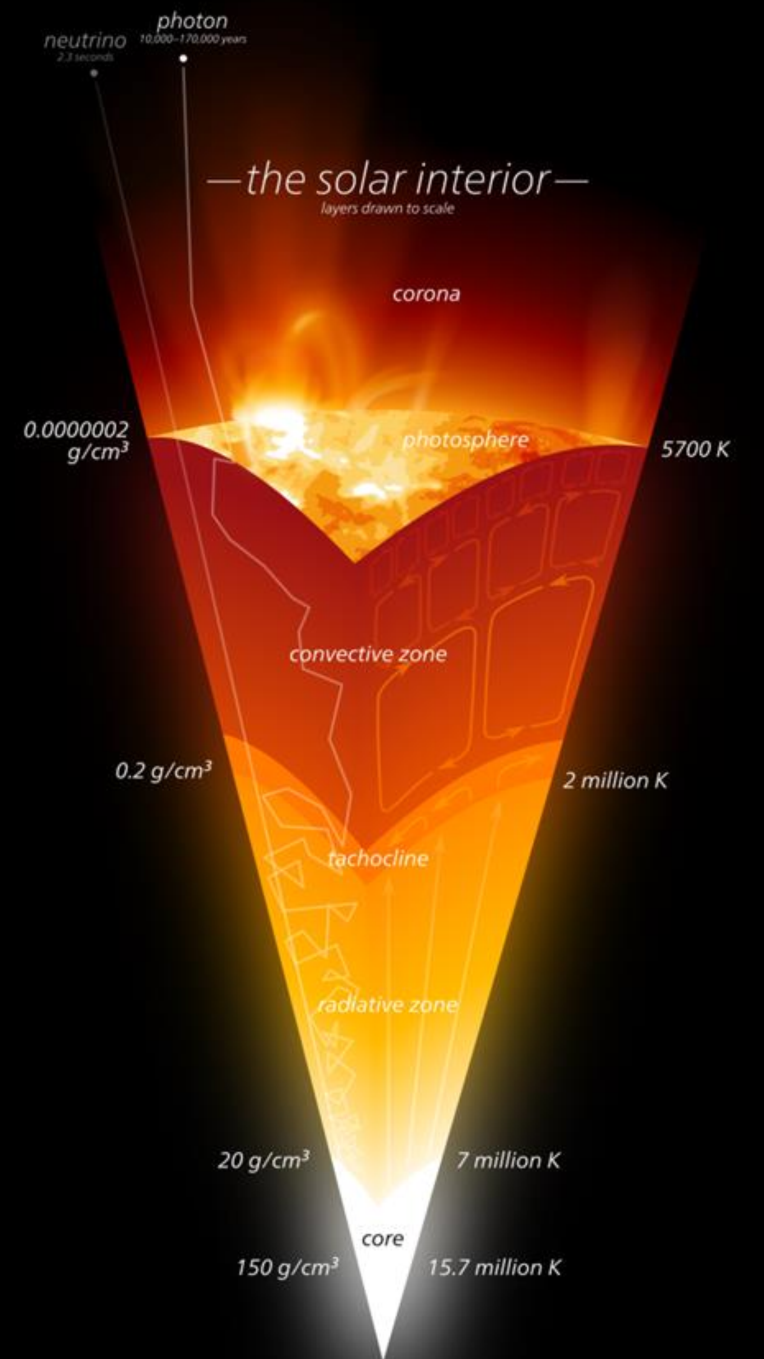
The topics which will be discussed in the given course

Stellar atmospheres

WHAT IS A STELLAR ATMOSPHERE?
WHY SHOULD WE CARE ABOUT IT?
WHAT CAN WE LEARN FROM OBSERVATIONS?

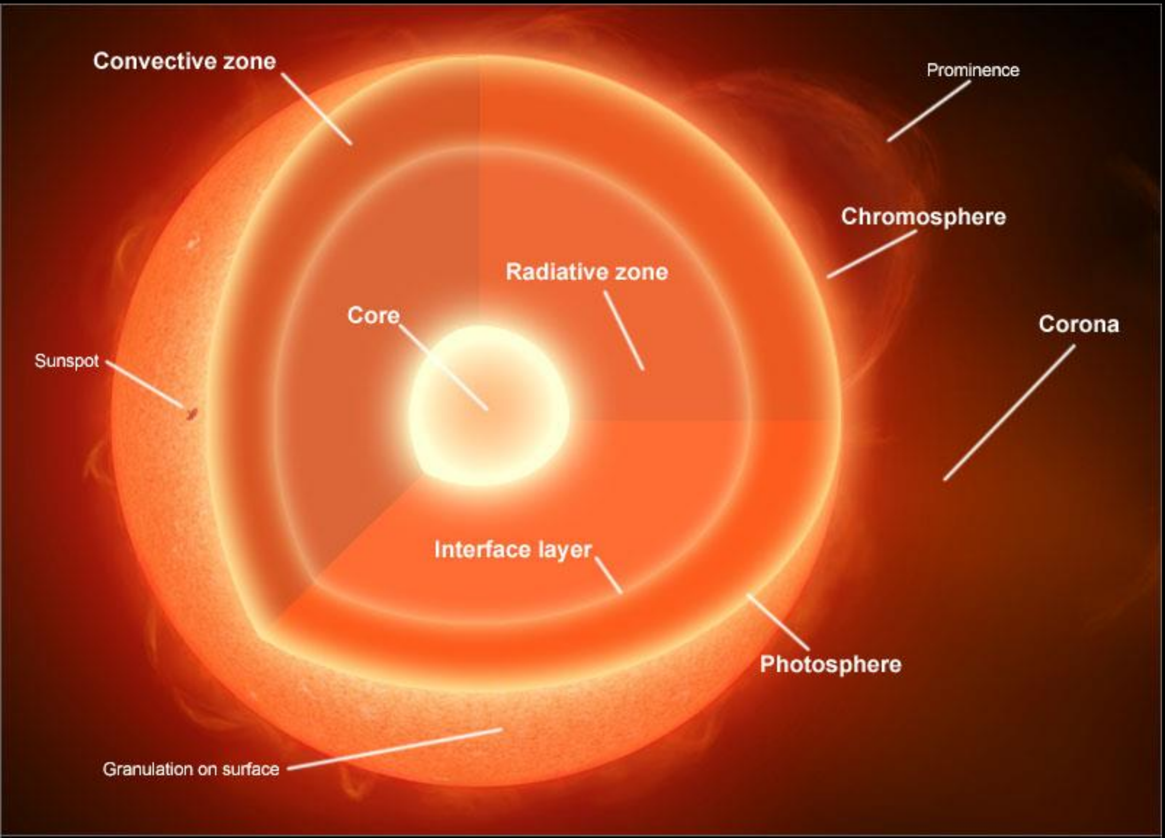
What is a stellar photosphere?

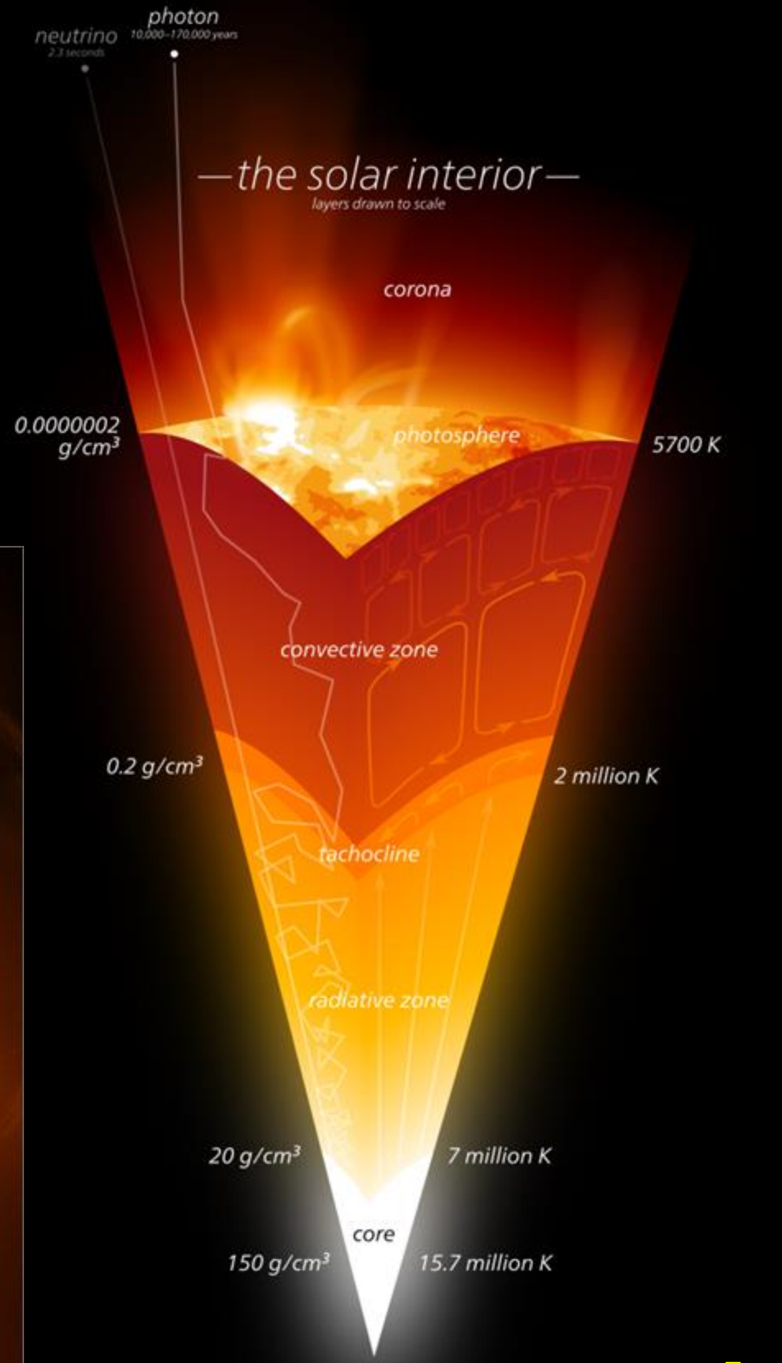
- Thin, tenuous transition zone between (invisible) stellar interior and (essentially vacuum) exterior.
- The "photosphere" is the visible disc, whilst the "atmosphere" also includes coronae and winds.
- In contrast with the interior, where convection may dominate, the energy transport mechanism of the atmosphere is radiation.
- Stellar atmospheres are primarily characterized by two parameters: (T_{eff}, $\log g$).



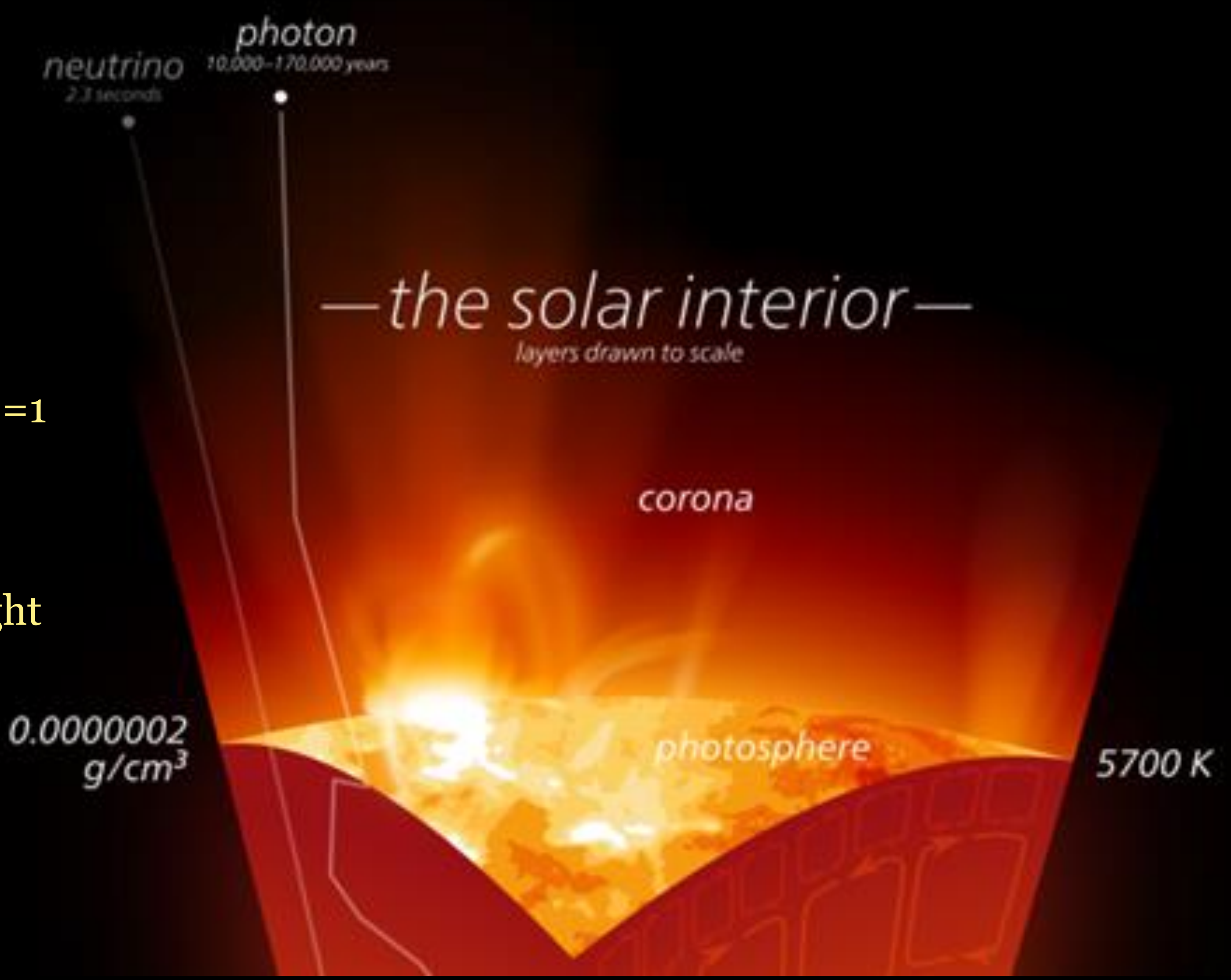
What is a stellar photosphere?

Thin zone between stellar interior and exterior:
ΔR_{sun}=a few$\times 10^7$ cm, $M_{atm} \sim 2\times 10^{21}$ g=$\sim 10^{-12}\ M_\odot$





Stellar atmospheres: why should we care?



The optical depth τ=1

about 2/3 of the light is absorbed

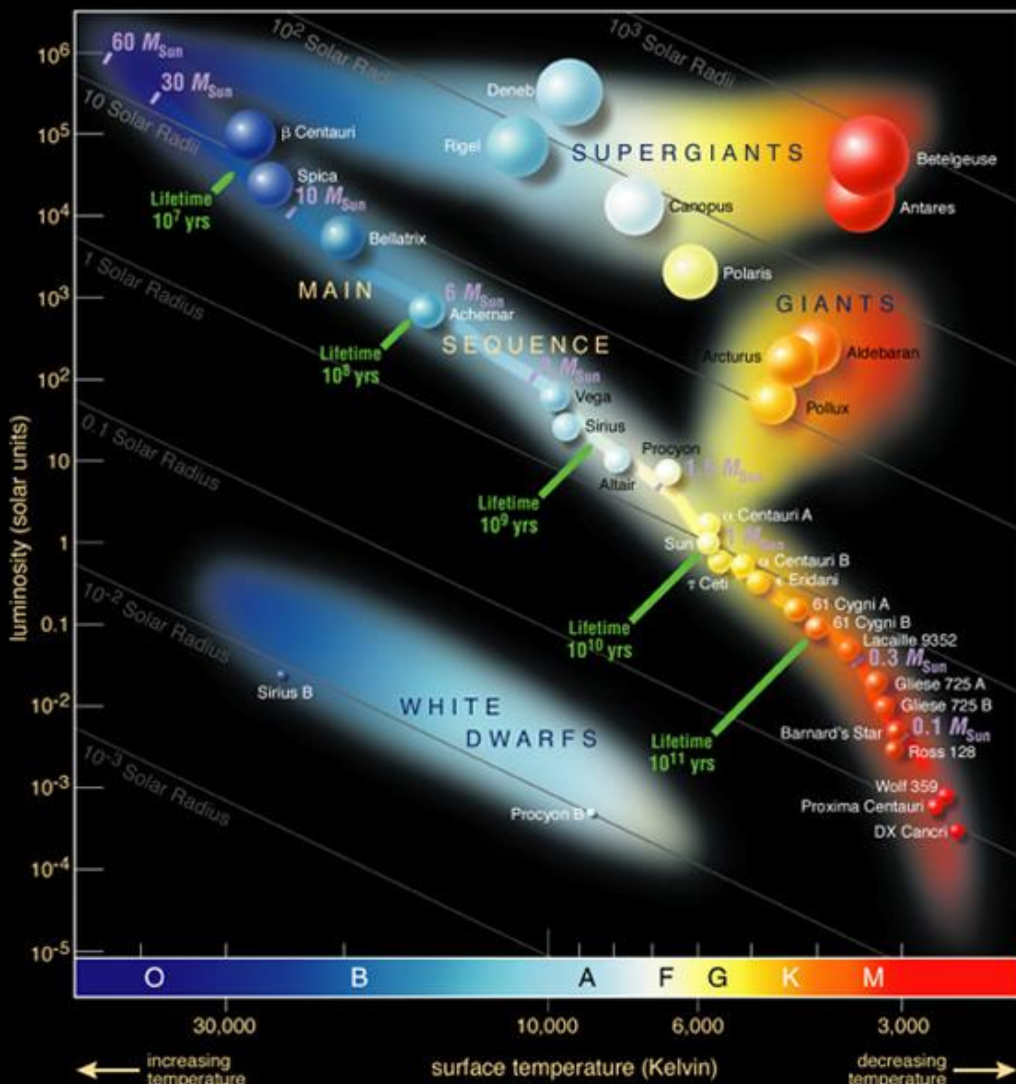
Stellar atmospheres?

- Stellar interiors are effectively invisible to external observers (apart for e.g. astroseismology) so **all** the information we receive from stars originates from their atmospheres. In particular, spectral lines also originate in a stellar atmosphere. Understanding how radiation interacts with matter affecting the emergent line and continuous spectrum is at the heart of this course.

- Knowledge of **plasma physics** (e.g. line broadening), **atomic physics** (microscopic interaction between light and matter), **radiative transfer** (macroscopic interaction between light and matter), **thermodynamics** (LTE vs non-LTE), **hydrodynamics** (velocity fields) yields stellar properties, chemical composition, outflow properties.

- Inputs for stellar/galactic evolution and structure.

Recap: what can we learn from observations?



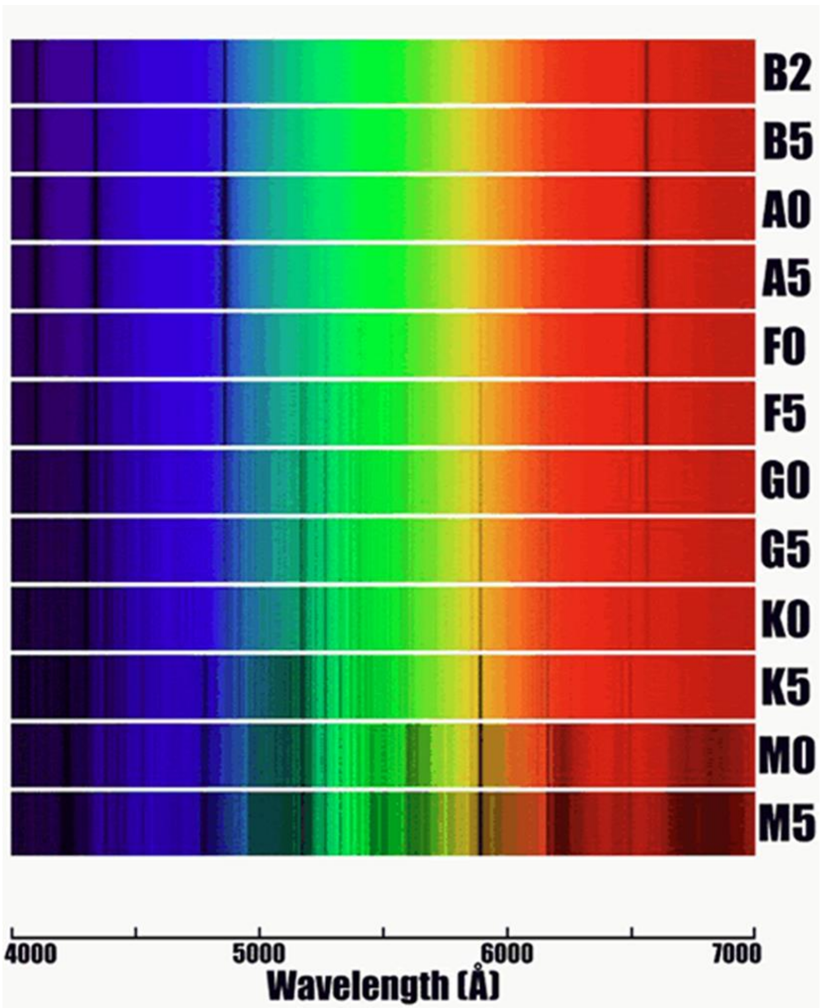
What can we learn from observations?

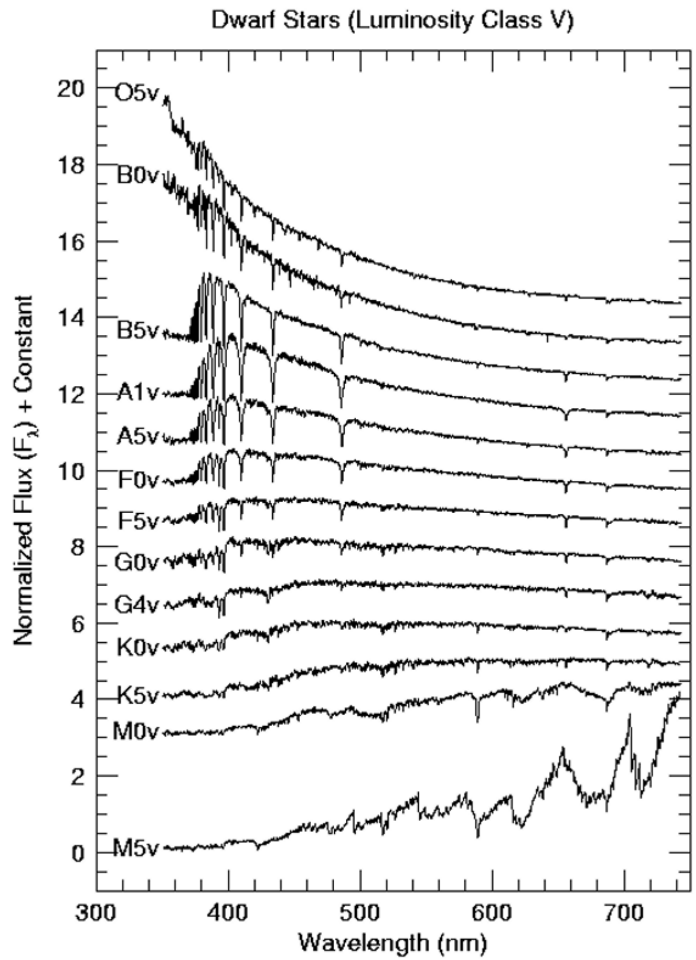
Please re-read Lecture 1 carefully.

Also, before the next class,
re-study Lectures 5 & 6 VERY carefully.
We will be based on that material a lot.

What can we learn from observations?

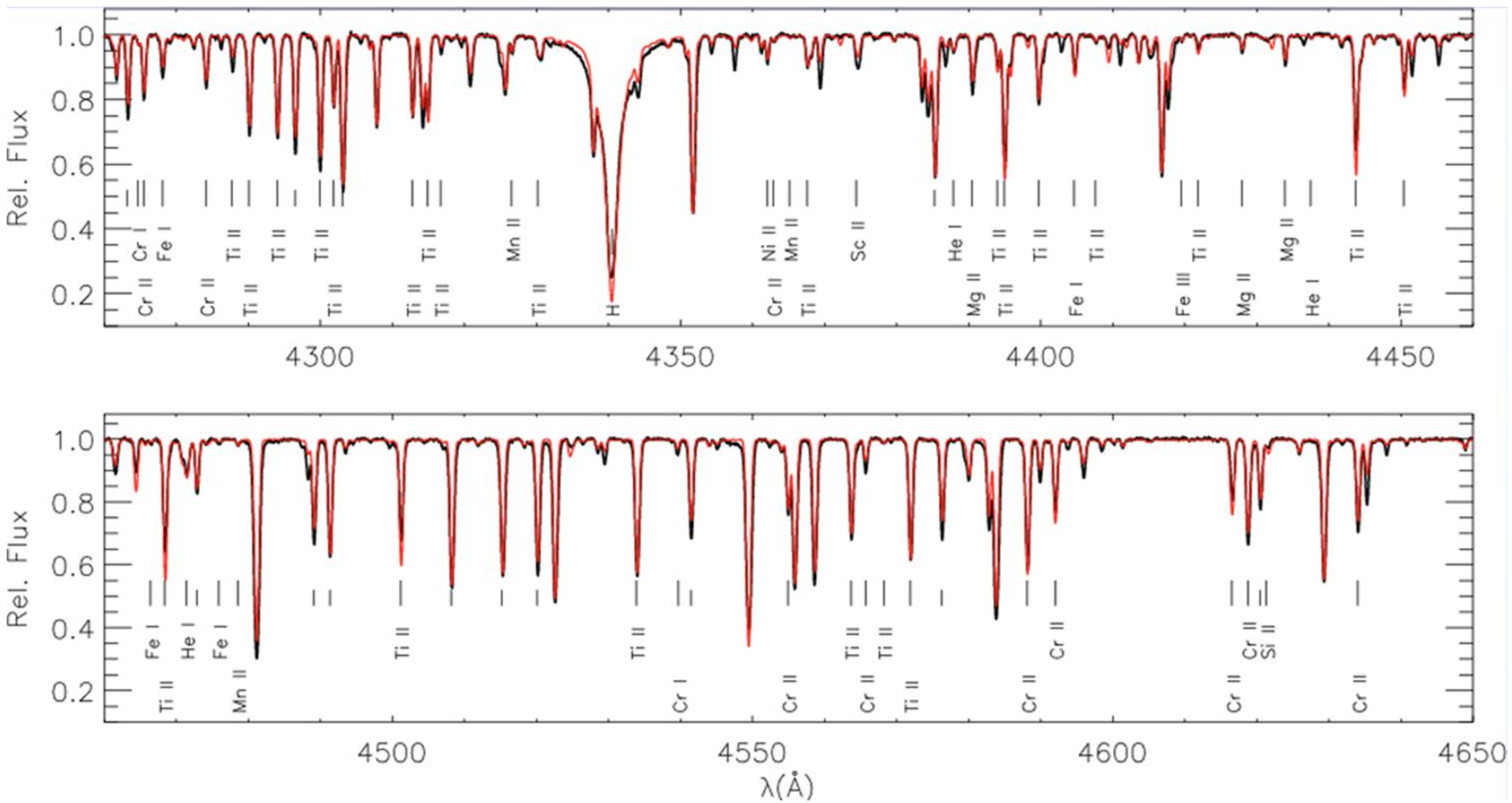
Temperature





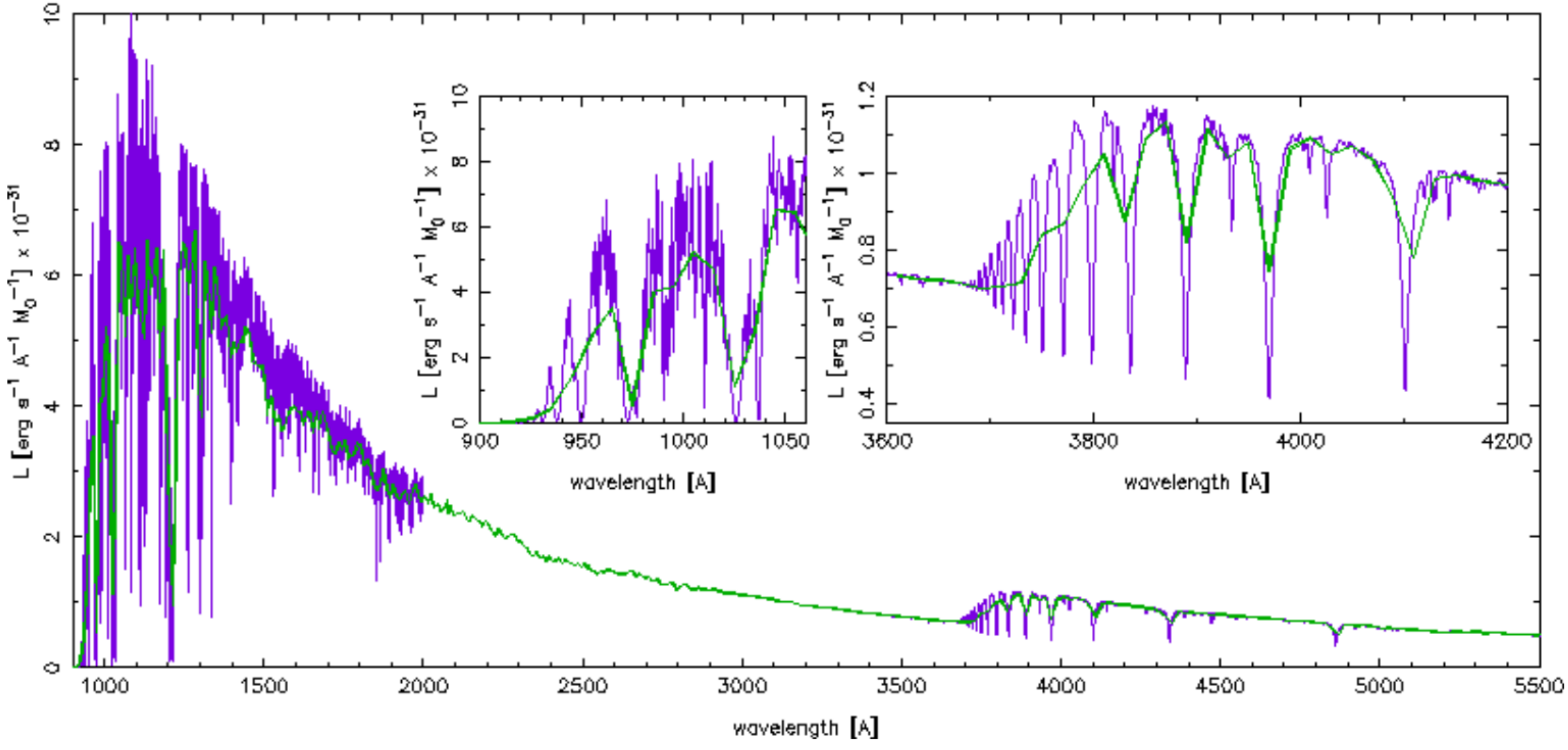
What can we learn from observations?

Surface gravity and stellar **abundances** also come from spectra:



Spectral Lines

Impact of Spectral Resolution



Primary star parameters (T_{eff}, log g)

- Primary star parameters are effective temperature $T_{\rm eff}$ and surface gravity log g, + chemical composition (metallicity):

  - **Effective temperature** (in K) is defined by $L=4\pi R^2\sigma T_{eff}^4$

    (here L - luminosity, R - stellar radius), related to *ionization*.

  - **Surface gravity** (cm/s²), $g = GM/R^2$, related to *pressure*.

- The Sun has $T_{\rm eff}$=5777K, log g=4.44 – its atmosphere is only a few hundred km deep, <0.1% of the stellar radius.
- A red giant has log g~1 (extended atmosphere), whilst a white dwarf has log g~8 (effectively zero atmosphere), and neutron stars have log g~14-15

Spectral Types

Morgan-Keenan (M-K) classification scheme orders stars via "OBAFGKM" spectral classes using ratios of line strength.

Only Bad Astronomers Forget Generally Known Mnemonics

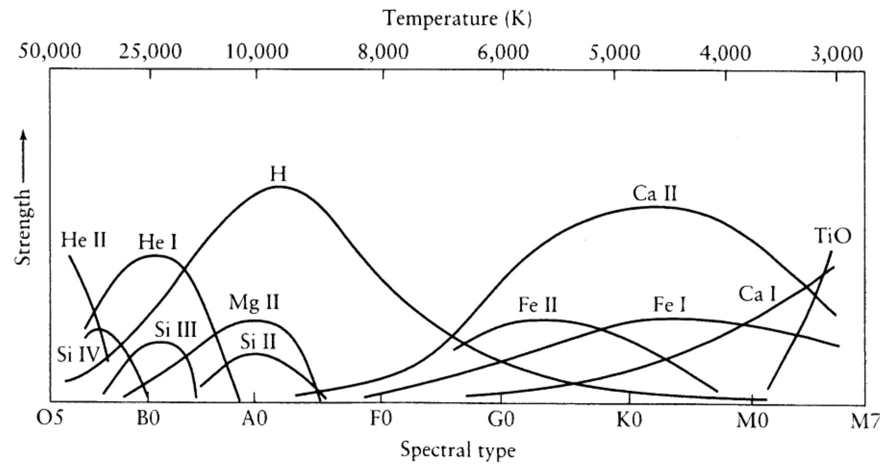
Oh, Be A Fine Girl/Guy, Kiss Me

Table 15.1. *MK spectral classes.*

MK spectral class	Class characteristics
O	Hot stars with He II absorption
B	He I absorption; H developing later
A	Very strong H, decreasing later; Ca II increasing
F	Ca II stronger; H weaker; metals developing
G	Ca II strong; Fe and other metals strong; H weaker
K	Strong metallic lines; CH and CN bands developing
M	Very red; TiO bands developing strongly

O-types have the bluest *B-V* & highest T_{eff}'s. OBA stars are early-type star, whilst cooler stars are late-type.

Spectral classes are each subdivided into (up to) ten divisions – e.g. O2 .. O9, B0, B1 .. B9, A0, A1 .. etc



Luminosity Class classification

- Luminosity class information is often added, based upon spectral line widths:

Ia	Most luminous supergiants
Ib	Less luminous supergiants
II	Luminous giant
III	Normal giants
IV	Subgiants
V	Main sequence stars (dwarfs)
VI	Subdwarfs
VII	White dwarfs

- Dwarfs have high pressures (large line widths) and supergiants have lower pressures (smaller line widths).

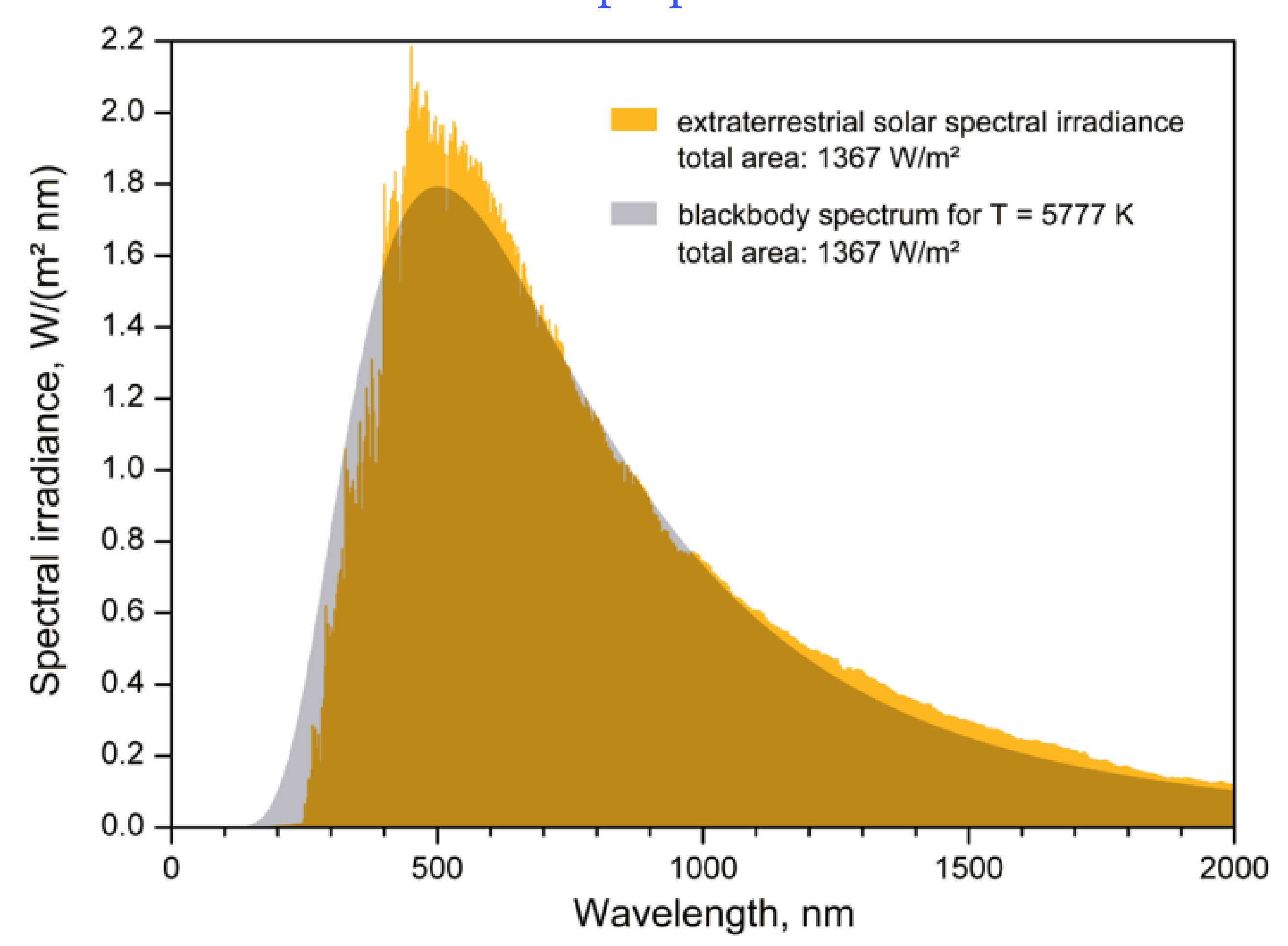
Luminosity Classes and Luminosity

- Line pairs for spectral classification:

Class	Line pairs for class	Class	Line pairs for luminosity
O5 ⇔ O9	4471 He I/4541 He II	O9 ⇔ B3	4116–21 (Si IV, He I)/4144 He I
B0 ⇔ B1	4552 Si III/4089 Si IV	B0 ⇔ B3	3995 N II/4009 He II
B2 ⇔ B8	4128–30 Si II/4121 He I	B1 ⇔ A5	Balmer line wings
B8 ⇔ A2	4471 He I/4481 Mg II	A3 ⇔ F0	4416/4481 Mg II
	4026 He I/3934 Ca II		
A2 ⇔ F5	4030–34 Mn I/4128–32	F0 ⇔ F8	4172/4226 Ca I
	4300 CH/4385		
F2 ⇔ K	4300 (G band)/4340 Hγ	F2 ⇔ K5	4045–63 Fe I/4077 Sr II
F5 ⇔ G5	4045 Fe I/4101 Hδ		4226 Ca I/4077 Sr II
	4226 Ca I/4340 Hγ	G5 ⇔ M	Discontinuity near 4215
G5 ⇔ K0	4144 Fe I/4101 Hδ	K3 ⇔ M	4215/4260, Ca I increasing
K0 ⇔ K5	4226 Ca I/4325		
	4290/4300		

Continuous Energy Distribution

Stars share some properties of black-bodies



Stefan – Boltzmann Law

Blackbody radiation is continuous and isotropic whose intensity varies only with wavelength and temperature.

Following empirical (Josef Stefan in 1879) and theoretical (Ludwig Boltzmann in 1884) studies of black bodies, there is a well-known relation between Flux and Temperature known as Stefan-Boltzmann law:

$$F = \sigma T^4$$

with $\sigma = 5.6705 \times 10^{-5}$ erg/cm^2/s/K^4

(Note that Bohm-Vitense refers to "astronomical flux", $H = F/\pi$, as "flux").

We will return to "different" types of fluxes later.

Flux

Flux (1)

- From an observational point of view, we are generally more interested in the energy flux or flux (L_λ, L) and the flux density (F_λ, F).
  Flux density gives the power of the radiation per unit area and hence has dimensions of erg s^{-1} cm^{-2} $Å^{-1}$ (or erg s^{-1} cm^{-2} Hz^{-1}).
  Observed flux densities are usually extremely small and therefore (especially in radio astronomy) flux densities are often expressed in units of the Jansky (Jy), where 1 Jy=10^{-23} erg s^{-1} cm^{-2} Hz^{-1}.

- You should be aware - and beware - that different authors define the terms flux density, flux and intensity differently, and they are sometimes used interchangeably!

- We will often call flux density as just flux.

- Standard definition:
  Flux describes any effect that appears to pass or travel through a surface or substance. In transport phenomena (radiative transfer, heat transfer, mass transfer, fluid dynamics), flux is defined as the rate of flow of a property per unit area, which has the dimensions [quantity]×$[time]^{-1}$×$[area]^{-1}$.
  - For example, the magnitude of a river's current, i.e. the amount of water that flows through a cross-section of the river each second is a kind of flux.

Flux (2)

In radiative transfer, flux is related to the intensity ("specific" is often omitted):

- Flux F_λ, is a measure of the net energy flow across an area $d\sigma$, over a time dt, in a $d\lambda$. The only directional significance is whether the energy crosses $d\sigma$ from the top or from the bottom. Then we can write:

The solid angle $d\omega$ appears for I_λ but not for F_λ

$$F_\lambda = \frac{\oint dE_\lambda}{d\lambda\, d\sigma\, dt}$$

Integrated over all directions.

substitute

$$I_\lambda = \frac{dE_\lambda}{\cos\theta\, d\lambda\, d\sigma\, d\omega\, dt}$$

$$F_\lambda = \oint I_\lambda \cos\theta\, d\omega \left[\frac{erg}{Å\, cm^2\, s}\right]$$

Thus, flux F_λ is the projection of the specific intensity I_λ in the radial direction (integrated over all solid angles)

The amount of energy going through 1 cm^2 per second per 1Å into the solid angle $d\omega$ in the direction inclined by the angle θ to the normal of the area.

Flux (3)

$d\omega = \sin\theta\, d\theta\, d\varphi$

Expressing $d\omega$ by means of θ and φ,

$$F_\lambda = \oint I_\lambda \cos\theta\, d\omega = \int_0^{2\pi} d\varphi \int_0^{\pi} I_\lambda \cos\theta \sin\theta\, d\theta$$

If there is no azimuthal dependence for I_λ then

$$F_\lambda = \oint I_\lambda \cos\theta\, d\omega = 2\pi \int_0^{\pi} I_\lambda \cos\theta \sin\theta\, d\theta$$

In the plane-parallel or spherical case, we do not find any dependence of I_λ on the longitude φ

$$F_\lambda = -2\pi \int_0^{\pi} I_\lambda \cos\theta\, d(\cos\theta)$$

Meaning of flux:

Radiation flux = netto energy going through area
Decomposition into two half-spaces:

$$F = -2\pi \int_0^{\pi} I_\lambda \cos\theta\, d(\cos\theta) = 2\pi \int_{-1}^{1} I(\mu)\, \mu\, d\mu \qquad \mu = \cos\theta$$

$$= 2\pi \int_0^1 I(\mu)\, \mu\, d\mu + 2\pi \int_{-1}^0 I(\mu)\, \mu\, d\mu$$

$$= 2\pi \int_0^1 I(\mu)\, \mu\, d\mu - 2\pi \int_0^1 I(-\mu) \mu\, d\mu = \boldsymbol{F^+ - F^-}$$

Netto = Outwards - Inwards

Special cases: at the surface of a star $F^- = 0$, so that $F = F^+$
at the centre of a star, isotropic radiation field: $F = 0$

Magnitude scale

- In practice, we often (historically) measure flux densities F (erg cm^{-2} s^{-1}) from astronomical objects via a logarithmic magnitude scale (like the eye and most other human senses).
- See the course “Observational Astronomy” (765640S) for more detail (lecture 9), here we discuss it shortly.

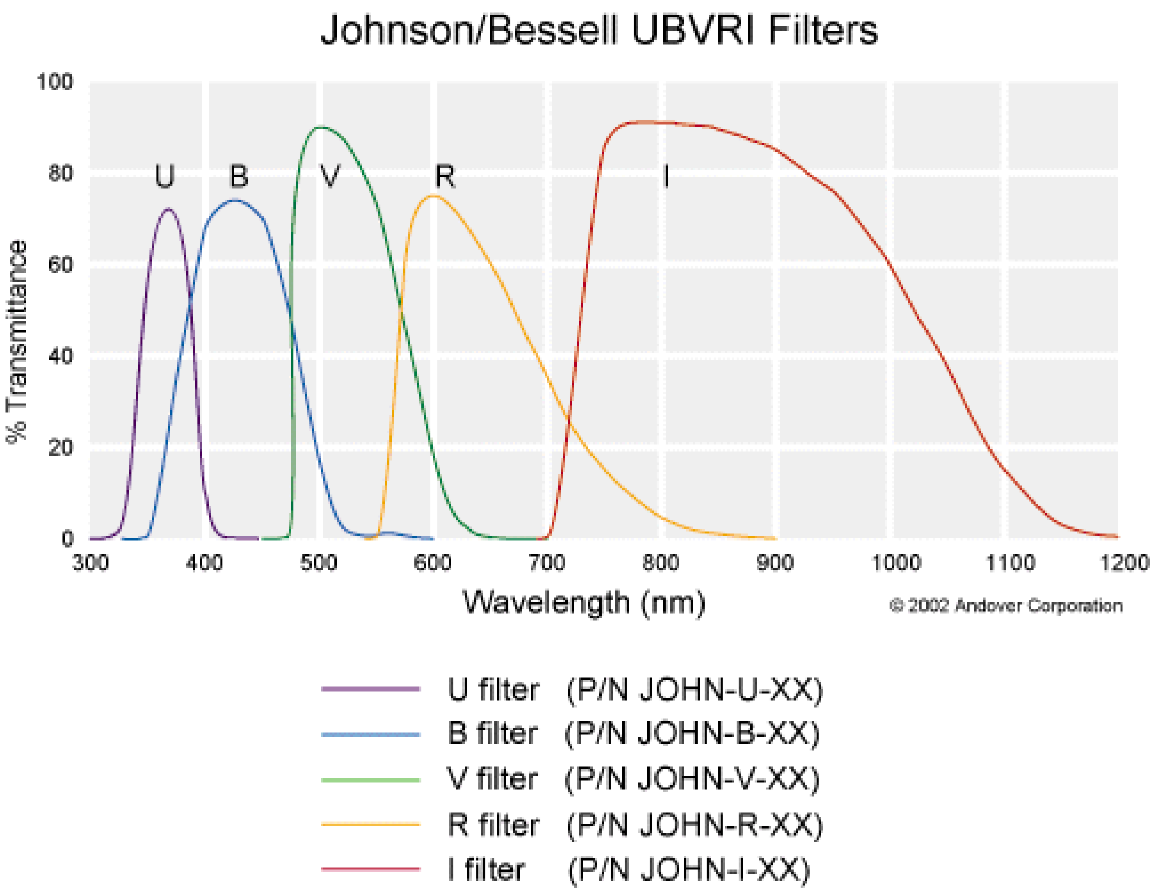
- $m_v - m_0 = -2.5 \log(F_v/F_0)$

In the Vega system, the star Vega (A0V) defines the photometric “zero point” m_0 at all wavelengths (U=B=V=R=I=0.0 mag etc).

Table 15.6. *Flux calibration for an A0 V star.*

Symbol	Flux (erg cm^{-2} s^{-1} Å^{-1})	λ_0 (μm)
U	4.22×10^{-9}	0.36
B	6.40×10^{-9}	0.44
V	3.75×10^{-9}	0.55
R	1.75×10^{-9}	0.71
I	8.4×10^{-10}	0.97

Standard broad-band filters



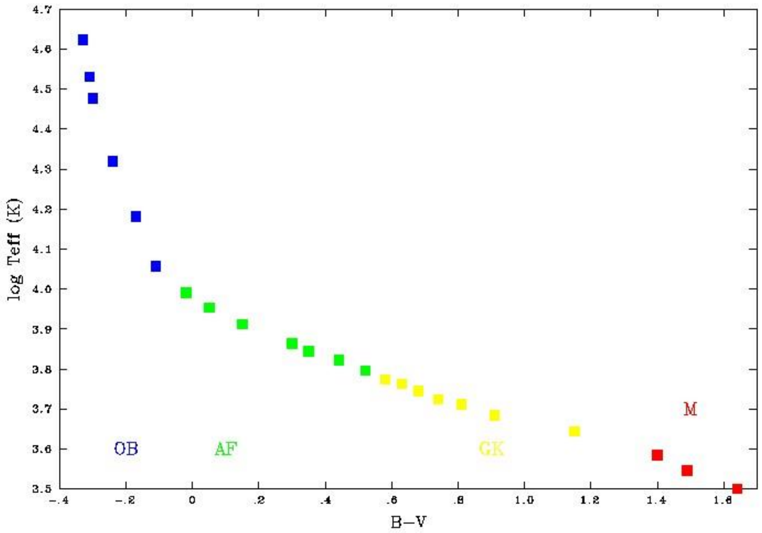
It is convenient to measure flux densities or magnitudes within some certain frequency or wavelength range. The total energy measured is then the integral of the source flux times some frequency dependent effective filter response. This last quantity includes all the factors that modify the energy arriving at the top of the Earth's atmosphere.

$$m = -2.5 \log \int_0^\infty F_\nu W(\nu)\, d\nu + \text{constant}$$

F_ν – a star SED
$W(\nu)$ – a filter passband

Colour index

- We can define a colour index as the difference between filters relative to Vega e.g. $B - V = m_B - m_V$, such that stars bluer than A0 have a negative B-V colour and stars redder than Vega have a positive colour e.g. $(B\text{-}V)_{Sun} = +0.65$ mag.



$$B - V = -2.5\log\left(\frac{\int F_\nu W_B(\nu)\,d\nu}{\int F_\nu W_V(\nu)\,d\nu}\right) + 0.710$$

$$U - B = -2.5\log\left(\frac{\int F_\nu W_U(\nu)\,d\nu}{\int F_\nu W_B(\nu)\,d\nu}\right) - 1.093.$$

e.g., for T_{eff}<10000K:

$$T = \frac{7090}{(B - V) + 0.71} K$$

More on magnitudes

- We define the absolute (visual) magnitude (M_V) as the apparent (visual) magnitude of a star of m_V lying at a distance of d=10pc: $M_V=m_V$ (10 pc).
- Because $F \propto d^{-2}$
  $M_V - m_V = -2.5 \log[F(10pc)/F(d)] = -5\log(d/10\ pc) = 5 - 5\log(d/pc)$
- For the Sun (d=4.85×10^{-6} pc), m_V=-26.75 and M_V=+4.82 mag.
  The "distance modulus" $M_v - m_v$=31.57 mag
- Because interstellar medium is not completely transparent, we write
  $M_V - m_V = 5 - 5\log(d/pc) - A_V$.
- The A_V term is due to interstellar extinction.
  Visually, $A_V \sim$ 3.1 E(B-V) for most sight lines.
  E(B-V)=B-V – $(B-V)_o$, i.e. the difference between the observed and intrinsic B-V colour.

Interstellar Extinction

Extinction is MUCH higher at shorter wavelengths, so IR observations of e.g. Milky Way disk probe much further. The extinction to the Galactic Centre (*d*=8kpc) is approx A_V=30 mag (5500A) versus A_K=3 mag (2μm).

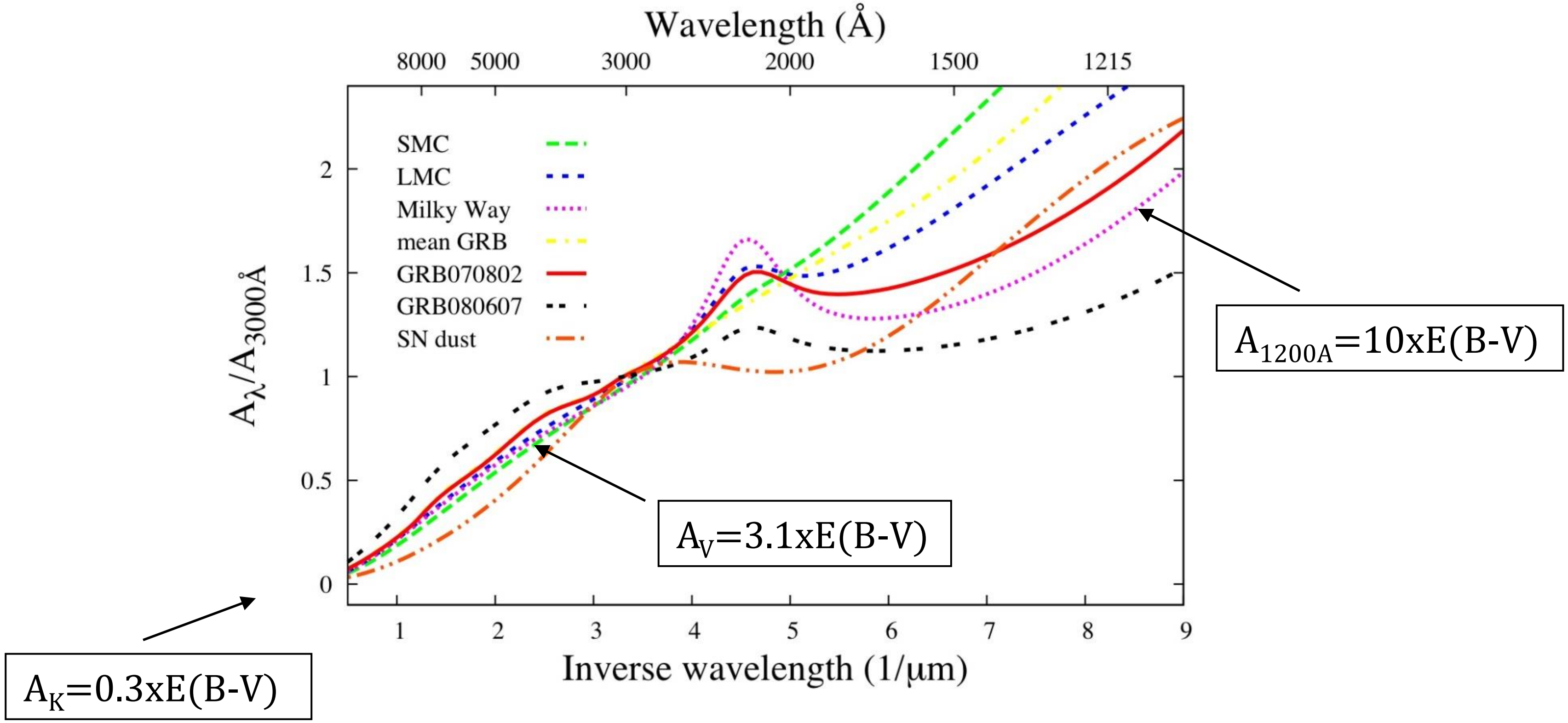
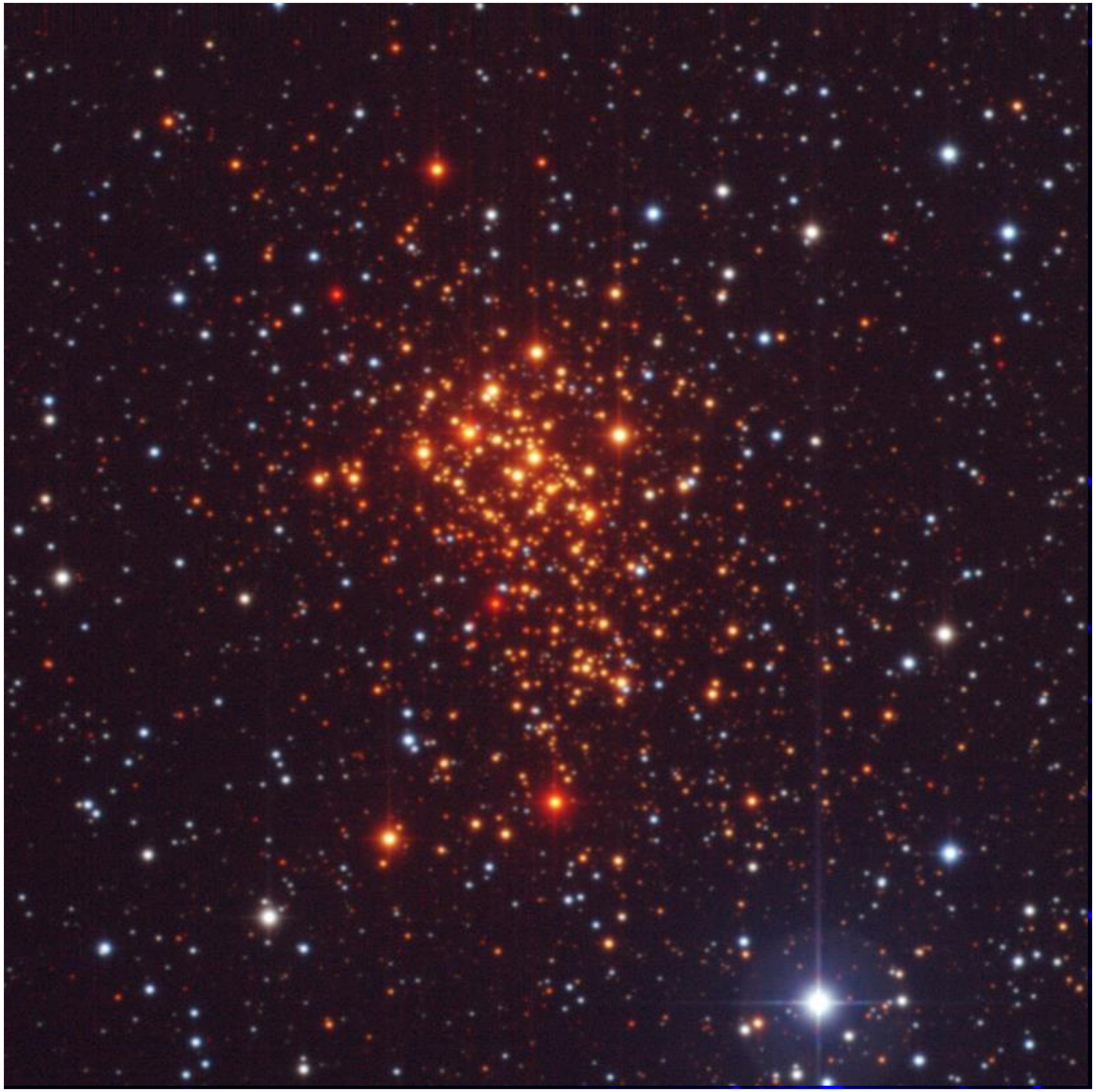


Illustration of interstellar extinction

V-band (5500Å)

R-band (7000Å)

I-band (9000Å)



VRI-composite
of highly reddened
cluster Wd1 (E_{B-V}~4)

Bolometric Flux

- The bolometric flux (erg cm^{-2} s^{-1}) from a star received at the top of the Earth's atmosphere is the integral of the spectral flux (measured at a frequency ν or a wavelength λ) over all frequencies or wavelengths:

$$F_{Bol} = \int_0^\infty F_\nu d\nu = \int_0^\infty F_\lambda d\lambda$$

- The luminosity (erg/s) is the bolometric flux from the star integrated over a full sphere (at distance d):

$$L = 4\pi d^2 F_{Bol}$$

- Since the Earth's atmosphere is opaque to UV and some IR radiation one cannot always directly measure the bolometric flux.

Bolometric Corrections

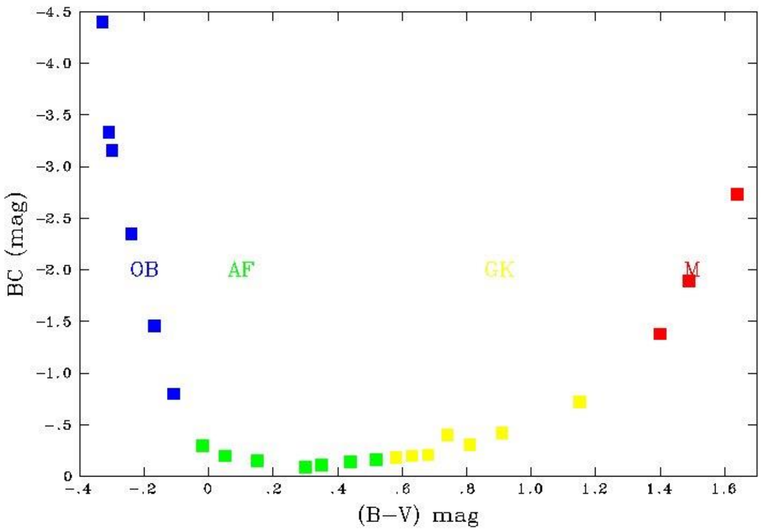
One can calculate bolometric corrections (BC), primarily from atmospheric models to correct measured fluxes (usually in the V band) for the total (bolometric) flux. Usually expressed in magnitudes:

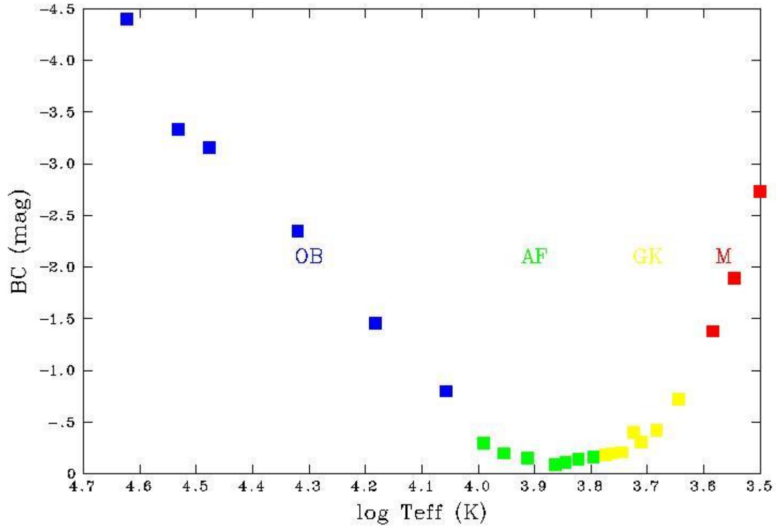
$$BC = M_{bol} - M_V \quad \text{with} \quad M_{bol} = 4.74 - 2.5 \log(L/L_{\odot})$$

BC=-0.08 mag for the Sun is a small correction since it emits most radiation in the visual. Hot OB stars have very negative BC's, since most of the energy is emitted in the UV, as are cool M stars with most energy emitted in the IR.

BC calibrations

Bolometric corrections can be estimated from intrinsic colours $(B\text{-}V)_o$ as shown here for dwarfs:





Or from the Spectral Type, using a T_{eff} – Spectral Type calibration. See the next slide...

Properties of Main-Sequence Stars

Table 15.7. *Calibration of MK spectral types.*

Sp	*M(V)*	*B − V*	*U − B*	*V − R*	*R − I*	T_{eff}	BC
MAIN SEQUENCE, V							
O5	−5.7	−0.33	−1.19	−0.15	−0.32	42 000	−4.40
O9	−4.5	−0.31	−1.12	−0.15	−0.32	34 000	−3.33
B0	−4.0	−0.30	−1.08	−0.13	−0.29	30 000	−3.16
B2	−2.45	−0.24	−0.84	−0.10	−0.22	20 900	−2.35
B5	−1.2	−0.17	−0.58	−0.06	−0.16	15 200	−1.46
B8	−0.25	−0.11	−0.34	−0.02	−0.10	11 400	−0.80
A0	+0.65	−0.02	−0.02	0.02	−0.02	9 790	−0.30
A2	+1.3	+0.05	+0.05	0.08	0.01	9 000	−0.20
A5	+1.95	+0.15	+0.10	0.16	0.06	8 180	−0.15
F0	+2.7	+0.30	+0.03	0.30	0.17	7 300	−0.09
F2	+3.6	+0.35	0.00	0.35	0.20	7 000	−0.11
F5	+3.5	+0.44	−0.02	0.40	0.24	6 650	−0.14
F8	+4.0	+0.52	+0.02	0.47	0.29	6 250	−0.16
G0	+4.4	+0.58	+0.06	0.50	0.31	5 940	−0.18
G2	+4.7	+0.63	+0.12	0.53	0.33	5 790	−0.20
G5	+5.1	+0.68	+0.20	0.54	0.35	5 560	−0.21
G8	+5.5	+0.74	+0.30	0.58	0.38	5 310	−0.40
K0	+5.9	+0.81	+0.45	0.64	0.42	5 150	−0.31
K2	+6.4	+0.91	+0.64	0.74	0.48	4 830	−0.42
K5	+7.35	+1.15	+1.08	0.99	0.63	4 410	−0.72
M0	+8.8	+1.40	+1.22	1.28	0.91	3 840	−1.38
M2	+9.9	+1.49	+1.18	1.50	1.19	3 520	−1.89
M5	+12.3	+1.64	+1.24	1.80	1.67	3 170	−2.73

From Allen's Astrophysical Quantities (4th edition)

Solve a problem

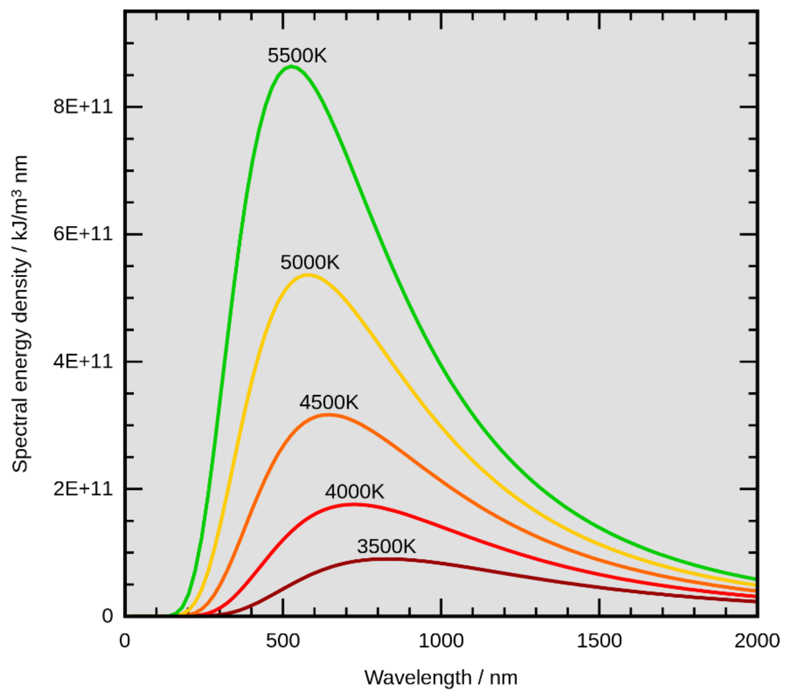
A B5V star in the LMC (distance 50kpc) has V=13.5 mag, B-V=-0.07 mag.

What is its bolometric luminosity, relative to the Sun?

Properties of the Planck law

- For increasing temperatures, the black body intensity increases for all wavelengths. The maximum in the energy distribution shifts to shorter λ (longer ν) for higher temperatures.
- λ_{max} **T = 2.98978 x10^{7}** Å K

  is Wien's displacement law for the maximum I_λ providing an estimate of the peak emission (λ_{max} =5175 Å for the Sun).



Rayleigh-Jeans and Wien approximations

At long wavelengths $\lambda >> \lambda_{max}$ (small frequencies $\nu << \nu_{max}$) the Planck formulae

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1} \qquad B_\lambda(T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}$$

can be approximated by the Rayleigh-Jeans law

$$B_\nu(T) \approx 2\frac{\nu^2}{c^2}kT\,, \quad B_\lambda(T) \approx 2ckT\lambda^{-4}$$

At short wavelengths $\lambda \le \lambda_{max}$ (large frequencies $\nu \ge \nu_{max}$), the Wien law is a good approximation

$$B_\nu(T) \approx 2\frac{h\nu^3}{c^2}e^{-\frac{h\nu}{kT}}\,, \quad B_\lambda(T) \approx 2\frac{hc^2}{\lambda^5}e^{-\frac{hc}{\lambda kT}}$$

Color and brightness temperatures

Define brightness temperature as $I_\nu = B_\nu(T_b)$

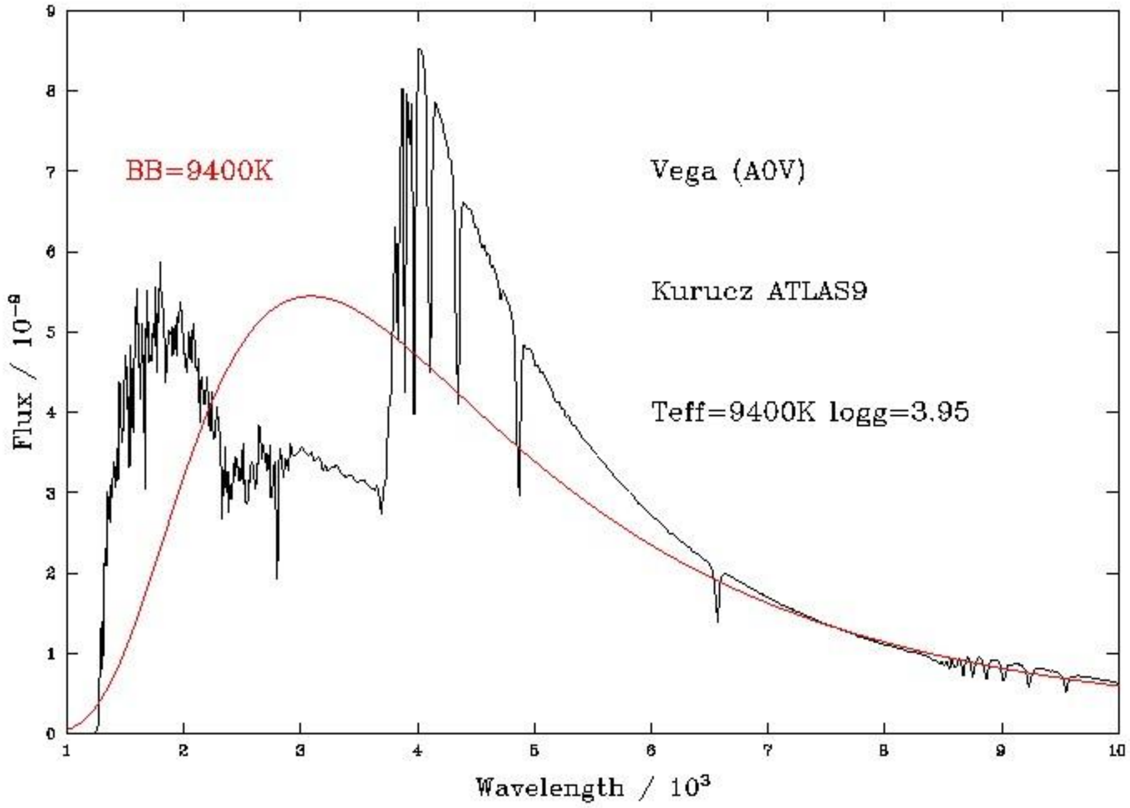
In radio band we get

$$I_\nu = 2\frac{\nu^2}{c^2}kT_b \quad \text{, so that } T_b = \frac{c^2}{2\nu^2 k} I_\nu \text{ for } h\nu << kT$$

Colour temperature T_c is obtained by "fitting" the observed spectrum with the Planck function ignoring normalization. It gives correctly the temperature of the black body source of unknown absolute scale of the intensity.

Are stars black bodies?

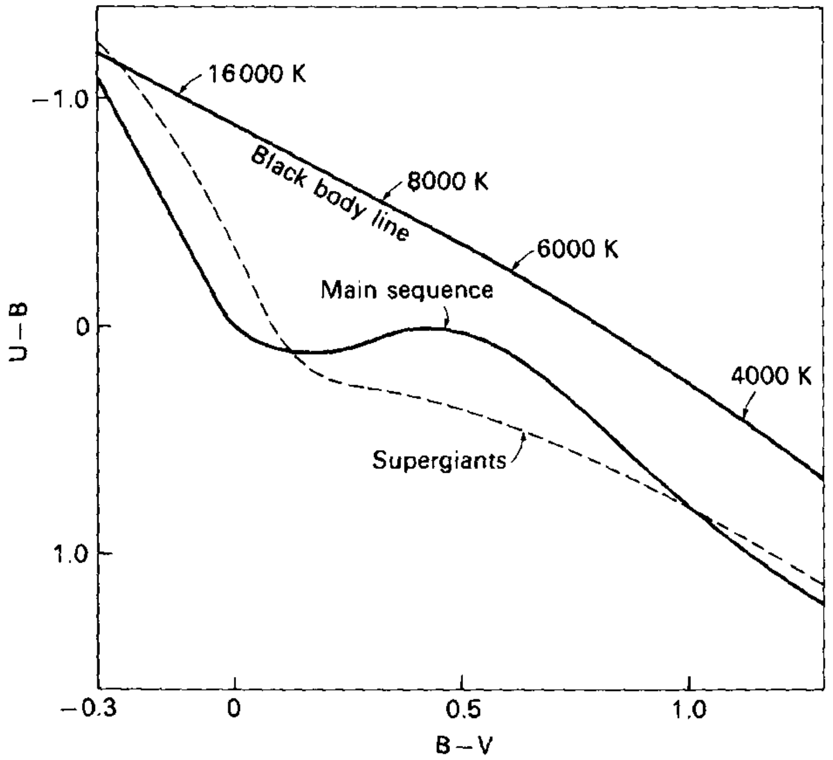
(e.g. UV-optical spectrophotometry of Vega)



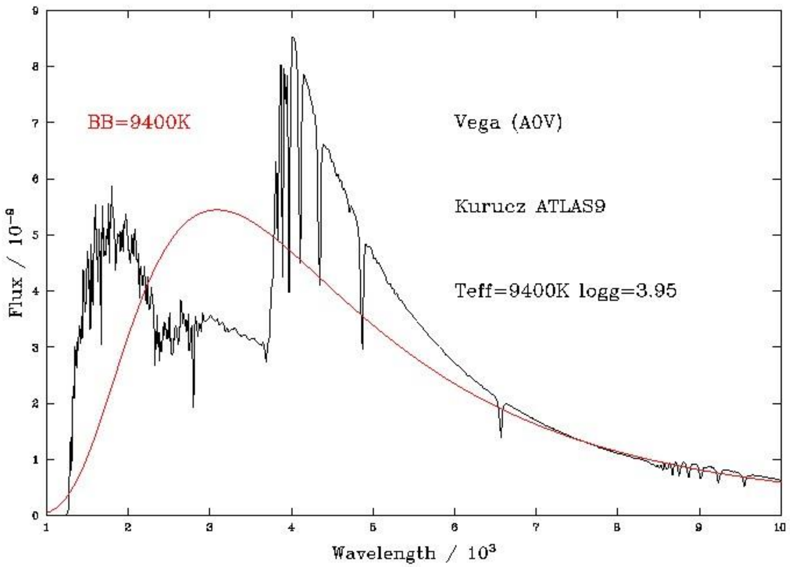
Not really

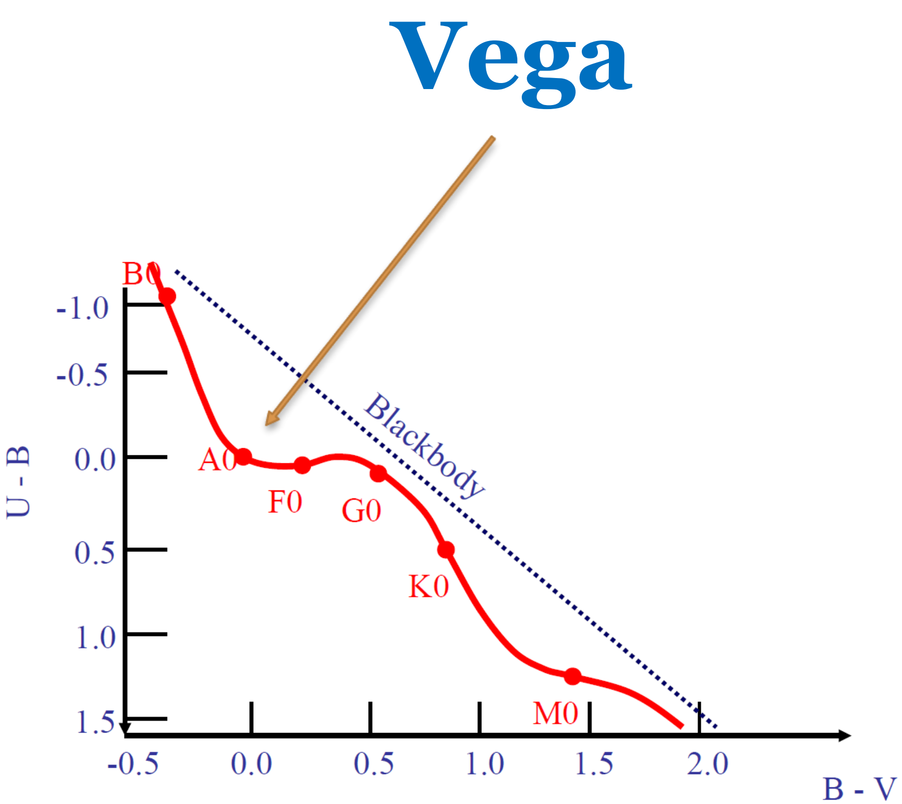
Stars do differ from black bodies

The observed flux distributions of real stars deviate from black body curves, as indicated here for the UBV colors of dwarfs and supergiants. This difference is due to sources of continuous and line opacity in the stellar photospheres and will be discussed later in this course.



Stars do differ from black bodies





Radiative transfer III

RADIATIVE TRANSFER EQUATION IN PLANE-PARALLEL ATMOSPHERE.
LIMB DARKENING.

Solar limb darkening

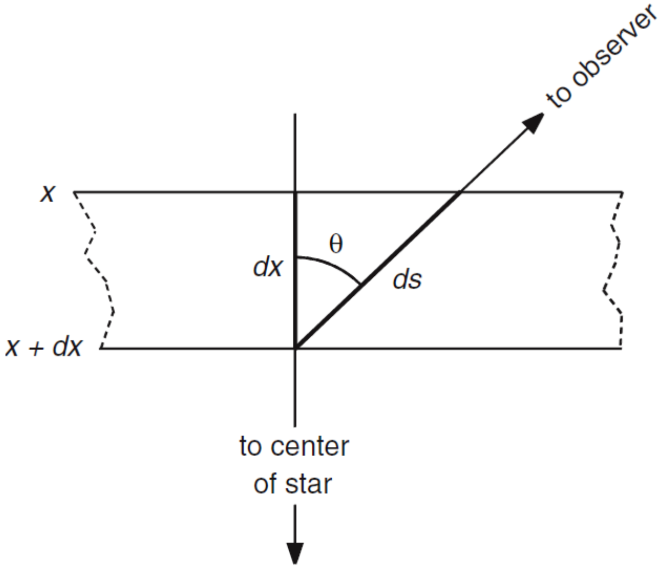
Transfer Equation for Stars

From lecture 6 (side 166):

The plane-parallel transfer equation
(for stars with thin photospheres)

$$\cos\theta \frac{dI_\lambda(\theta)}{d\tau_\lambda} = I_\lambda(\theta) - S_\lambda$$

The cos(θ) term is because the optical depth is measured along the radial direction *x* and not along the line of sight, i.e $d\tau_\lambda = -\kappa_\lambda \rho \, dx$



We are looking from the outside in, along direction x

Surface Intensity

- To derive the intensity at the **surface**, we can multiply the plane-parallel transfer equation by an integrating factor $e^{-\tau/\cos\theta} = e^{-u}$,

$$\frac{dI_\lambda(\theta)}{du} e^{-u} - I_\lambda(\theta)\ e^{-u} = -S_\lambda e^{-u}$$

- This can be written as

$$\frac{d(I_\lambda(\theta)e^{-u})}{du} = -S_\lambda e^{-u}$$

- Integrating du from 0 to infinity

$$[I_\lambda(\theta)e^{-u}]_0^\infty = -\int_0^\infty S_\lambda\,(\tau_\lambda)e^{-u}du$$

$$I_\lambda(0,\theta) = \int_0^\infty S_\lambda\,(\tau_\lambda)e^{-u}du$$

Limb darkening

Let us assume a linear source function: $S_\lambda(\tau_\lambda) = a_\lambda + b_\lambda \tau_\lambda$

We then derive: $I_\lambda(0,\theta) = \int_0^\infty S_\lambda(\tau_\lambda) e^{-u} du = \int_0^\infty a_\lambda e^{-u} du + \int_0^\infty b_\lambda \tau_\lambda e^{-u} du$

Recall $u = \tau/\cos(\theta)$, so $\tau = u \cos(\theta)$ and $I_\lambda(0,\theta) = a_\lambda \int_0^\infty e^{-u} du + b_\lambda \cos\theta \int_0^\infty u\, e^{-u} du$

Using the standard integral $\int_0^\infty u^n e^{-u} du = n!$

we obtain $$I_\lambda(0,\theta) = a_\lambda + b_\lambda \cos\theta = S_\lambda(\tau_\lambda = \cos\theta)$$

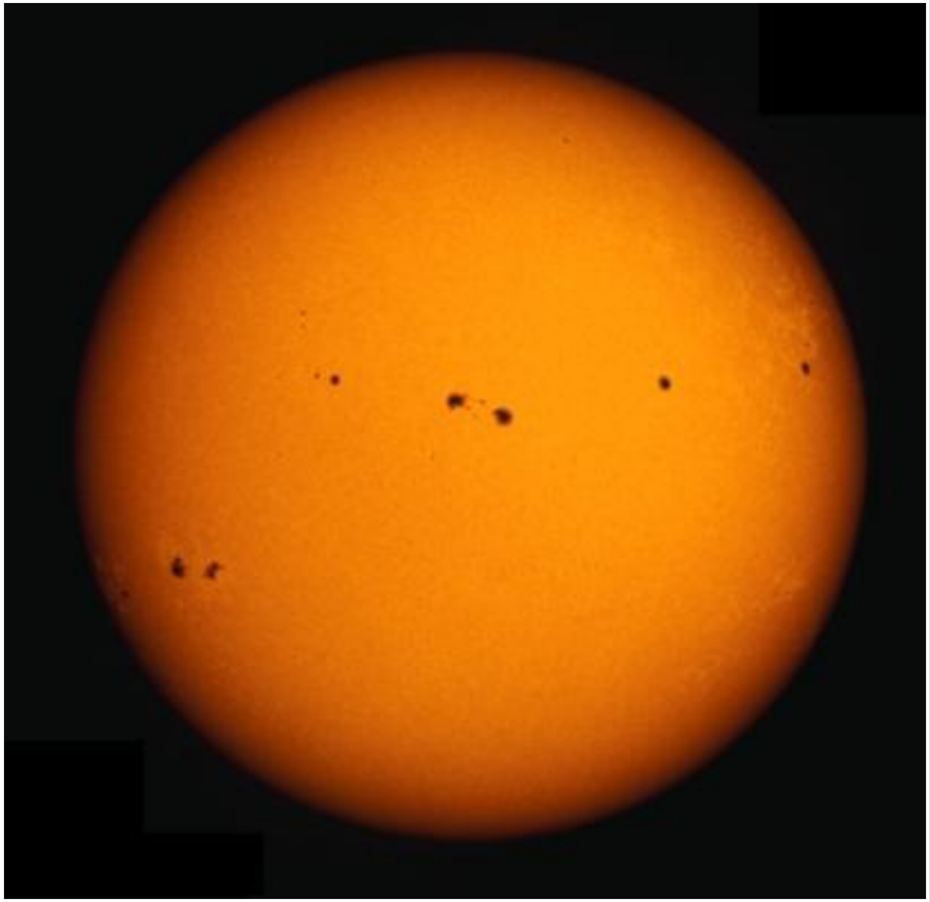
Thus, in the linear approximation for the Source function, the optical depth lies between 0 and 1. From the centre of the star we see radiation leaving the star perpendicular to the surface: $I_\lambda(0,0°)=a_\lambda+b_\lambda$, whilst at the limb the starlight leaves the surface at an angle $I_\lambda(0,90°)=a_\lambda$.

Limb darkening (less light from the limb versus the centre, if $b_\lambda>0$).

Solar limb darkening

- This optical image of the Sun clearly shows limb darkening. We see into the atmosphere down to a depth of $\tau=1$.
- Limb darkening exists because the continuum source function decreases outward:
  $S_\lambda(\tau_\lambda) = a_\lambda + b_\lambda \tau_\lambda$,
  both a_λ and $b_\lambda > 0$.

- As we look towards the limb, we see higher photospheric layers, which are less bright.



Schematic of limb darkening

Schematic illustration of limb darkening – penetration of different lines of sight (thick lines) to "unit optical depth" (dashed lines) corresponds to different depths in the photosphere, depending on θ. Radiation seen at θ_2 is characteristic of higher (cooler) layers than the radiation seen at position θ_1