Basic physics of star formation

Consider the forces acting on a "star forming unit" within a molecular cloud - a molecular cloud core:

- **Gravity** - act to collapse the cloud
- **Pressure**
- **Magnetic fields**
- **"Bulk motions"**

(Pressure, magnetic fields and "bulk motions":) sources of support against collapse to form a star

If somehow we form a core in which gravity dominates over all other forces, collapse will occur on the dynamical or free-fall time:

$$t_{dyn} = \frac{\pi}{2\sqrt{2}}\left(\frac{R^3}{GM}\right)^{1/2} = \left(\frac{3\pi}{32}\right)^{1/2}\frac{1}{\sqrt{G\bar{\rho}}}$$

... for a cloud of mass M, radius R, and mean density $\bar{\rho}$.

The Jeans Mass

Ignore for now magnetic fields and bulk motions. The **Jeans mass** is the minimum mass a cloud must have if gravity is to overwhelm pressure and initiate collapse.

Borderline case is the one where the cloud is in hydrostatic equilibrium:

$$\frac{dP(r)}{dr} = -\frac{Gm}{r^2}\rho(r)$$

To derive an estimate of the Jeans mass, consider a cloud of mass M and radius R:

- approximate derivative dP/dr by $-P/R$
- assume pressure is that of an ideal gas: $P = \frac{\Re \rho T}{\mu}$

Substitute:

$$-\frac{\Re \rho T}{\mu\, R} = -\frac{GM}{R^2}\rho \quad \rightarrow \quad M = \frac{\Re}{\mu G} T R$$

Can eliminate R in favor of the density ρ using $M = \frac{4}{3}\pi R^3 \rho$ and we get a final expression for **the Jeans mass**:

$$M_J = \left(\frac{\Re}{\mu G}\right)^{3/2} \left(\frac{3}{4\pi}\right)^{1/2} T^{3/2} \rho^{-1/2}$$

This is a basic formula for star formation. Numerical constants can vary depending on the details of the derivation.

Mass scale of star formation

$$M_J = \left(\frac{\Re}{\mu G}\right)^{3/2} \left(\frac{3}{4\pi}\right)^{1/2} T^{3/2} \rho^{-1/2}$$

Observationally, stars form from cold dense **molecular** gas with typical density $\rho \sim 10^{-19}$ g cm^{-3} and temperature $T \sim 10$ K, take $\mu = 2$ for molecular hydrogen **What ?**
Put all these numbers in the Jeans mass formula and get

$\Re = 8.314 \times 10^{7}$ erg mol^{-1}K^{-1}

$$M_\mathrm{J} = 7.8\times10^{32}\ \mathrm{g} \approx 0.4\ M_\odot$$

…which matches the typical mass of stars in the Galaxy!

Level of agreement here is "too good to be true", however we can conclude that the Jeans mass in these conditions is about a Solar mass and sets the basic mass scale for star formation.

The Jeans length

We can likewise define a characteristic length scale — **the Jeans length** — by eliminating mass rather than radius from the previous expression:

$$M = \frac{\Re}{\mu G} T R \quad \rightarrow \quad \frac{4}{3}\pi R^3 \rho = \frac{\Re}{\mu G} T R$$

$$R_J = \left(\frac{\Re}{\mu G}\right)^{1/2} \left(\frac{3}{4\pi}\right)^{1/2} T^{1/2} \rho^{-1/2}$$

For the same density / temperature as before, $R_J \sim 1.2 \times 10^{17}$ cm = 10^4 AU

Free-fall timescale for a cloud of this density is:

$$t_{dyn} \sim \frac{1}{\sqrt{G\bar{\rho}}} \sim 10^{13} \text{ s} \sim 10^5 \text{ yr}$$

Star formation in these conditions should create solar mass stars within a few hundred thousand years.

The Jeans density

The above condition can also be stated as a condition on the density, which must be larger than **the Jeans density**:

$$M_J = \left(\frac{\Re}{\mu G}\right)^{3/2} \left(\frac{3}{4\pi}\right)^{1/2} T^{3/2} \rho^{-1/2} \rightarrow \boxed{\rho_J = \left(\frac{\Re}{\mu G}\right)^{3} \frac{3T^3}{4\pi M^2}}$$

For a typical cloud mass of $1000\ M_\odot$ and temperature $50\ K$,

$$\rho_J \sim 1.8\times10^{-24}\ \mathrm{g\ cm^{-3}}$$

corresponding to a number density of

$$n_J = \frac{\rho_J}{\mu m_p} = \frac{1.8 \times 10^{-24}}{2 \times 1.67 \times 10^{-24}} \approx 0.5\ \mathrm{cm}^{-3}$$

Thus, the typical observed density of molecular clouds, 10^2–10^4 cm^{-3}, is several orders of magnitude higher than the Jeans density and, according to the criterion we have just formulated, the clouds should be unstable to gravitational collapse.

Since the clouds exist and appear to be long lived, another source of pressure, other than thermal pressure, must be present. It is currently believed that the dominant pressure is provided by turbulence, magnetic fields, or both.

Interstellar cloud collapse and fragmentation

What happens during the collapse?

Jeans mass formula:

$$M_J \propto T^{3/2} \rho^{-1/2}$$

Initially: gas is optically thin, the cloud is transparent to far-infrared radiation and thus cools efficiently, so that the early stages of the collapse are isothermal (T constant).

If T stays constant, the density of the collapsing cloud increases, its Jeans mass decreases.

- Gravity becomes even more dominant over pressure
- The stability criterion within the cloud may now also be violated:
  M_J drops - allows for possibility that the cloud might break up into smaller fragments,
  so that the cloud starts to fragment into smaller pieces, each of which continues to collapse.
- The fragmentation process probably continues until the mass of the smallest fragments (dictated by the decreasing Jeans mass) is less than $0.1\ M_\odot$.

Formation of a proto-stellar core

The increasing density of the collapsing cloud fragment eventually makes the gas opaque to infrared photons. As a result, radiation is trapped within the central part of the cloud, leading to heating the cloud and an increase in gas pressure. It changes the nature of the collapse from an "isothermal" phase to an "adiabatic" phase. By the definition of an adiabat,

$$P \propto \rho^{\gamma} \rightarrow \quad T \propto \rho^{\gamma-1}$$

Substituting this into the Jeans equation we obtain

$$M_J \propto T^{3/2}\rho^{-1/2} = \rho^{\frac{3}{2}(\gamma-1)}\rho^{-1/2} = \rho^{\frac{(3\gamma-4)}{2}}$$

For γ=5/3, $M_J \propto \rho^{1/2}$.

In other words, the Jeans mass no longer decreases with increasing density!
Thus, the cloud is no longer unstable to fragmentation. As a result, the cloud core comes into hydrostatic equilibrium and the dynamical collapse is slowed to a quasistatic contraction. At this stage we may start to speak of a protostar.

The contraction will now proceed slowly, at a pace determined by the rate at which thermal energy is radiated away. The gravitational energy is converted to dissociation of H_2, which uses up 4.5 eV per molecule, and ionization of hydrogen, which takes 13.6 eV per atom (see below).

Accretion

The surrounding gas keeps falling onto the protostellar core, so that the next phase is dominated by accretion. Since the contracting clouds contain a substantial amount of angular momentum, the infalling gas forms an accretion disc around the protostar. These accretion discs are a ubiquitous feature of the star formation process and are observed around most very young stars, mostly at infrared and sub-millimeter wavelengths.

The accretion of gas generates gravitational energy. As with other accretion, half the energy goes into further heating of the core, and half is radiated away, providing the luminosity of the protostar:

$$L_{acc} = \frac{GM\dot{M}}{2R}$$

where M and R are the mass and radius of the core and $\dot{M}$ is the mass accretion rate. Meanwhile, the core heats up almost adiabatically since the accretion timescale $\tau_{\mathrm{acc}} = M/\dot{M}$ is much smaller than the thermal timescale τ_{KH}.

Dissociation and ionization

- The gas initially consists of molecular hydrogen and behaves like an ideal gas, such that $\gamma > 4/3$ and the protostellar core is dynamically stable:

$$M_J \propto \rho^{\frac{(3\gamma-4)}{2}}$$

- Eventually, the core reaches a temperature of ~ 2000 K and begins to dissociate the molecular hydrogen, which is analogous to ionization and leads to a strong increase of the specific heat and a decrease of γ_{ad} below the critical value of $4/3$.
- Hydrostatic equilibrium is no longer possible, and a renewed phase of dynamical collapse follows. The collapse releases energy, which further dissociates molecules without a significant rise in temperature. When H_2 is completely dissociated into atomic hydrogen, the star settles into a new hydrostatic equilibrium. Somewhat later, the same thing happens when the temperature rises enough to ionize hydrogen (and then helium, at ~10^4 K, and then helium again, at ~8×10^4 K).
- When ionization of the protostar is complete it settles back into hydrostatic equilibrium at a **much reduced** radius.
- Note also that the temperature required to doubly ionize helium (~ 80000 K) is far less than that required to fuse hydrogen.
- In fact, throughout most of this time, the temperature is cool enough so that H^- (**hydrogen with two electrons** rather than one) is the dominant source of opacity. This means that the star is fully convective and on the Hayashi track which we will discuss soon.

Pre-main sequence phase

- Finally, the accretion slows down and eventually stops and the protostar is revealed as a pre-main sequence star. Its luminosity is now provided by gravitational contraction and, according to the virial theorem, its internal temperature rises as

$$T \propto M^{2/3} \rho^{1/3}$$ **Prove it!**

- The surface cools and a temperature gradient builds up, transporting heat outwards.
- Further evolution takes place on the thermal timescale τ_{KH}.
- The temperature of this early-evolution proto-star is roughly 4000 K. A size of $R \sim 35\ R_{\odot}$. Then it implies a luminosity of $\sim 10^3\ L_{\odot}$. Note however, that observed pre-main sequence stars have radii substantially smaller than 35 $R_{\odot}$, indicating that additional energy must have been radiated (or otherwise lost) in the contraction process.
- At these low temperatures, the opacity is very high, rendering radiative transport inefficient and making the protostar convective throughout.
- The properties of such fully convective stars must be examined more closely.

Summary of star formation on large scales

If the internal gas pressure is not strong enough to prevent gravitational collapse of a region filled with matter, then the Jeans instability may occur, causing the collapse of interstellar gas clouds and subsequent star formation.

Jeans' mass - minimum mass of a gas cloud of temperature T and density ρ that will collapse under gravity:

$$M_J = \left(\frac{\Re}{\mu G}\right)^{3/2} \left(\frac{3}{4\pi}\right)^{1/2} T^{3/2} \rho^{-1/2}$$

Several stages of collapse:

- Initial isothermal collapse - still optically thin
- The creation of proto-stellar cores with surrounding discs
- Collapse slows or halts once gas becomes optically thick - heats up so pressure becomes important again
- Second phase of free-fall collapse as hydrogen molecules are broken up - absorbs energy and robs cloud of pressure support
- Finally forms protostar with radius of 5 - 10 $R_{\odot}$ and the remnants of a disc.

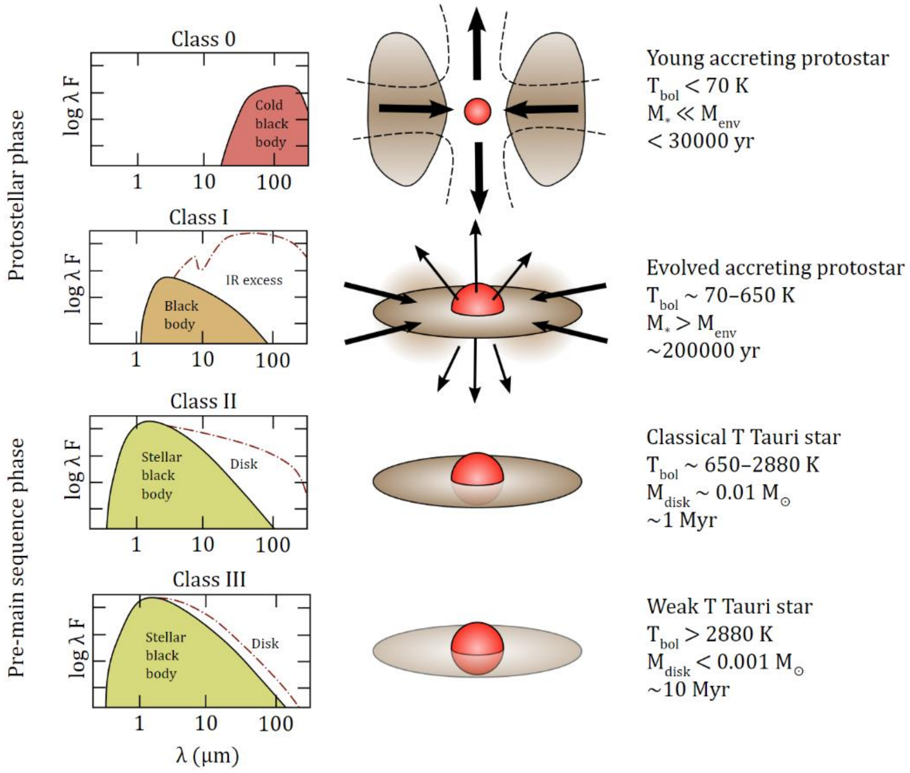
All this happens very rapidly – $t_{dyn} \sim 10^5$ yr – not easy to observe

Identification of Young Stars

Observationally, young stellar objects (YSOs) can be classified

- By infrared excess in their spectral energy distributions (SEDs): Young stars are built up by accretion, and the accretion disc can linger for some time after the nuclear reactions have started. The dust and material in the disk can reprocess some of the light into the IR.
- By X-ray identification: Conservation of angular momentum guarantees that young stars will be rotating quickly. If the star has a convective envelope, then differential rotation will cause an increased magnetic dynamo effect, leading to increased flare activity and X-rays.
- Hα emission: The same mechanism that can create X-rays may also result in Hα emission. In addition, there may still be residual Hα emission from the accretion disc.
- Lithium absorption: Lithium can be burned during the pre-main sequence phase. Stars with lithium absorption in their spectrum must be young (see below).

Four main classes of object have been identified.



On the left, the stellar flux is depicted (shaded area) and the contribution from the disc (dotted line). On the right the corresponding geometry of the object is shown.

Class 0 source

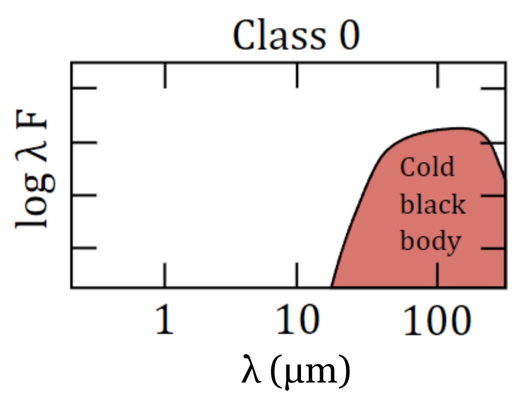
Young accreting protostars. Observationally, this is a source whose SED peaks in the far-infrared or mm part of the spectrum. No flux in the near-infrared (at a few microns). Effective temperature is several tens of degrees Kelvin.

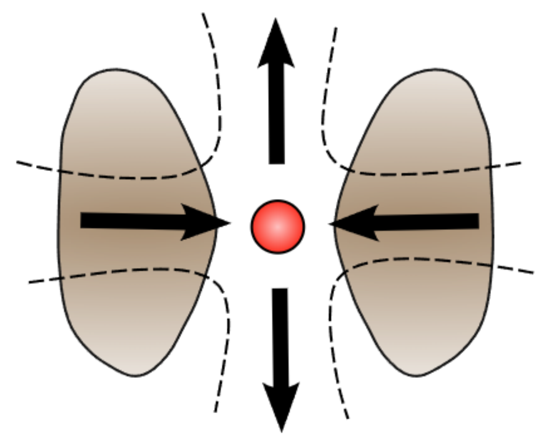
What are **Class 0** sources? Earliest observed stage of star formation…

- Still very cool - not much hotter than molecular cloud cores. Implies extreme youth.
- Deeply embedded in gas and dust, any shorter wavelength radiation is absorbed and re-radiated at longer wavelengths before escaping.
- Fairly small numbers - consistent with short duration of the initial collapse.
- Outflows are seen - suggests a protostar is forming.

$t < 30000$ yr

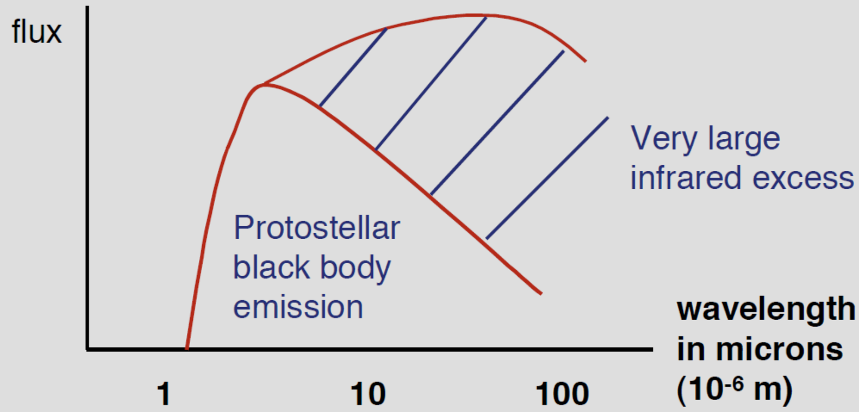
$T_{bol} < 70$ K
$M_* \ll M_{env}$





Class I source

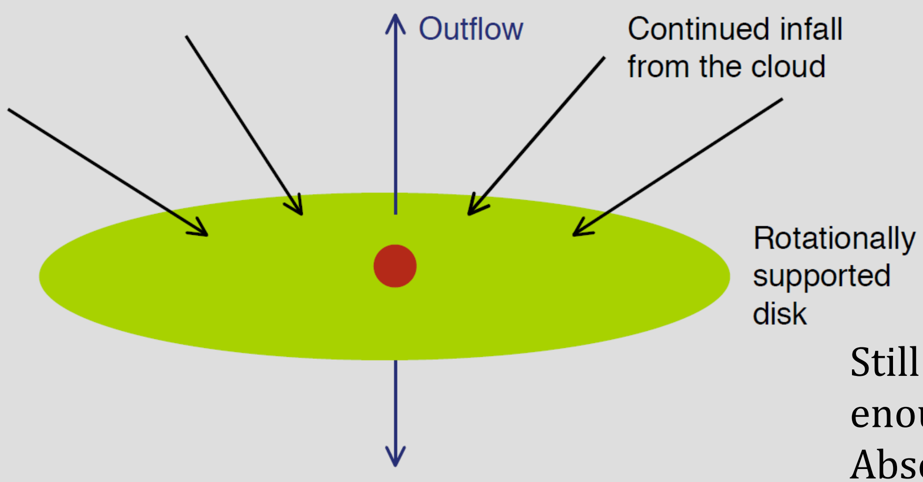
Class I sources – evolved accreting protostars – also have SEDs that rise into the mid and far IR. But they differ from Class 0 in having detectable near infrared flux.
Still not seen at visible wavelengths.



$t \approx 200000$ yr

$M_* > M_{env}$

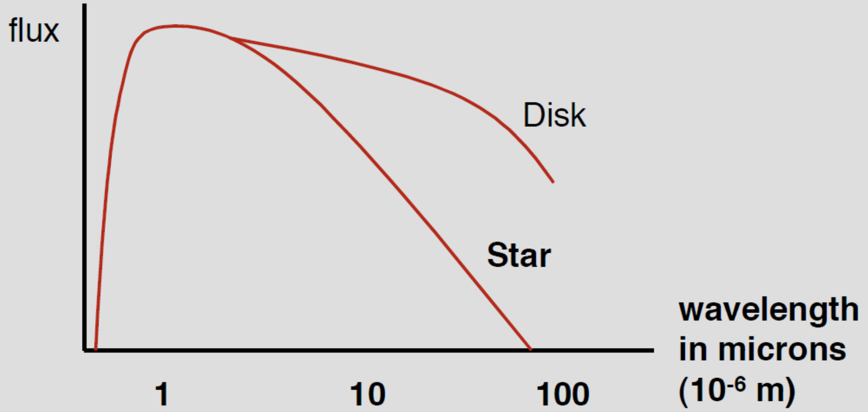
$T_{bol} \sim 70 - 650$ K



Still can't see the star itself, but dust has cleared enough to see the hot gas and dust close to the star. Absorption and re-radiation of this near-infrared flux by the dust in the envelope produces the far-infrared peak.

Class II source: classical T Tauri stars

Class II: Flat or falling SEDs in the mid-infrared. Optically visible pre-main-sequence stars.
Also called classical T Tauri stars, after the prototype star T Tauri in the Taurus star forming region.



- By this stage almost all of the collapsing cloud has settled onto the star or onto a disc surrounding the star.
- From most angles we can see the young star directly.
- Disk slowly drains onto the star over several million years.

$t \approx 1$ Myr

$M_{\text{disk}} \sim 0.01\, M_{\odot}$

$T_{\text{bol}} \sim 650 - 2880$ K

Class III source: weak-lined T Tauri stars

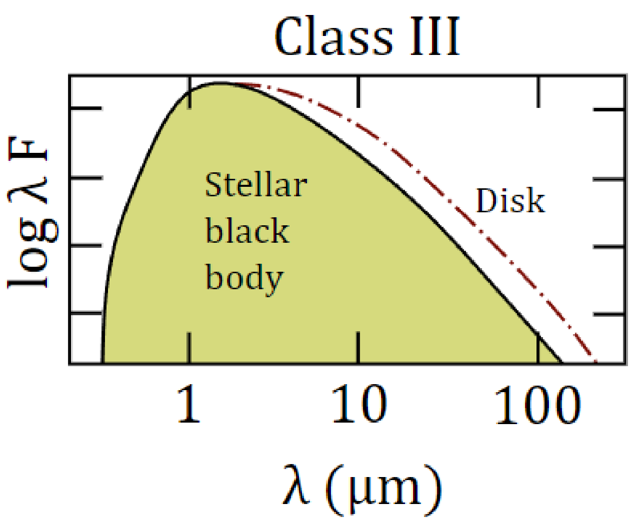
Class III sources: Fairly normal stellar SEDs, but more luminous than main-sequence stars of the same effective temperature (i.e. they lie above the main sequence).

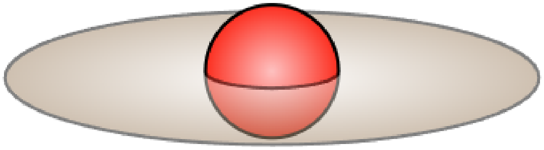
Also, they are more active (e.g. in X-rays) than ordinary main-sequence stars.

$t \approx 10$ Myr

$M_{disk} < 0.001\ M_{\odot}$

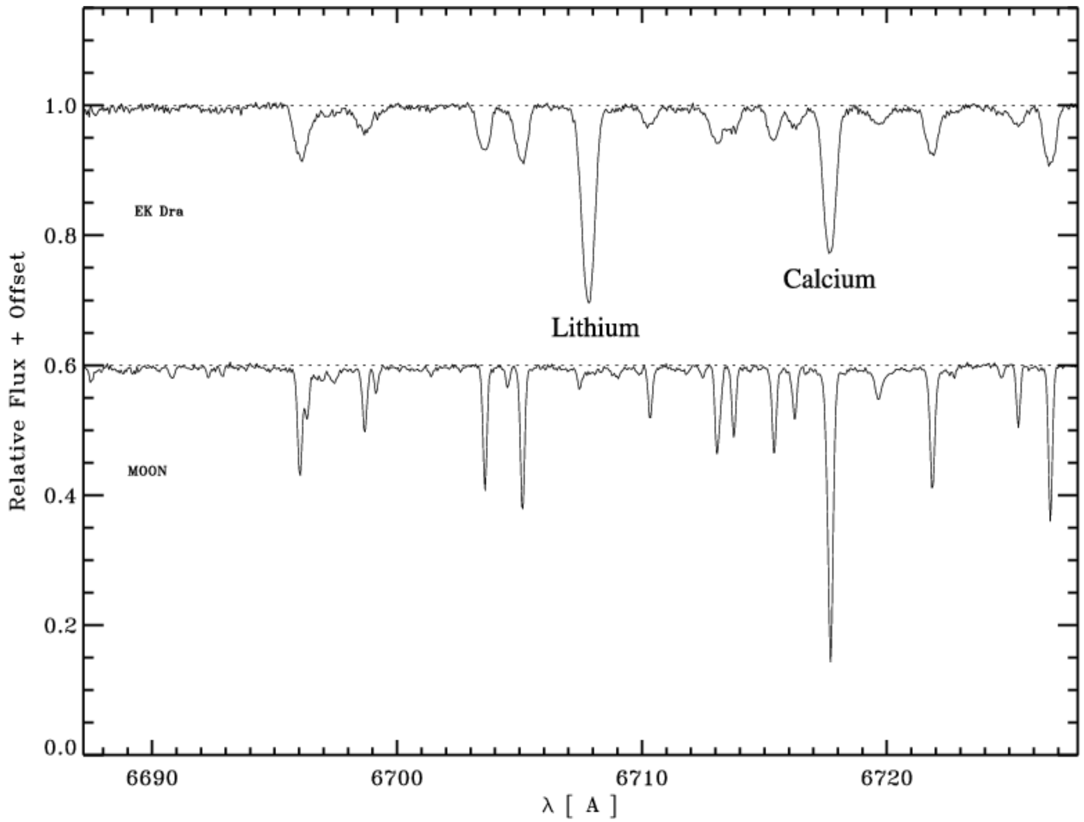
$T_{bol} > 2880$ K



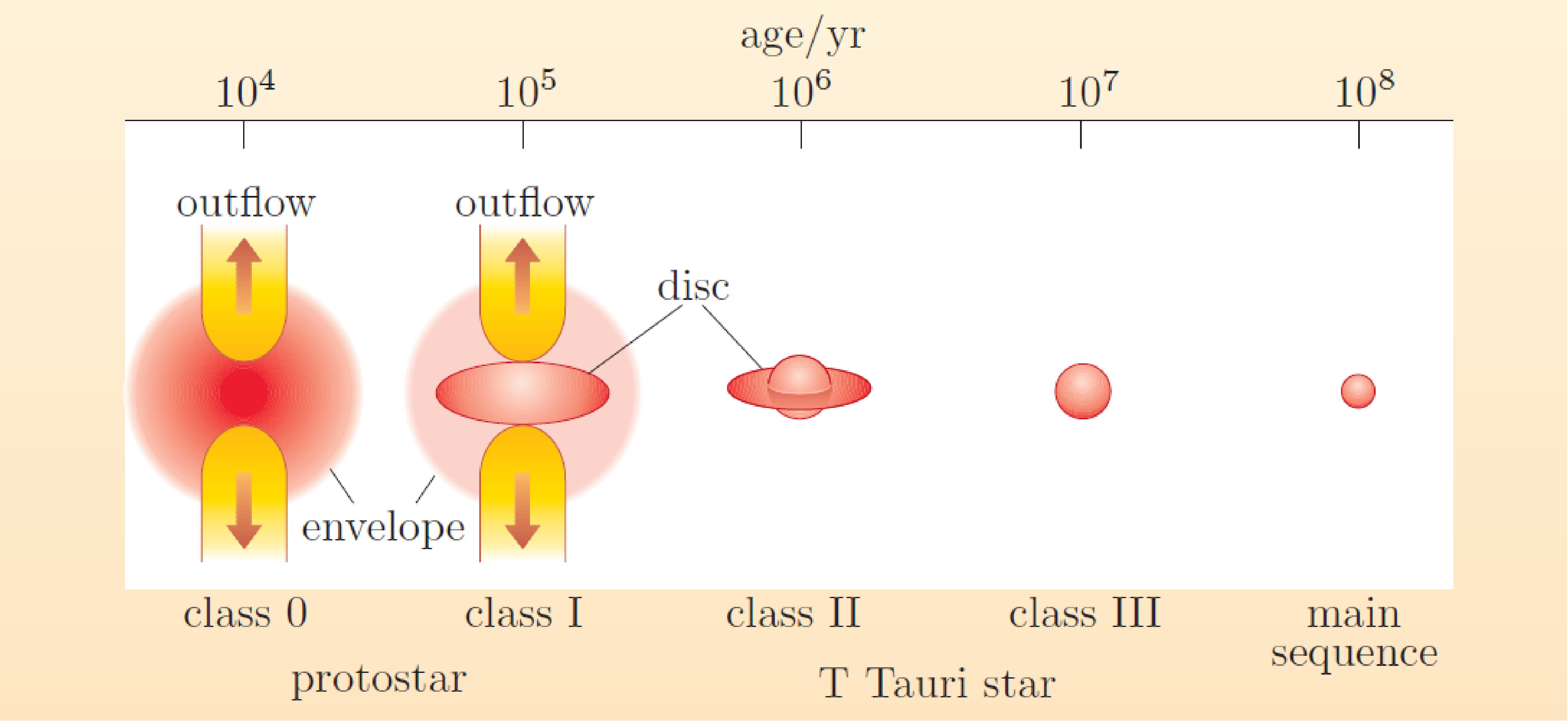


Lithium absorption

Lithium can be burned during the pre-main sequence phase. Moreover, in stars with convective envelopes, the temperature at the bottom of the convective layers is sufficient for lithium burning. Stars with lithium absorption in their spectrum must be young.



Protostars and pre-main-sequence stars



A schematic diagram (from Ryan & Norton) of the evolution of a pre-main-sequence star from the protostar stage, through the T Tauri stage, to the stage where the star becomes a genuine main-sequence object.

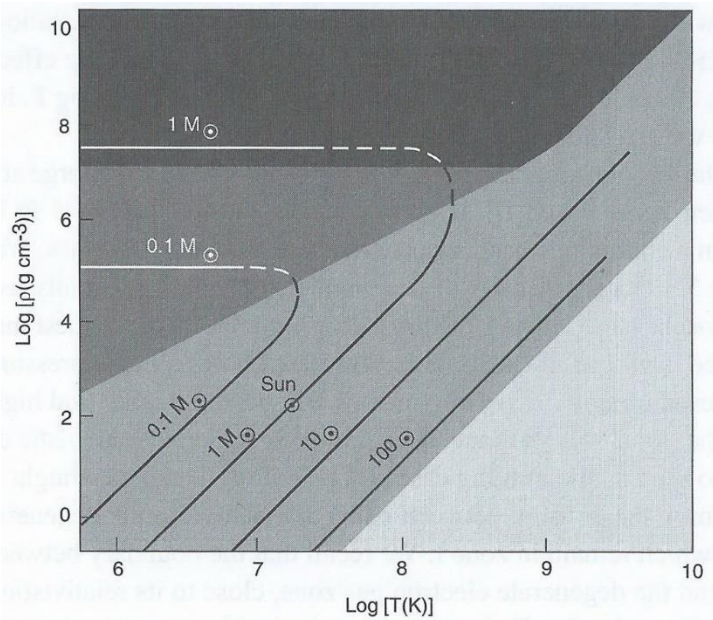
Pre-main sequence star evolution

The birth-line for pre-main sequence stars is usually taken as a transition from Class I to Class II (T Tauri stars).

At this stage, in addition to the radiation emitted by accreting material as it strikes the stellar surface, the star itself also radiates. However, since the protostar is initially not hot enough to burn hydrogen, it has no internal source of nuclear energy to balance out this radiation, and it is forced to contract on a Kelvin-Helmholtz timescale.

(It can burn deuterium, but this all gets used up on a timescale well under the KH timescale.)

This contracting state represents the "initial condition" for a calculation of stellar evolution. In terms of the ($\log T$, $\log \rho$) plane describing the centre of the star, we already know what this configuration looks like: the star lies somewhere on the low T, low ρ side of its mass track, and it moves toward the hydrogen burning line on a KH timescale.



We would also like to know what it looks like on the H-R diagram, since this is what we can **actually observe**. Therefore, we want to understand the movement of the star in the ($\log T_{eff}$, $\log L$) plane.

Kelvin–Helmholtz Contraction

There are two phases during this Kelvin–Helmholtz contraction:

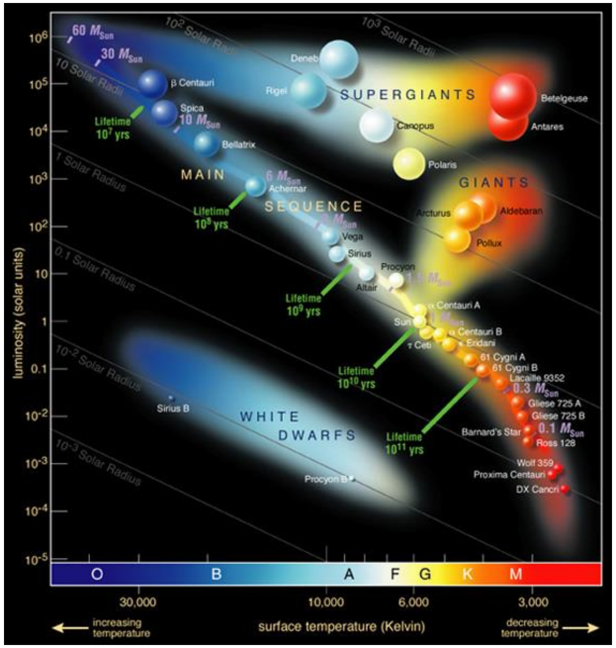
- the **Hayashi** phase, and
- the **Henyey** phase.

The overall timescale for the process is the Kelvin–Helmholtz timescale, as the time for a star to collapse to the main sequence assuming its luminosity is provided solely by energy liberated as a result of gravitational collapse.

Work by Chushiro Hayashi in the 1960s showed that a star **cannot** achieve hydrostatic equilibrium if its outer layers are too cool. Two important concepts in understanding Hayashi's work are opacity and convection.

Hayashi showed that there is a boundary on the right-hand side of the H–R diagram cooler than which hydrostatic equilibrium is impossible, and hence stable stars cannot exist. It lies at an effective surface temperature of around $T_{eff} \approx$ 3000 to 5000 K (depending on the star's mass, chemical composition and luminosity).

Objects to the right of that boundary are **out** of hydrostatic equilibrium, and collapse rapidly until the surface temperature reaches the value corresponding to stability. The boundary is almost vertical in the H–R diagram, so collapsing protostars moves vertically along the path which is called a **Hayashi track** and enter the H–R diagram near that boundary.



Hayashi track (1)

$$\left[\frac{GM}{-\xi_1^2(d\theta/d\xi)_{\xi_1}}\right]^{(n-1)}\left(\frac{R}{\xi_1}\right)^{3-n} = \frac{[K(n+1)]^n}{4\pi G}$$

To figure this out, we can approximate the protostellar interior as a polytrope with $P = K_p\rho^{1+\frac{1}{n}}$, or

$$\log P = \log K_p + \left(\frac{n+1}{n}\right)\log\rho$$

Recalling way back to the discussion of polytropes, the polytropic constant K_P is related to the mass and radius of the star by

$$K_p \propto M^{(n-1)/n}R^{(3-n)/n} \Longrightarrow \log K_p = \left(\frac{n-1}{n}\right)\log M + \left(\frac{3-n}{n}\right)\log R + \text{const}$$

so, we have

$$\log P = \left(\frac{n-1}{n}\right)\log M + \left(\frac{3-n}{n}\right)\log R + \left(\frac{n+1}{n}\right)\log\rho + \text{const}$$

Now consider the photosphere of the star, at radius R, where it radiates away its energy into space.
If the density at the photosphere is ρ_R, then hydrostatic balance requires that

$$\frac{dP}{dr} = -\rho_R\frac{GM}{R^2} \qquad \Longrightarrow \qquad P_R = \frac{GM}{R^2}\int_R^\infty \rho dr$$

where P_R is the pressure at the photosphere

Hayashi track (2)

$$\frac{dP}{dr} = -\rho_R \frac{GM}{R^2} \quad \Rightarrow \quad P_R = \frac{GM}{R^2}\int_R^\infty \rho dr$$

where P_R is the pressure at the photosphere and we have assumed that GM/R^2 is constant across the photosphere, which is a reasonable approximation since the photosphere is a very thin layer.
The photosphere is the place where the optical depth τ drops to a value below 1 (we will soon learn that the "surface" of a star, which we "see", and which has temperature $T_{\rm eff}$ (by definition) lies at τ=2/3).
Thus, we know that at the photosphere

$$\tau = \kappa \int_R^\infty \rho dr \approx 1$$

where we are also approximating that κ is constant at the photosphere. Putting this together, we have

$$P_R = \frac{GM}{\kappa R^2} \quad \Rightarrow \quad \log P_R = \log M - 2\log R - \log \kappa + \text{constant}$$

For simplicity we will approximate κ as a power-law of the form $\kappa = \kappa_0 \rho T_{\rm eff}^b$, where $T_{\rm eff}$ is the star's effective temperature, i.e. the temperature at its photosphere:

$$\log P_R = \log M - 2\log R - \log \rho - b \log T_{\rm eff} + \text{constant}$$

Finally, we know that the ideal gas law applies at the stellar photosphere, so we have

$$\log P_R = \log \rho_R + \log T_{\rm eff} + \text{constant}$$

$$P = NkT = \frac{\rho}{\mu m_p} kT$$

and we have the standard relationship between luminosity and temperature

$$\log L = 2\log R + 4\log T_{\rm eff} + \text{constant}$$

$$L \propto R^2\, T^4$$

Hayashi track (3)

We now have four equations:

$$\log P_R = \left(\frac{n-1}{n}\right)\log M + \left(\frac{3-n}{n}\right)\log R + \left(\frac{n+1}{n}\right)\log \rho + \text{constant}$$
$$\log P_R = \log M - 2\log R - \log \rho - b\log T_{\text{eff}} + \text{constant}$$
$$\log P_R = \log \rho_R + \log T_{\text{eff}} + \text{constant}$$
$$\log L = 2\log R + 4\log T_{\text{eff}} + \text{constant}$$

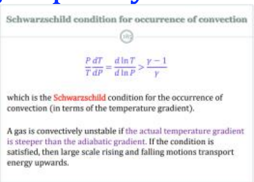
And the four unknowns $\log T_{\text{eff}}$, $\log L$, $\log \rho_{\text{R}}$, and $\log P_{\text{R}}$.
Solving these equations (and skipping over the algebra), we obtain

$$\log L = \left(\frac{9-2n+b}{2-n}\right)\log T_{\text{eff}} + \left(\frac{2n-1}{2-n}\right)\log M + \text{constant}$$

Thus, to figure out the slope of a young star's track in the HR diagram, we need only specify n and b.

Stars at the Hayashi boundary are fully convective. The reason is easy to understand:

1) First, the opacity of protostellar matter decreases with temperature, so cool objects have high opacity.
2) Second, high opacity leads to steep radiative temperature gradients, and
3) Third, steep radiative temperature gradients lead to convective instability → (Slide 185)



Putting these three things together, **the coolest stars are more likely to be unstable to convection**.

Hayashi track (4)

Thus, protostars are fully convective, so $n = 1.5$.

Protostars are fully convective due to their high opacities, and they are initially quite cold, ~4000 K.

This makes their opacity very different from that of main-sequence stars.

We will discuss opacities in detail in the following lectures, but now I just note that in main sequence stars like the Sun, the opacity is mostly free-free or, at high temperatures, electron scattering.
At the low temperatures of protostars, however, there are too few free electrons for either of this to be significant, and instead the main opacity source is bound-bound. One species particularly dominates: **H^-, that is hydrogen with two electrons** rather than one (we discussed it a few lectures ago).

The H^- opacity is very different than other opacities. It strongly increases with temperature, rather than decreases, because higher temperatures produce more free electrons via the ionization of metal atoms with low ionization potentials, which in turn can combine with hydrogen to make more H^-.

Once the temperature passes several thousand K, H^- ions start falling apart and the opacity decreases again, but in the crucial temperature regime where protostars find themselves (4000 K), opacity increases extremely strongly with temperature: $\kappa_{H^-} \propto \rho T^4$ is a reasonable approximation, giving $b = 4$.

Hayashi track (5)

$$\log L = \left(\frac{9-2n+b}{2-n}\right)\log T_{\text{eff}} + \left(\frac{2n-1}{2-n}\right)\log M + \text{const}$$

Thus, plugging in n=1.5 and b=4, we get

$$\log L = 20\log T_{\text{eff}} + 4\log M + \text{const}$$

Thus, the slope is 20, extremely large.

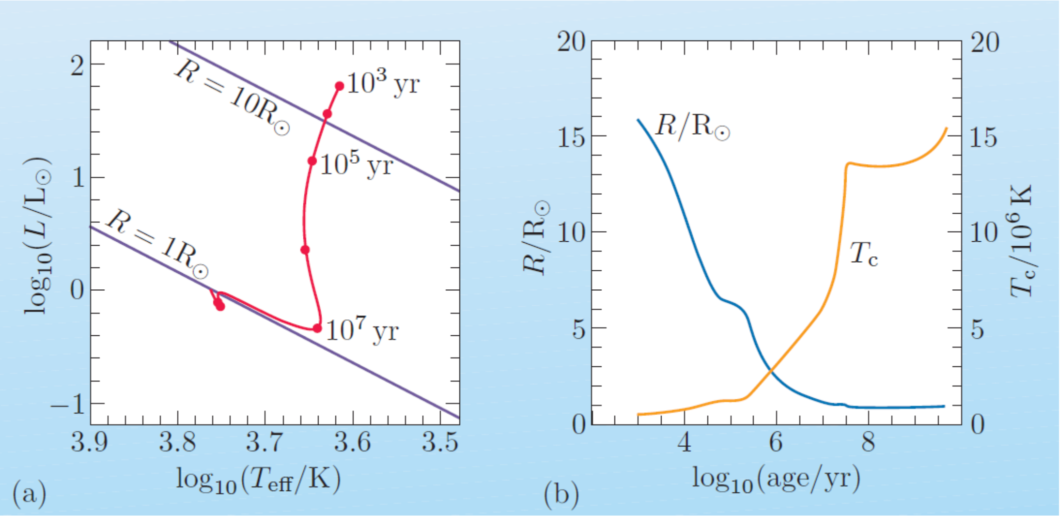
Stars in this phase of contraction make a nearly vertical track in the H–R diagram, **the Hayashi track**.

Remember, T_{eff} and radius of a star R are related by $L = 4\pi R^2\sigma T^4$. Since the star's temperature changes very little during this stage, the luminosity is proportional to the square of the radius.

As R is still decreasing due to contraction, the luminosity **decreases** significantly.

Stars of different masses have Hayashi tracks that are slightly offset from one another due to the $4 \log M$ term, but they are all vertical.

(from Ryan & Norton)



(a) H–R diagram with Hayashi track for the Sun. The red dots indicate elapsed times of 10^3, 10^4, 10^5, 10^6, 10^7, 10^8 and 10^9 years.
(b) Evolution of radius and core temperature with time.

The Henyey contraction

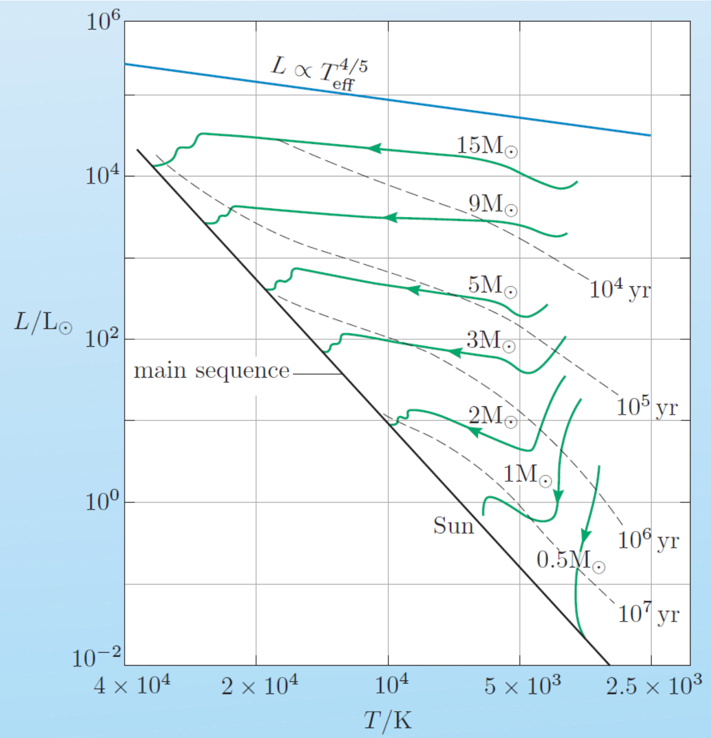
$$\log L = \left(\frac{9-2n+b}{2-n}\right)\log T_{\text{eff}} + \left(\frac{2n-1}{2-n}\right)\log M + \text{const}$$

Contraction along the Hayashi track ends once the star contracts and heats up enough for H^- opacity not to dominate, so that b is no longer a large positive number. Once b becomes 0 or smaller, as the opacity changes over to other sources, the track flattens, and the star contracts toward the main sequence at roughly fixed luminosity but increasing temperature.

This is known as **a Heyney track**.

As the protostar continues to contract, its core gets hotter and its opacity decreases, because for a Kramers opacity, $\kappa \propto \rho T^{-3.5}$. As a result of the decreasing opacity, the radiative temperature gradient becomes shallower and the condition for convection eases. Eventually the core becomes non-convective (radiative).

It can be shown that during the Henyey contraction, the slope becomes close to 4/5 (almost flat).



Only stars with masses $\sim M_{\odot}$ or less have Hayashi phases, and only stars of mass $M \leq 0.5 M_{\odot}$ reach the main sequence at the bottom of their Hayashi tracks.

More massive stars are "born" hot enough so that they are already too warm to be dominated by H^-.