Stellar evolution



THE ZERO-AGE MAIN SEQUENCE STELLAR EVOLUTION FROM OBSERVATIONS EVOLUTION DURING THE MAIN SEQUENCE POST-MS EVOLUTION OF LOW-MASS STARS POST-MS EVOLUTION OF HIGH-MASS STARS

The Zero-Age Main Sequence

- Kelvin–Helmholtz contraction continues until the central temperature becomes high enough for nuclear fusion reactions. Once the energy generated by hydrogen fusion compensates for the energy loss at the surface, the star stops contracting and settles on the zero-age main sequence (ZAMS) if its mass is above the hydrogen burning limit of $\sim 0.08 M_{\odot}$.
- Because contraction is slowest when both *R* and *L* are small, the pre-main sequence lifetime is dominated by the final stages of contraction, when the star is already close to the ZAMS.
- Since the nuclear energy source is much more concentrated towards the centre than the gravitational energy released by overall contraction, the transition from contraction to hydrogen burning requires a **rearrangement** of the internal structure.
- When we look at a population of stars that are at many different ages, and thus at many random points in their lives, we expect the number of stars we see in a given population to be proportional to the fraction of its life that a star spends as a member of that population. Since the main sequence is the most heavily populated part of the H–R diagram and the hydrogen nuclear burning phase is the longest evolutionary phase, it seems natural to assume that main sequence stars are burning hydrogen.

The Main Sequence stars (1)

Main-sequence stars obey several relations:

- Main sequence stars obey a mass-luminosity relation, with $L \propto M^{\eta}$. The slope η changes slightly over the range of masses; between 1 and $10M_{\odot}$, $\eta \approx 3.88$. The relation flattens out at higher masses, due to the contribution of radiation pressure in the central core. This helps support the star, and decreases the central temperature slightly. The relation also flattens significantly at the very faint end of the luminosity function. This is due to the increasing important of convection for stellar structure.
- Main sequence stars obey a mass-radius relation. However, the relation displays a significant break around $1M_{\odot}$; $R \propto M^{\beta}$, with $\beta \approx 0.57$ for M > $1M_{\odot}$, and $\beta \approx 0.8$ for M < $1M_{\odot}$. This division marks the onset of a convective envelope. Convection tends to increase the flow of energy out of the star, which causes the star to contract slightly. As a result, stars with convective envelopes lie below the mass-radius relation for non-convective stars. This contraction also increases the central temperature (via the virial theorem) and also moves the star above the nominal mass-luminosity relation.
- The depth of the convective envelope (in terms of $M_{\rm env}/M$) increases with decreasing mass. Stars with $M \sim 1M_{\odot}$ have extremely thin convective envelopes, while stars with $M < 0.3M_{\odot}$ are entirely convective. Nuclear burning ceases around $0.08M_{\odot}$.
- The interiors of stars are extremely hot (T > 10^6 K). The fall-off to surface temperatures (T~ 10^4 K) takes place in a very thin region near the surface.

Some Relations for the MS stars

Mass-Radius relation

M-R and M-L relations





The Main Sequence stars (2)

Main-sequence stars obey several relations (cont):

- The region of nuclear energy generation is restricted to a very small mass range near the center of the star. The rapid fall-off of ε_n with radius reflects the extreme sensitivity of energy generation to temperature.
- Stars with masses below $\sim 1 M_{\odot}$ generate most of their energy via the proton-proton chain. Stars with more mass than this create most of their energy via the CNO cycle. This changeover causes a shift in the homology relations for the stellar interior.
- CNO burning exhibits an extreme temperature dependence. Consequently, those stars that are dominated by CNO fusion have very large values of $L/4\pi r^2$ in the core. This results in a large value of $\nabla_{\text{rad}} \equiv \frac{d \ln T}{d \ln P}$, and convective instability. In this region, convective energy transport is extremely efficient, and $\nabla \approx \nabla_{\text{ad}}$.
- Because of the extreme temperature sensitivity of CNO burning, nuclear reactions in high mass stars are generally confined to a very small region, much smaller than the size of the convective core.
- As the stellar mass increases, so does the size of the convective core (due again to the large increase in ε_n with temperature). Supermassive stars with M ~100 M_{\odot} would be entirely convective.
- A star's position on the ZAMS depends on both its mass and its initial helium abundance. Stars with higher initial helium abundances have higher luminosities and effective temperatures. The higher mean molecular weight translates into lower core pressures. Helium rich stars therefore are more condensed, which (through the virial theorem) mean they have higher core temperatures and larger nuclear reaction rates.

Main-sequence lifetimes

330

- Stars arrive at the main sequence chemically homogeneous because of convective mixing during the protostellar phase.
- The time of arrival on the main sequence is known as the ZAMS zero-age main sequence.
- "Where" it ends up depends only on mass and chemical composition.
- Approximate MS lifetimes: $\tau_{MS} \approx 10^{10} (M/M_{\odot})^{-2.5}$ yr
- Stars of all masses live on the main-sequence, but subsequent evolution differs enormously.
- Most of stars form in clusters, open and globular. Because they born at same time, age of cluster will show on the H–R diagram as the upper end, or turn-off of the main-sequence.
- We can use this as a tool (clock) for measuring age of star clusters. Stars with lifetimes less than cluster age, have left main sequence. Stars with main-sequence lifetimes longer than age, still sit on the main-sequence.

Mass, M_{\odot}	Time, yr
0.1	$6 imes 10^{12}$
0.5	7×10^{10}
1.0	1×10^{10}
1.25	4×10^9
1.5	2×10^9
3.0	2×10^8
5.0	7×10^7
9.0	2×10^7
15	1×10^7
25	6×10^{6}

Stellar Evolution from observations

Globular cluster Messier 55

H-R diagram







Stellar Evolution from observations

Different globular clusters have different age, and accordingly different turn-off points.



Stellar Evolution from observations

- An interesting effect can be observed in young star clusters.
- Recall, the time needed for a protostar to reach the ZAMS depends on its mass. This time is basically the Kelvin-Helmholtz contraction timescale. Since contraction is slowest when both *R* and *L* are small, the pre-main sequence lifetime is dominated by the final stages of contraction, when the star is already close to the ZAMS.
- An estimate of the PMS lifetime is $\tau_{PMS} \approx 10^7 (M/M_{\odot})^{-2.5} \text{ yr}$
- Thus, massive protostars reach the ZAMS much earlier than lower-mass stars, and they can even leave the MS while low-mass stars still lie above and to the right of it.

Table 8.1 Evolutionary lifetimes (years)

M/M_{\odot}	<i>I</i> –2	2–3	3-4	4–5
15	6.7(2)	2.6(4)	1.3(4)	6.0(3)
9	1.4(3)	7.8(4)	2.3(4)	1.8(4)
5	2.9(4)	2.8(5)	7.4(4)	6.8(4)
3	2.1(5)	1.0(6)	2.2(5)	2.8(5)
2.25	5.9(5)	2.2(6)	5.0(5)	6.7(5)
1.5	2.4(6)	6.3(6)	1.8(6)	3.0(6)
1.25	4.0(6)	1.0(7)	3.5(6)	1.0(7)
1.0	8.9(6)	1.6(7)	8.9(6)	1.6(7)
0.5	1.6(8)			

Note: powers of 10 are given in parentheses.



Pleiades, a young open star cluster

Massive protostars reach the ZAMS much earlier than lower-mass stars, and they can even leave the MS while low-mass stars still lie above and to the right of it.

Pleiades, a young open star cluster, $\sim 10^8$ yr:



B-V UNCORRECTED INDEX

(B-V) CORRECTED INDEX

Tracks

Stellar evolutionary <u>tracks</u> are trajectories of individual stars in the H-R diagram, which trace the evolution of a **given** mass star as a function of time.

Consider stars of different masses but with the same age. Let's make a plot of $Log(L/L_{\odot})$ vs. Log T_{eff} for an age of 1Gyr. The result is an <u>isochrone</u>.

Describe evolution of a single star with time. Tables of stellar parameters as function of *T*: (luminosity, temperature, surface gravity; core temperature, core composition, current mass, etc, etc).



Isochrones

An isochrone: a curve which traces the properties of stars as a function of mass for a given age.

Don't be confused with an evolutionary **track** which shows the properties of a star as a function of age for a **fixed** mass.

Isochrones are particularly useful for star clusters - all stars born at the same time with the same composition.

The best way to check stellar evolutionary calculations is to compare calculated isochrones and an observed H-R diagram of a cluster.

Important - think about what we are looking at when we observe a cluster. We see a "freezeframe" picture at a particular age. We see how stars of different masses have evolved up to that fixed age (this is **not** equivalent to an evolutionary track).



Convective regions on the ZAMS

Occurrence of convective regions (gray shading) on the ZAMS in terms of fractional mass coordinate m/M as a function of stellar mass, for detailed stellar models with a composition X = 0.70, Z = 0.02.

The solid (red) lines show the mass shells inside which 50% and 90% of the total luminosity are produced.

The dashed (blue) lines show the mass coordinate where the radius r is 25% and 50% of the stellar radius R.

We can distinguish three types of ZAMS stars:

- completely convective, for $M < 0.35 M_{\odot}$
- radiative core + convective envelope, for $0.35 M_{\odot} < M < 1.2 M_{\odot}$
- convective core + radiative envelope, for $M > 1.2 M_{\odot}$.



(After Kippenhahn & Weigert)

Evolution During the Main Sequence

During the MS phase H is converted into He in the core. The temperature in the core can only change very little, because fusion is a strong function of *T* with $\varepsilon \sim T^4$ for P-P chain and $\sim T^{18}$ for the CNO-cycle. Even a small change in *T* would result in a large change in ε and in *L*, which is not allowed by the hydrostatic equilibrium requirement. So nuclear fusion acts like a thermostat in the center of the star. The CNO cycle is a better thermostat than the pp chain.

Even while on the main sequence, the composition of a star's core is changing, thus μ_c increases. a Sun-like star

Recall, when Z is negligible: $\mu = 4/(3 + 5X)$

For the solar abundance X=0.73, $\mu = 0.6$.

When all H gets converted into He, we then have $\mu = 1.3$. It more than doubles! But

$$P_{\rm gas} = \frac{\rho kT}{\mu m_p} = \frac{\Re \rho T}{\mu}$$

If T_c remains constant during the MS-phase but μ_c increases, then $P_c/\rho_c \sim T_c/\mu_c$ must decrease. So, either P_c decreases or ρ_c increases as more H is converted into He. It turns out that both effects occur. Actually, T_c is also slightly raising.



Stellar Evolution on the Main Sequence

- In Hydrostatic Equilibrium the central pressure is set by the weight of the layers above.
- So, as the central pressure decreases during the MS phase the outer layers of the star must expand.
- So, when μ increases in the center the radius must increase.
- At the same time, the nuclear energy generation increases and then so does the luminosity (μ-effect).
- This causes a slow increase of the star's luminosity over the whole MS phase.
- Because *R*² increases more than *L*, the effective temperature decreases. This implies that the stars move up and to the right in the HRD during H-fusion in the core.



Gradual change of the Sun's parameters

- How much has the solar luminosity changed over time? Calculations show that the Sun's ZAMS luminosity was about 25-30% less than it is today, which has/had implications for the Earth.
- Hence stars of a given mass but different ages populate the main-sequence with a width of ~0.5 dex [for decimal exponent]





Change of the Sun's parameters

Table 13.1. Distribution of mass, temperature, pressure, density and
luminosity for the young sun at the age of 5.4×10^7 years when it had
$R = 6.14 \times 10^{10} cm = R_{\odot Z}, L = 2.66 \times 10^{33} erg s^{-1} and T_{eff} = 5610 K.$
(These data were provided by C. Proffitt.)

$r/R_{\odot Z}$	M_r/M_{\odot}	T [K]	P_g [dyn cm ⁻²]	ho [g cm ⁻³]	L/L_{\odot}	r/R_{\odot}
0	0	13.62 (6)	1.49 (17)	8.02(1)	0	0
0.014	1.00(-4)	13.62 (6)	1.48 (17)	8.01 (1)	0.001	0.012
0.018	2.22(-4)	13.60 (6)	1.48 (17)	7.99 (1)	0.003	0.016
0.035	1.64(-3)	13.49 (6)	1.45 (17)	7.89(1)	0.020	0.031
0.057	7.23(-3)	13.23 (6)	1.38 (17)	7.67(1)	0.076	0.051
0.081	1.99(-2)	12.84 (6)	1.28 (17)	7.33 (1)	0.164	0.072
0.098	3.42(-2)	12.49 (6)	1.19 (17)	7.03(1)	0.233	0.087
0.115	5.32(-2)	12.09 (6)	1.10(17)	6.69(1)	0.309	0.101
0.125	6.71(-2)	11.84 (6)	1.04(17)	6.45(1)	0.358	0.110
0.138	8.75(-2)	11.50 (6)	9.59 (16)	6.14(1)	0.418	0.122
0.147	1.05(-1)	11.24 (6)	9.00 (16)	5.90(1)	0.461	0.130
0.158	1.26(-1)	10.94 (6)	8.33 (16)	5.61 (1)	0.506	0.140
0.178	1.69(-1)	10.40 (6)	7.16 (16)	5.07(1)	0.575	0.157
0.198	2.18(-1)	9.85 (6)	6.03 (16)	4.51 (1)	0.625	0.174
0.219	2.75(-1)	9.28 (6)	4.93 (16)	3.92(1)	0.655	0.193
0.263	3.99(-1)	8.18 (6)	3.10 (16)	2.80(1)	0.682	0.232
0.424	7.63(-1)	5.26 (6)	4.11 (15)	5.81 (0)	0.692	0.374
0.635	9.45(-1)	3.13 (6)	2.94 (14)	7.01(-1)	0.690	0.560
0.731	9.74(-1)	2.33 (6)	9.15 (13)	2.94(-1)	0.690	0.645
0.745	9.78(-1)	2.16 (6)	7.56 (13)	2.62(-1)	0.690	0.658
0.843	9.93(-1)	1.18 (6)	1.65 (13)	1.05(-1)	0.690	0.744
1.00	1.00	5.61 (3)	()		0.690	0.884

The numbers in brackets give the powers of 10.

Table 13.2. *Distribution of mass, temperature, pressure, density, luminosity, and abundances of H, He, C and N in the present sun according to Bahcall and Ulrich (1988).*

	0	T	P	0					
r/R_{\odot}	M_r/M_{\odot}	[K]	$[dyn cm^{-2}]$	$[g \text{ cm}^{-3}]$	L/L_{\odot}	Н	He	С	Ν
0.0	0.0	1.56 (7)	2.29 (17)	1.48 (2)	0.0	0.341	0.639	2.61 (-5)	6.34(-3)
0.024	0.0014	1.55 (7)	2.21 (17)	1.42 (2)	0.012	0.359	0.621	2.50(-5)	6.22(-3)
0.048	0.0108	1.49(7)	1.99 (17)	1.26(2)	0.085	0.408	0.571	2.24(-5)	5.98(-3)
0.071	0.0307	1.42(7)	1.72 (17)	1.08(2)	0.217	0.467	0.513	1.98(-5)	5.84(-3)
0.095	0.0654	1.33 (7)	1.41 (17)	8.99(1)	0.400	_0.530	0.450	1.71(-5)	5.78(-3)
0.115	0.1039	1.25 (7)	1.18 (17)	7.64 (1)	0.553	0.577	0.403	1.50(-5)	5.77(-3)
0.135	0.1500	1.17(7)	9.60 (16)	6.45(1)	0.688	0.615	0.364	1.68(-5)	5.77(-3)
0.149	0.186	1.12 (7)	8.25 (16)	5.72 (1)	0.766	0.637	0.342	1.84(-4)	5.57 (-3)
0.162	0.222	1.07(7)	7.11 (16)	5.10(1)	0.826	0.654	0.325	1.09(-3)	4.52(-3)
0.174	0.258	1.02(7)	6.14(16)	4.55(1)	0.872	0.667	0.312	2.39(-3)	3.00(-3)
0.188	0.300	9.74 (6)	5.16 (16)	3.99(1)	0.912	0.679	0.301	3.42(-3)	1.80(-3)
0.211	0.370	9.00 (6)	3.84 (16)	3.18(1)	0.954	0.692	0.288	4.01(-3)	1.11(-3)
0.235	0.440	8.32 (6)	2.81 (16)	2.51(1)	0.978	0.699	0.280	4.12(-3)	9.86(-4)
0.259	0.510	7.67 (6)	2.00(16)	1.94(1)	0.992	0.704	0.274	4.13(-3)	9.66(-4)
0.318	0.655	6.39 (6)	8.69 (15)	1.01(1)	1.000	0.708	0.271	4.14(-3)	9.63(-4)
0.504	0.900	3.88 (6)	6.59 (14)	1.27(0)	1.000	0.710	0.271	4.14(-3)	9.63(-4)
0.752	0.985	1.82 (6)	2.98 (13)	1.22(-1)	1.00	0.710	0.271	4.14(-3)	9.63(-4)
0.886	0.998	6.92(5)	2.60 (12)	2.84(-2)	1.00	0.710	0.271	4.14(-3)	9.63(-4)
0.920	0.999	4.54 (5)	8.95 (11)	1.50(-2)	1.00	0.710	0.271	4.14(-3)	9.63(-4)
1.000	1.000	5.77 (3)		(-)	1.00	0.710	0.271	4.14 (-3)	9.63 (-4)

The numbers in brackets give the powers of 10.

Gradual change in size of the Sun



Now 30% brighter, 12% larger, 3% hotter

The main-sequence phase of the Sun

- 50% of mass is within radius $0.25R_{\odot}$
- Only 1% of total mass is in the convection zone and above
- Pressure increases steeply in the centre





Post-MS evolution through helium burning

After the main-sequence phase, stars are left with a hydrogen-exhausted core surrounded by a still hydrogen-rich envelope. To describe the evolution after the main sequence, it is useful to make a division based on the mass. Note that all masses are approximate, boundaries overlap depending on definition.

- **Red dwarfs:** stars whose main-sequence lifetime exceeds the present age of the Universe (estimated as $1-2 \times 10^{10}$ yr). Models yield an upper mass limit of $0.7M_{\odot}$ of stars that must still be on main-sequence, even if they are as old as the Universe.
- Low-mass stars: stars in the region $0.7 \le M \le 2 M_{\odot}$. After shedding considerable amount of mass, they will end their lives as white dwarfs and possibly planetary nebulae. We will follow the evolution of a $1M_{\odot}$ star in more detail.
- Intermediate mass stars: stars of mass $2 \le M \le 8-10 M_{\odot}$. Similar evolutionary paths to low-mass stars, but always at higher luminosity. Give planetary nebula and higher mass white dwarfs. Complex behaviour on the AGB (asymptotic giant branch).
- *High mass (or massive) stars:* $M > 8-10 M_{\odot}$. Distinctly different lifetimes and evolutionary paths, huge variation.

The evolution of low-mass stars

345

The Terminal-Age MS and Subgiant Branch

The cores of $1M_{\odot}$ stars become He rich. Fusion is most efficient in the centre, where T is highest.

- As He content increases, core shrinks and heats up \rightarrow He rich core grows
- The *T* is not high enough for the triple- α process
- H-burning continues in a shell around the core, and as *T* increases, the CNO process can occur in the shell
- As $\varepsilon_{CNO} \sim T^{18}$ energy generation is concentrated in the regions of highest T and highest H content (in shell $T \sim 20 \times 10^6$ K)
- The envelope expands using the gravitational energy released during core contraction and thermal energy generated by efficient H shell burning.
- This expansion **terminates** the life at the main-sequence
- Luminosity remains approximately constant, hence $T_{\rm eff}$ must decrease, star

moves along the red subgiant branch.



The red-giant phase of a Sun-like Star

- The shell source slowly burns, moving through the star, as the He core grows in mass and contracts. But the star cannot expand and cool indefinitely.
- When the temperature of the outer layers reach <5000 K the envelopes become fully convective.
- This enables greater luminosity to be carried by the outer layers and hence quickly forces the star almost vertically in the HR diagram. Despite its cooler temperature, its luminosity increases enormously due to its large size.





- The star approaches the Hayashi line, and a small increase in the He core mass causes a relatively large expansion of the envelope.
- It is now a **red giant**, extending out as far as the orbit of Mercury.

The He-flash and core He-burning

- The helium core does not reach threshold *T* for further burning as it ascends the RGB, and as it is not producing energy it continues to contract until it becomes degenerate.
- At tip of the RGB the e⁻ in core are completely **degenerate**. *P* is due to **degenerate** electron pressure, which is independent of *T*.
- *T* is defined mainly by the energy distribution of the heavy particles (He nuclei). Gravitational collapse is resisted by e⁻ degeneracy pressure.
- At $T \sim 10^8$ K, triple- α reactions start in the very dense core. They generate energy, heating core, and kinetic energy of He nuclei increases, increasing the energy production. Energy generation and heating under degenerate conditions leads to runway the He flash.
- During the He-flash, the core temperature changes within seconds. The rapid increase in *T* leads the e⁻ again following Maxwellian velocity distribution and degeneracy is removed. The pressure increases and core expands. The luminosity actually drops as most of the released energy goes into expansion.



The star finds a new equilibrium configuration with an expanded **non**-degenerate core which is hot enough to burn He. The H-burning shell source is also expanded and has lower *T* and density and generates less energy than before. The star sits in the Red Clump (metal rich stars) or the Horizontal Branch (metal poor stars).

Observations of these evolution stages



The AGB and thermal pulses

- The triple- α reaction is even more *T*-dependent ($\epsilon \sim T^{40}$), hence energy generation is even more centrally condensed. Note, the H-burning shell is still generating energy.
- The core will soon consist only of C+O, and in a similar way to before, the CO-core grows in mass and contracts, while a He-burning shell source develops.
- These two shell sources force expansion of the envelope and the star evolves up the red giant branch a second time - this is called the asymptotic giant branch (AGB).





- For high metallicity stars, the AGB coincides closely with the first RGB.
- For globular clusters (typical heavy element composition 100 times lower than solar) they appear separated.

The stellar wind

351

- Large radiation pressure (and dust formation) at tip of AGB probably drives mass-loss. Particles may absorb photons from radiation field and be accelerated out of the gravitational potential well. Observations of red giants and supergiants (more massive evolved stars) give the mass loss rate in the range 10^{-9} to $10^{-4} M_{\odot}$ yr⁻¹.
- Mass-loss is generally classified into two types of wind.
 - 1. Stellar wind: described by empirical formula (Dieter Reimers), linking mass, radius, luminosity with simple relation and a constant from observations. Typical wind rates are of order $10^{-6} M_{\odot} \text{ yr}^{-1}$

$$\dot{M} = 10^{-13} \frac{L}{L_{\odot}} \frac{R}{R_{\odot}} \frac{M_{\odot}}{M} \, \mathrm{M_{\odot} \, yr^{-1}}$$

- 2. A superwind: a stronger wind, leading to stellar ejecta observable as a shell surrounding central star
- The existence of a superwind is suggested by two independent variables. The high density observed within the observed shells of stellar ejecta, and relative paucity of very bright stars on the AGB.
- The latter comes from the number of AGB stars expected compared to observed is >10. Hence a process prevents them completing their movement up the AGB, while losing mass at the Reimer's rate.

The planetary nebula phase

- This is a superwind which removes the envelope mass before the core has grown to it's maximal possible size. Direct observations of some stars indicate mass-loss rates of order $10^{-6} M_{\odot} \text{ yr}^{-1}$. Probably this is due to pulsational instability and thermal pulses (unstable He shell burning) in envelope e.g. Mira-type variables.
- Superwind causes ejection of outer layers of gas which form planetary nebula.
- The cores collapse into C-O white dwarfs.
- Core mass at a tip of AGB $\sim 0.6-1 M_{\odot}$ and most white dwarfs have masses close to this.
- Planetary nebula nucleus is still burning H in a thin shell above C-O core.



The Death of a Low-Mass Star







- Planetary Nebula is ejected (25 - 60% of mass ejected) and expands for about 10⁵ yrs before dissipating into the surroundings.
- An envelope is about the size of our solar system.
- The ejected envelope is ionized by the hot white dwarf and glows.



- Planetary Nebulae are very common (20,000 to 50,000 in the Milky Way).
- Planetary nebulae can have many shapes.
- As the dead core of the star cools, the nebula continues to expand, and dissipates into the surroundings.

The end point: white and black dwarfs

- Once the nebula has gone, the remaining core is extremely dense and extremely hot, but quite small.
- It is luminous only due to its high temperature.
- Core is all carbon, very dense degenerate.
- Carbon ignition never occurs
- As the white dwarf cools, its size does not change significantly; it simply gets dimmer and dimmer, and finally ceases to glow.
- Estimated cooling time for the white dwarf is longer than the current age of the Universe => no black dwarfs yet.



Summary of 1 M_{\odot} evolution

356,

Approximate typical timescales:

Phase	T (yrs)	Steps →
Main-sequence	9×10 ⁹	1-4
Subgiant	3×10 ⁹	5-7
Red Giant Branch	1×10 ⁹	8-9
Red clump	1×10 ⁸	10
AGB evolution	$\sim 5 \times 10^{6}$	11-15
PNe	$\sim 1 \times 10^{5}$	
WD cooling	>8×10 ⁹	

