

Stellar evolution codes



- A stellar evolution code — a piece of software that can construct a model for the interior of a star, and then evolve it over time.
- Stellar evolution codes are often complicated to use. **Rich Townsend** from the University of Wisconsin-Madison created **EZ-Web**, a simple, web-based interface to a code that can be used to calculate models over a wide range of masses and metallicities: <http://ftp.astro.wisc.edu/~townsend/static.php?ref=eZ-web>
- Read **carefully** the description of the program on its webpage and play with it.
- To construct and evolve a model, enter parameters into the form, and then submit the calculation request to the server.

Submit a Calculation

Initial Mass	<input type="text" value="1.0"/>
Metallicity	<input type="text" value="0.02"/> ▼
Maximum Age	<input type="text" value="0"/>
Maximum Number of Steps	<input type="text" value="0"/>
Create Detailed Structure Files?	<input checked="" type="checkbox"/>
Use CGS units?	<input checked="" type="checkbox"/>
Email Address	<input type="text" value="vitaly.neustroev@oulu.fi"/>

EZ-Web: Output File Formats

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Detailed structure files are text (ASCII) files containing one line for each grid point of the model. Each line is divided into 36 columns, containing the following data:

Summary files have the filename 'summary.txt'. They are text (ASCII) files containing one line for each time step. Each line is divided into 23 columns, containing the following data:

Column Number	Datum	Description
1	i	Step number
2	t	Age (years)
3	M	Mass (M_{\odot})
4	$\text{Log}_{10} L$	Luminosity (L_{\odot})
5	$\text{Log}_{10} R$	Radius (R_{\odot})
6	$\text{Log}_{10} T_s$	Surface temperature (K)
7	$\text{Log}_{10} T_c$	Central temperature (K)
8	$\text{Log}_{10} \rho_c$	Central density (kg m^{-3})
9	$\text{Log}_{10} P_c$	Central pressure (N m^{-2})
10	ψ_c	Central electron degeneracy parameter
11	X_c	Central hydrogen mass fraction
12	Y_c	Central helium mass fraction
13	$X_{C,c}$	Central carbon mass fraction
14	$X_{N,c}$	Central nitrogen mass fraction
15	$X_{O,c}$	Central oxygen mass fraction
16	T_{dyn}	Dynamical timescale (seconds)
17	T_{KH}	Kelvin-Helmholtz timescale (years)
18	T_{nuc}	Nuclear timescale (years)
19	L_{pp}	Luminosity from PP chain (L_{\odot})
20	L_{CNO}	Luminosity from CNO cycle (L_{\odot})
21	$L_{3\alpha}$	Luminosity from triple-alpha reactions (L_{\odot})
22	L_Z	Luminosity from metal burning (L_{\odot})
23	L_{ν}	Luminosity of neutrino losses (L_{\odot})
24	M_{He}	Mass of helium core (M_{\odot})
25	M_C	Mass of carbon core (M_{\odot})
26	M_O	Mass of oxygen core (M_{\odot})
27	R_{He}	Radius of helium core (R_{\odot})
28	R_C	Radius of carbon core (R_{\odot})
29	R_O	Radius of oxygen core (R_{\odot})

Column Number	Datum	Description
1	M_r	Lagrangian mass coordinate (M_{\odot})
2	r	Radius coordinate (R_{\odot})
3	L_r	Luminosity (L_{\odot})
4	P	Total pressure (N m^{-2})
5	ρ	Density (kg m^{-3})
6	T	Temperature (K)
7	U	Specific internal energy (J kg^{-1})
8	S	Specific entropy ($\text{J K}^{-1} \text{kg}^{-1}$)
9	C_p	Specific heat at constant pressure ($\text{J K}^{-1} \text{kg}^{-1}$)
10	Γ_1	First adiabatic exponent
11	∇_{ad}	Adiabatic temperature gradient
12	μ	Mean molecular weight (see note below)
13	n_e	Electron number density (m^{-3})
14	P_e	Electron pressure (N m^{-2})
15	P_r	Radiation pressure (N m^{-2})
16	∇_{rad}	Radiative temperature gradient
17	∇	Material temperature gradient
18	v_c	Convective velocity (m s^{-1})
19	κ	Rosseland mean opacity ($\text{m}^2 \text{kg}^{-1}$)
20	ϵ_{nuc}	Power per unit mass from all nuclear reactions, excluding neutrino losses (W kg^{-1})
21	ϵ_{pp}	Power per unit mass from PP chain (W kg^{-1})
22	ϵ_{CNO}	Power per unit mass from CNO cycle (W kg^{-1})
23	$\epsilon_{3\alpha}$	Power per unit mass from triple-alpha reaction (W kg^{-1})
24	$\epsilon_{\nu,\text{nuc}}$	Power loss per unit mass in nuclear neutrinos (W kg^{-1})
25	ϵ_{ν}	Power loss per unit mass in non-nuclear neutrinos (W kg^{-1})
26	ϵ_{grav}	Power per unit mass from gravitational contraction (W kg^{-1})
27	X	Hydrogen mass fraction (all ionization stages)
28	—	Molecular hydrogen mass fraction
29	X^+	Singly-ionized hydrogen mass fraction
30	Y	Helium mass fraction (all ionization stages)
31	Y^+	Singly-ionized helium mass fraction
32	Y^{++}	Doubly-ionized helium mass fraction
33	X_C	Carbon mass fraction
34	X_N	Nitrogen mass fraction
35	X_O	Oxygen mass fraction
36	ψ	Electron degeneracy parameter



summary.txt

structure 0000.txt

00000	0.00000000E+00	1.00000000E+00	-1.54655858E-01	-5.27369474E-02	3.74939481E+00	7.12628879E+00	1.89374032E+00	1.71500221E+01	-1.72642659E+00	6.9
00001	0.00000000E+00	1.00000000E+00	-1.4912128E-01	-4.7791210E-02	3.74830842E+00	7.12713579E+00	1.89359124E+00	1.71500910E+01	-1.72840030E+00	6.9
00002	1.00000000E+05	9.99999999E-01	-1.49130943E-01	-4.7800510E-02	3.74805703E+00	7.12713579E+00	1.89359124E+00	1.71500910E+01	-1.72840030E+00	6.9
00003	1.00000000E+05	9.99999999E-01	-1.49116611E-01	-4.77974786E-02	3.74803918E+00	7.12713579E+00	1.89359124E+00	1.71500910E+01	-1.72840030E+00	6.9
00004	0.23000000E+05	9.99999997E-01	-1.49108609E-01	-4.7796525E-02	3.74831096E+00	7.12713579E+00	1.89359124E+00	1.71500910E+01	-1.72840030E+00	6.9
00005	3.18400000E+05	9.99999978E-01	-1.49105478E-01	-4.7795451E-02	3.74831096E+00	7.12713579E+00	1.89359124E+00	1.71500910E+01	-1.72840030E+00	6.9
00006	4.22000000E+05	9.99999978E-01	-1.49102247E-01	-4.7794376E-02	3.74831096E+00	7.12713579E+00	1.89359124E+00	1.71500910E+01	-1.72840030E+00	6.9
00007	5.46900000E+05	9.99999963E-01	-1.49107610E-01	-4.77957983E-02	3.74831108E+00	7.12712843E+00	1.89356774E+00	1.71510855E+01	-1.72824622E+00	6.9
00008	6.9752920E+05	9.99999952E-01	-1.49108163E-01	-4.7795643E-02	3.74831240E+00	7.12712421E+00	1.89360565E+00	1.71510781E+01	-1.72814580E+00	6.9
00009	8.74952420E+05	9.99999939E-01	-1.49097483E-01	-4.7792551E-02	3.74831240E+00	7.12711850E+00	1.89360565E+00	1.71511941E+01	-1.7281135E+00	6.9
00010	1.0894940E+06	9.99999920E-01	-1.4908937E-01	-4.77947786E-02	3.74831455E+00	7.12710048E+00	1.89370761E+00	1.71511571E+01	-1.72788672E+00	6.9
00011	1.34793411E+06	9.99999900E-01	-1.4907580E-01	-4.77925941E-02	3.74831682E+00	7.12709846E+00	1.89379152E+00	1.71512178E+01	-1.72762874E+00	6.9
00012	1.6752939E+06	9.9999881E-01	-1.4906047E-01	-4.77877308E-02	3.74831735E+00	7.12712252E+00	1.89360742E+00	1.71511039E+01	-1.72829797E+00	6.9
00013	2.0292511E+06	9.9999859E-01	-1.4905369E-01	-4.7785369E-02	3.74831767E+00	7.12722384E+00	1.89369916E+00	1.71510525E+01	-1.72828862E+00	6.9
00014	2.4783018E+06	9.9999832E-01	-1.4904634E-01	-4.77846553E-02	3.74831414E+00	7.12722883E+00	1.89383108E+00	1.71511894E+01	-1.72947167E+00	6.9
00015	3.0079164E+06	9.9999781E-01	-1.4903744E-01	-4.77911374E-02	3.74831324E+00	7.12730314E+00	1.89329595E+00	1.71510525E+01	-1.7297591E+00	6.9
00016	3.6517559E+06	9.9999720E-01	-1.4911554E-01	-4.7798273E-02	3.74830777E+00	7.12732148E+00	1.89310946E+00	1.71510810E+01	-1.7292948E+00	6.9
00017	4.42718647E+06	9.9999647E-01	-1.4913813E-01	-4.78009077E-02	3.74830742E+00	7.12732026E+00	1.8932107E+00	1.71510810E+01	-1.7294060E+00	6.9
00018	5.3652777E+06	9.9999569E-01	-1.4913513E-01	-4.7805771E-02	3.74830864E+00	7.12728762E+00	1.8935191E+00	1.71510941E+01	-1.7293992E+00	6.9
00019	6.4583322E+06	9.9999492E-01	-1.4906506E-01	-4.77907936E-02	3.74831858E+00	7.12727168E+00	1.8936117E+00	1.71511688E+01	-1.72878195E+00	6.9
00020	7.7849999E+06	9.9999408E-01	-1.4908908E-01	-4.7782878E-02	3.74832638E+00	7.12726872E+00	1.8940086E+00	1.71512556E+01	-1.7278557E+00	6.9
00021	9.3843998E+06	9.9999318E-01	-1.4854759E-01	-4.76816228E-02	3.74830922E+00	7.12722983E+00	1.89549437E+00	1.71522878E+01	-1.7266407E+00	6.9
00022	1.1381288E+07	9.9999210E-01	-1.4819711E-01	-4.75912311E-02	3.74830717E+00	7.12723429E+00	1.89523491E+00	1.71527982E+01	-1.7265920E+00	6.9
00023	1.3645136E+07	9.9999097E-01	-1.47522899E-01	-4.7493580E-02	3.74830858E+00	7.12723439E+00	1.89608781E+00	1.71537496E+01	-1.7255025E+00	6.9
00024	1.6274211E+07	9.9998985E-01	-1.4672179E-01	-4.7321210E-02	3.7483198E+00	7.12721791E+00	1.89831289E+00	1.71549733E+01	-1.7160972E+00	6.9
00025	2.3649426E+07	9.9998852E-01	-1.45568095E-01	-4.7244747E-02	3.74829238E+00	7.12729882E+00	1.89973001E+00	1.71670735E+01	-1.7100932E+00	6.9
00026	2.8418056E+07	9.9998720E-01	-1.4439181E-01	-4.7181853E-02	3.74830842E+00	7.12729444E+00	1.9015819E+00	1.71548169E+01	-1.7126430E+00	6.9
00027	3.414238E+07	9.9998585E-01	-1.43291075E-01	-4.7108187E-02	3.74829588E+00	7.12729885E+00	1.9031761E+00	1.7161581E+01	-1.7107252E+00	6.9
00028	4.1111656E+07	9.9998451E-01	-1.42291171E-01	-4.6998180E-02	3.7483051E+00	7.12728546E+00	1.9053229E+00	1.7163836E+01	-1.7052617E+00	6.9
00029	4.9253322E+07	9.9998317E-01	-1.41291171E-01	-4.6901830E-02	3.7483051E+00	7.12728546E+00	1.9053229E+00	1.7163836E+01	-1.7052617E+00	6.9
00030	5.9114878E+07	9.9998183E-01	-1.40291171E-01	-4.6810555E-02	3.7483051E+00	7.12728546E+00	1.9053229E+00	1.7163836E+01	-1.7052617E+00	6.9
00031	7.0128941E+07	9.9998049E-01	-1.39291171E-01	-4.6720280E-02	3.7483051E+00	7.12728546E+00	1.9053229E+00	1.7163836E+01	-1.7052617E+00	6.9
00032	8.2525473E+07	9.9997915E-01	-1.38291171E-01	-4.6630005E-02	3.7483051E+00	7.12728546E+00	1.9053229E+00	1.7163836E+01	-1.7052617E+00	6.9
00033	9.6464646E+07	9.9997781E-01	-1.37291171E-01	-4.6540005E-02	3.7483051E+00	7.12728546E+00	1.9053229E+00	1.7163836E+01	-1.7052617E+00	6.9
00034	1.1228588E+08	9.9997647E-01	-1.36291171E-01	-4.6450005E-02	3.7483051E+00	7.12728546E+00	1.9053229E+00	1.7163836E+01	-1.7052617E+00	6.9
00035	1.3147667E+08	9.9997513E-01	-1.35291171E-01	-4.6360005E-02	3.7483051E+00	7.12728546E+00	1.9053229E+00	1.7163836E+01	-1.7052617E+00	6.9
00036	1.5248652E+08	9.9997379E-01	-1.34291171E-01	-4.6270005E-02	3.7483051E+00	7.12728546E+00	1.9053229E+00	1.7163836E+01	-1.7052617E+00	6.9
00037	1.7596865E+08	9.9997245E-01	-1.33291171E-01	-4.6180005E-02	3.7483051E+00	7.12728546E+00	1.9053229E+00	1.7163836E+01	-1.7052617E+00	6.9
00038	2.0248021E+08	9.9997111E-01	-1.32291171E-01	-4.6090005E-02	3.7483051E+00	7.12728546E+00	1.9053229E+00	1.7163836E+01	-1.7052617E+00	6.9
00039	2.3254292E+08	9.9996977E-01	-1.31291171E-01	-4.6000005E-02	3.7483051E+00	7.12728546E+00	1.9053229E+00	1.7163836E+01	-1.7052617E+00	6.9
00040	2.6674147E+08	9.9996843E-01	-1.2997693E-01	-4.5878248E-02	3.75213996E+00	7.12713579E+00	1.9310807E+00	1.7187700E+01	-1.6675380E+00	6.7
00041	3.0473187E+08	9.9996709E-01	-1.2897693E-01	-4.5809814E-02	3.75213996E+00	7.12713579E+00	1.9310807E+00	1.7187700E+01	-1.6675380E+00	6.7
00042	3.4671947E+08	9.9996575E-01	-1.2797693E-01	-4.5741380E-02	3.75213996E+00	7.12713579E+00	1.9310807E+00	1.7187700E+01	-1.6675380E+00	6.7
00043	3.9271947E+08	9.9996441E-01	-1.2697693E-01	-4.5672946E-02	3.75213996E+00	7.12713579E+00	1.9310807E+00	1.7187700E+01	-1.6675380E+00	6.7
00044	4.4271947E+08	9.9996307E-01	-1.2597693E-01	-4.5604512E-02	3.75213996E+00	7.12713579E+00	1.9310807E+00	1.7187700E+01	-1.6675380E+00	6.7
00045	4.9671947E+08	9.9996173E-01	-1.2497693E-01	-4.5536078E-02	3.75213996E+00	7.12713579E+00	1.9310807E+00	1.7187700E+01	-1.6675380E+00	6.7
00046	5.5471947E+08	9.9996039E-01	-1.2397693E-01	-4.5467644E-02	3.75213996E+00	7.12713579E+00	1.9310807E+00	1.7187700E+01	-1.6675380E+00	6.7
00047	6.1671947E+08	9.9995905E-01	-1.2297693E-01	-4.5399210E-02	3.75213996E+00	7.12713579E+00	1.9310807E+00	1.7187700E+01	-1.6675380E+00	6.7
00048	6.8271947E+08	9.9995771E-01	-1.2197693E-01	-4.5330776E-02	3.75213996E+00	7.12713579E+00	1.9310807E+00	1.7187700E+01	-1.6675380E+00	6.7
00049	7.5271947E+08	9.9995637E-01	-1.2097693E-01	-4.5262342E-02	3.75213996E+00	7.12713579E+00	1.9310807E+00	1.7187700E+01	-1.6675380E+00	6.7
00050	8.2671947E+08	9.9995503E-01	-1.1997693E-01	-4.5193908E-02	3.75213996E+00	7.12713579E+00	1.9310807E+00	1.7187700E+01	-1.6675380E+00	6.7
00051	9.0471947E+08	9.9995369E-01	-1.1897693E-01	-4.5125474E-02	3.75213996E+00	7.12713579E+00	1.9310807E+00	1.7187700E+01	-1.6675380E+00	6.7
00052	9.8671947E+08	9.9995235E-01	-1.1797693E-01	-4.5057040E-02	3.75213996E+00	7.12713579E+00	1.9310807E+00	1.7187700E+01	-1.6675380E+00	6.7
00053	1.07271947E+09	9.9995101E-01	-1.1697693E-01	-4.4988606E-02	3.75213996E+00	7.12713579E+00	1.9310807E+00	1.7187700E+01	-1.6675380E+00	6.7
00054	1.16271947E+09	9.9994967E-01	-1.1597693E-01	-4.4920172E-02	3.75213996E+00	7.12713579E+00	1.9310807E+00	1.7187700E+01	-1.6675380E+00	6.7
00055	1.25771947E+09	9.9994833E-01	-1.1497693E-01	-4.4851738E-02	3.75213996E+00	7.12713579E+00	1.9310807E+00	1.7187700E+01	-1.6675380E+00	6.7
00056	1.35771947E+09	9.9994699E-01	-1.1397693E-01	-4.4783304E-02	3.75213996E+00	7.12713579E+00	1.9310807E+00	1.7187700E+01	-1.6675380E+00	6.7
00057	1.46271947E+09	9.9994565E-01	-1.1297693E-01	-4.4714870E-02	3.75213996E+00	7.12713579E+00	1.9310807E+00	1.7187700E+01	-1.6675380E+00	6.7
00058	1.57271947E+09	9.9994431E-01	-1.1197693E-01	-4.4646436E-02	3.75213996E+00	7.12713579E+00	1.9310807E+00	1.7187700E+01	-1.6675380E+00	6.7
00059	1.68771947E+09	9.9994297E-01	-1.1097693E-01	-4.4578002E-02	3.75213996E+00	7.12713579E+00	1.9310807E+00	1.7187700E+01	-1.6675380E+00	6.7
00060	1.80771947E+09	9.9994163E-01	-1.0997693E-01	-4.4509568E-02	3.75213996E+00	7.12713579E+00	1.9310807E+00	1.7187700E+01	-1.6675380E+00	6.7
00061	1.93271947E+09	9.9994029E-01	-1.0897693E-01	-4.4441134E-02	3.75213996E+00	7.12713579E+00	1.9310807E+00	1.7187700E+01	-1.6675380E+00	6.7
00062	2.06271947E+09	9.9993895E-01	-1.0797693E-01	-4.4372700E-02	3.75213996E+00	7.12713579E+00	1.9310807E+00	1.7187700E+01	-1.6675380E+00	6.7
00063	2.19771947E+09	9.9993761E-01	-1.0697693E-01	-4.4304266E-02	3.75213996E+00	7.12713579E+00	1.9310807E+00	1.7187700E+01	-1.6675380E+00	6.7
00064	2.33771947E+09	9.9993627E-01	-1.0597693E-01	-4.4235832E-02						

EZ-Web: Limitations

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- **EZ-Web** and the underlying EZ code have a number of limitations which restrict their validity. In some cases, the results can be misleading or inaccurate, and users should be aware of this if using EZ-Web for research purposes.
- As an alternative to **EZ-Web**, consider using **MESA-Web** — a web-based interface to the fully-featured **MESA** stellar evolution code. **MESA-Web** can produce models which are suitable for detailed scientific investigations: <http://user.astro.wisc.edu/~townsend/static.php?ref=mesa-web-submit>

Schematic stellar evolution

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THE (TEMPERATURE, DENSITY) DIAGRAM
ZONES OF THE EQUATION OF STATE
ZONES OF NUCLEAR BURNING
EVOLUTION OF A STAR IN THE (LOG P, LOG P) PLANE

Introduction

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- We have derived all the differential equations that define uniquely the equilibrium properties of a star of a given mass and composition. We know how to solve them.
- Our task now is to combine the knowledge acquired so far into a general picture of the evolution of stars.
- We will consider the schematic evolution of a star, as seen from its centre. The centre is the point with the highest pressure and density, and (usually) the highest temperature, where nuclear burning proceeds fastest. Therefore, the centre is the **most evolved** part of the star, and it **sets** the pace of evolution, with the outer layers lagging behind.
- The stellar centre is characterized by the central density ρ_c , pressure P_c and temperature T_c and the composition (usually expressed in terms of μ and/or μ_e). These quantities are related by the equation of state (EOS).
- We can thus represent the evolution of a star by an evolutionary track in the (P_c, ρ_c) diagram or the (T_c, ρ_c) diagram.
- Since the only property that distinguishes the evolutionary track of a star from that of any other star of the same composition is its **mass**, we may expect to obtain different lines in the (T_c, ρ_c) plane for different masses.
- The (T_c, ρ_c) plane will be divided into zones dominated by different equations of state and different nuclear processes.

Zones of the equation of state

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- As the ranges of density and temperature typical of stellar interiors span many orders of magnitude, **logarithmic** scales will be used for both.
- By considering the EOS we can derive the evolution of the central temperature. This is obviously crucial for the evolution track of a star because nuclear burning requires T_c to reach certain (high) values.
- We have previously encountered various regimes for the EOS:
 - The most common EOS is that of an ideal gas: $P = \frac{\Re T \rho}{\mu} = K_0 \rho T$
 - If radiation pressure is dominant, then the equation of state changes to $P = \frac{aT^4}{3}$
 - At high densities and relatively low temperatures, the electrons become degenerate, and since their contribution to the pressure is dominant, the EOS is replaced by $P = K_1 \rho^{5/3}$. This is independent of temperature. **More accurately**: the complete degeneracy implied by this relation is only achieved when $T_c \rightarrow 0$.
 - For still higher densities, when relativistic effects play an important role, the EOS changes to the form $P = K_2 \rho^{4/3}$.

EOS in the $(\log P, \log \rho)$ plane

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- The transition from one state to the other is, of course, **gradual** with the change in density and temperature, but an approximate boundary may be traced in the $(\log P, \log \rho)$ plane.
- The boundaries may be defined by the requirement that **the pressure obtained from a one EOS be equal to that obtained another**. For example, the boundary between the ideal gas zone and the non-relativistic-degeneracy zone may be obtained, $K_0 \rho T = K_1 \rho^{5/3}$, which defines a straight line with a slope of 1.5:

$$\log \rho = 1.5 \log T + \text{constant.}$$

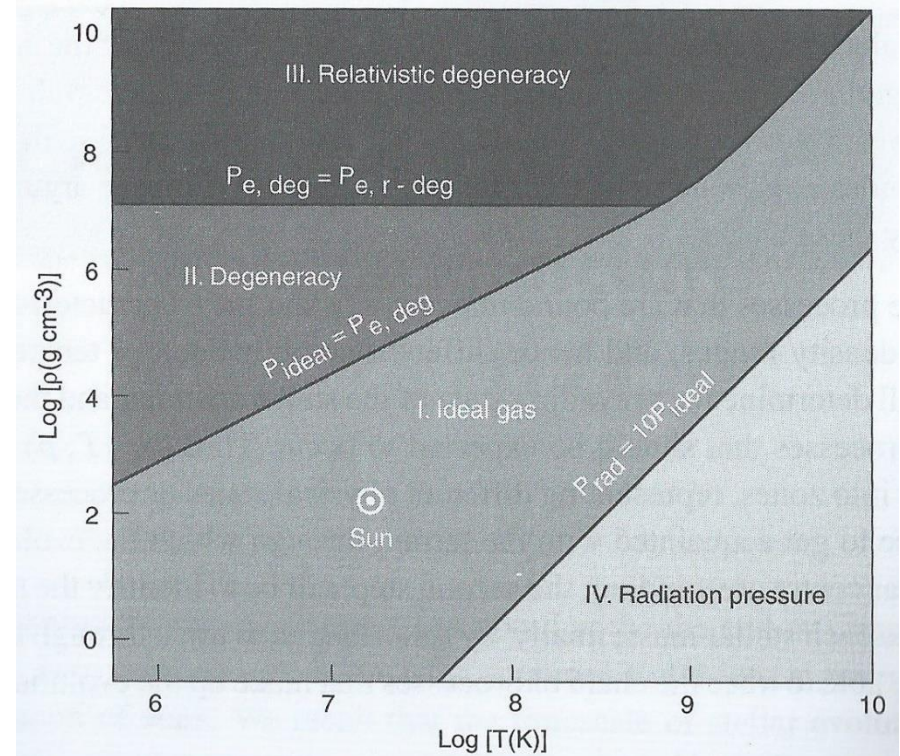


Figure from Prialnik

Zones of nuclear burning

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The nuclear energy generation rate is a sensitive function of the temperature, which can be written as

$$\varepsilon \approx \varepsilon_0 \rho^\lambda T^n$$

where for most nuclear reactions (those involving two nuclei) $\lambda=1$, while n depends mainly on the masses and charges of the nuclei involved and usually $n \gg 1$.

For H-burning by the pp-chain, $n \approx 4$ and for the CNO-cycle which dominates at somewhat higher temperature, $n \approx 18$.

For He-burning by the triple-alpha reaction, $n \approx 40$ (and $\lambda=2$ because three particles are involved). For C-burning and O-burning reactions n is even larger.

As discussed in previous lectures, the consequences of this strong temperature sensitivity are that

- each nuclear reaction takes place at a particular, nearly constant temperature, and
- nuclear burning cycles of subsequent heavier elements are well separated in temperature

The threshold given by $\varepsilon \approx \varepsilon_{min}$ is

$$\log \rho = -\frac{n}{\lambda} \log T + \frac{1}{\lambda} \log \frac{\varepsilon_{min}}{\varepsilon_0}$$

On one side of the threshold the rate of nuclear burning may be assumed negligible, and on the other side – considerable.

Nuclear burning in the $(\log P, \log \rho)$ plane

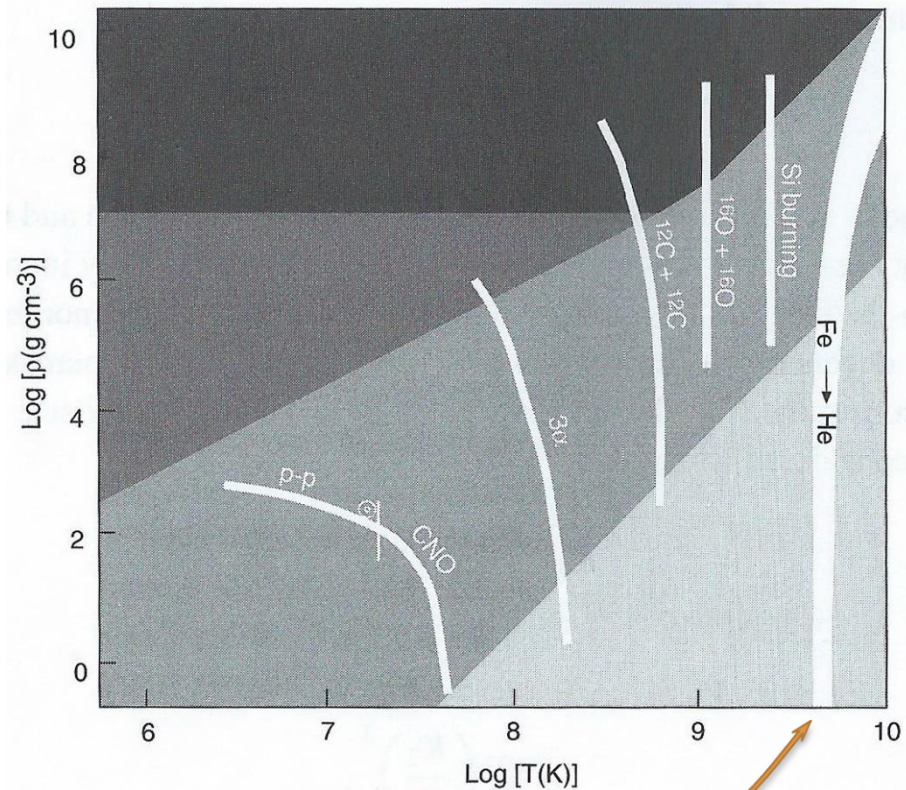
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The transformation of hydrogen into the iron group elements comprises **five** major stages:

- **hydrogen** burning into helium either by the p-p chain or by the CNO cycle;
- **helium** burning into carbon by the 3 α reaction;
- **carbon** burning;
- **oxygen** burning;
- **silicon** burning.

Nucleosynthesis ends with iron.

Iron nuclei heated to very high temperatures are disintegrated by energetic photons into helium nuclei. This energy **absorbing** process reaches equilibrium, with the relative abundance of iron to helium nuclei determined by the values of temperature and density. A threshold may be defined for the process of iron photodisintegration, as a strip in the $(\log P, \log \rho)$ plane, by the requirement that the number of helium and iron nuclei be approximately equal.



The evolutionary path of the central point

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Are the centre of a star of given mass M may assume any combination of temperature and density values, or these values are in some way constrained by M ?

We now regard the $(\log P, \log \rho)$ plane as a $(\log P_c, \log \rho_c)$ plane, referring to the stellar centre.

Assuming a polytropic configuration for a star in hydrostatic equilibrium, the central density is related to the central pressure by equation

$$P_c = (4\pi)^{1/3} B_n G M^{2/3} \rho_c^{4/3}$$

This relation is only weakly dependent on the polytropic index n , especially for stable configurations, for which n varies between 1.5 and 3 and the coefficient B_n between 0.157 and 0.206 (see the table above), and it is independent of K . As we noted before, this relation provides a good approximation to hydrostatic equilibrium for any configuration.

Additionally, P_c is related to ρ_c and T_c by the EOS (we have different ones). Combining each of them with the above relation, we can eliminate P_c to obtain a relation between ρ_c and T_c .

For example, for a star of mass M , whose central point is found in the ideal gas zone I, we obtain the relation between ρ_c and T_c

$$\rho_c = \frac{K_0^3}{4\pi B_n^3 G^3} \frac{T_c^3}{M^2}$$

For a star of given mass, the central density varies as the central temperature cubed.

The central point in the $(\log P, \log \rho)$ plane

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- For a star of mass M , whose central point is found in the ideal gas **zone I**, we obtain the relation

$$\rho_c = \frac{K_0^3}{4\pi B_n^3 G^3} \frac{T_c^3}{M^2}$$

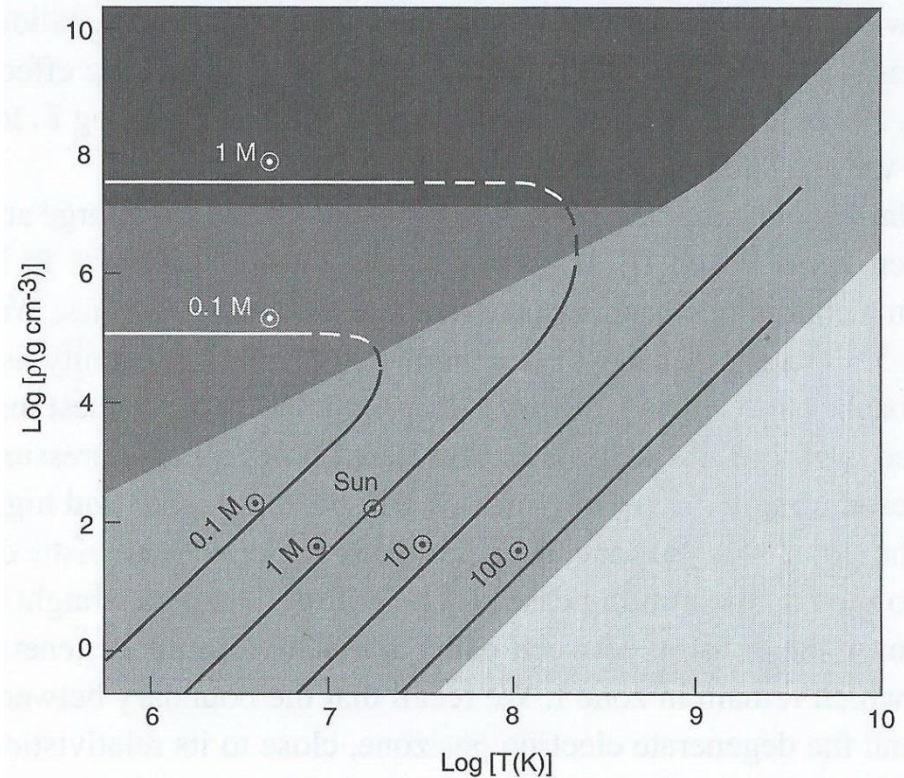
On logarithmic scales, it becomes a straight line with a slope of 3. Different masses define different parallel lines.

- If at the centre of a star the electrons are strongly but non-relativistically degenerate, the central point is found in **zone II**. Then the relation is

$$\rho_c = \left(\frac{B_{1.5} G}{K_1} \right)^3 M^2$$

which replaces the ideal gas relation. Here ρ_c is independent of T_c and the corresponding line in the $(\log P_c, \log \rho_c)$ plane is horizontal and increases with mass M .

- Zones I and II are the only stable regions in the $(\log P, \log \rho)$ plane. Hence, there is no need to consider the others.

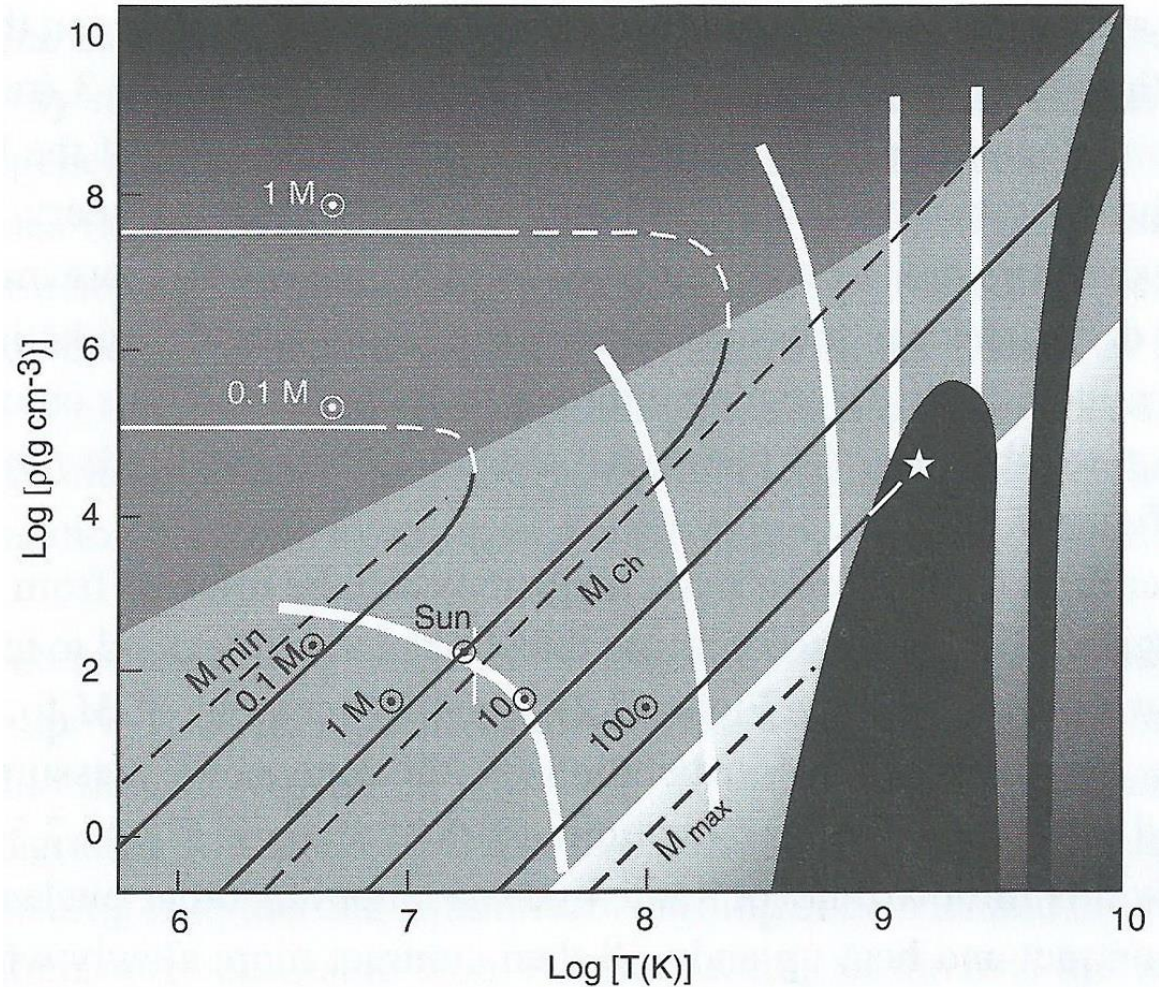


For relatively low masses, the relations will merge at the boundary between zones I and II, resulting in a continuous bending path characteristic of each mass.

Evolution of a star in the $(\log P, \log \rho)$ plane

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Stars are limited to a rather narrow mass range of $0.1 M_{\odot}$ to $\sim 100 M_{\odot}$. The lower limit is set by the minimum temperature required for nuclear burning, and the upper limit by the requirement of dynamical stability.



Star formation

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BASIC PHYSICS OF STAR FORMATION
JEANS INSTABILITY
THE JEANS MASS, LENGTH, DENSITY
STEPS OF STAR FORMATION

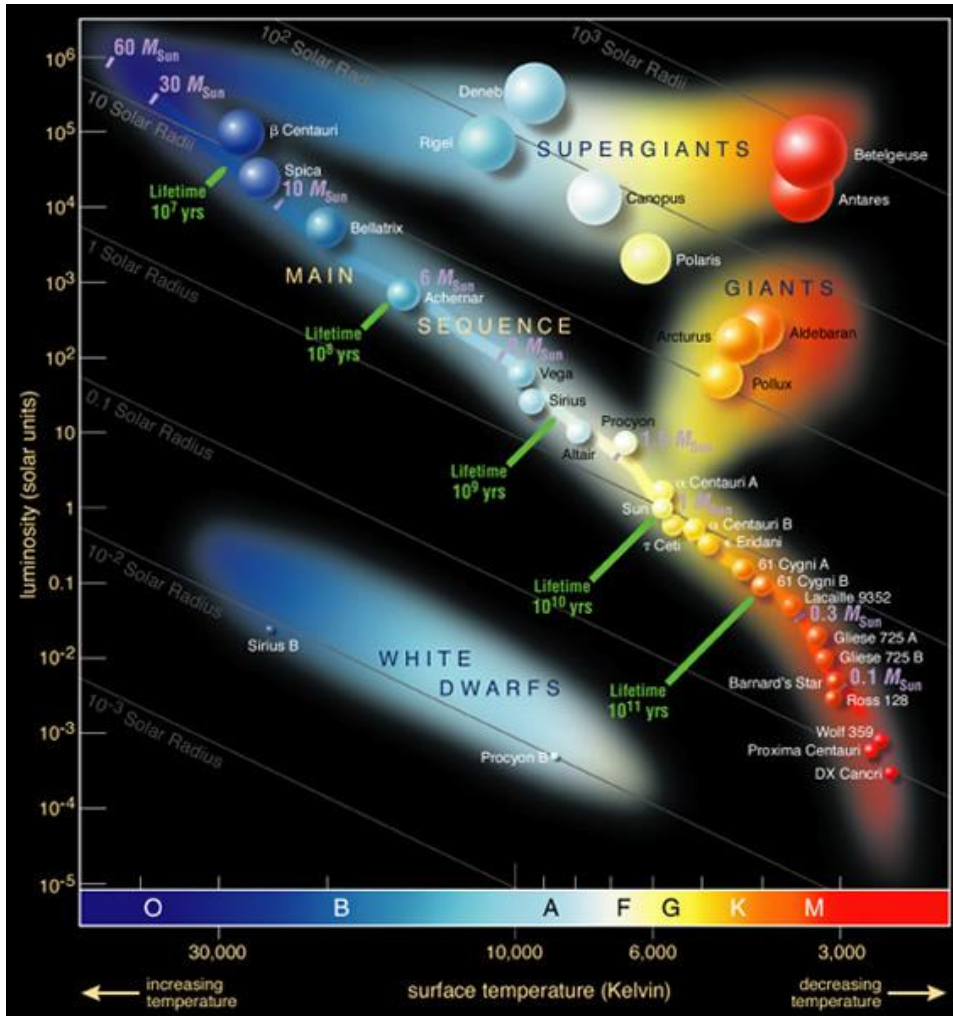
Introduction

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- Today we will deal with early phases in the evolution of stars, as they evolve towards and during the main-sequence phase.
- We start with a very brief (and incomplete) overview of the formation of stars.

The Hertzsprung–Russell diagram

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Stars are like people in that they are born, grow up, mature, and die. A star's mass determines what life path it will take.

The HR diagram shows the relationship between the stars' luminosities versus their effective temperatures.

Different evolutionary stages correspond to different positions at HR diagram.

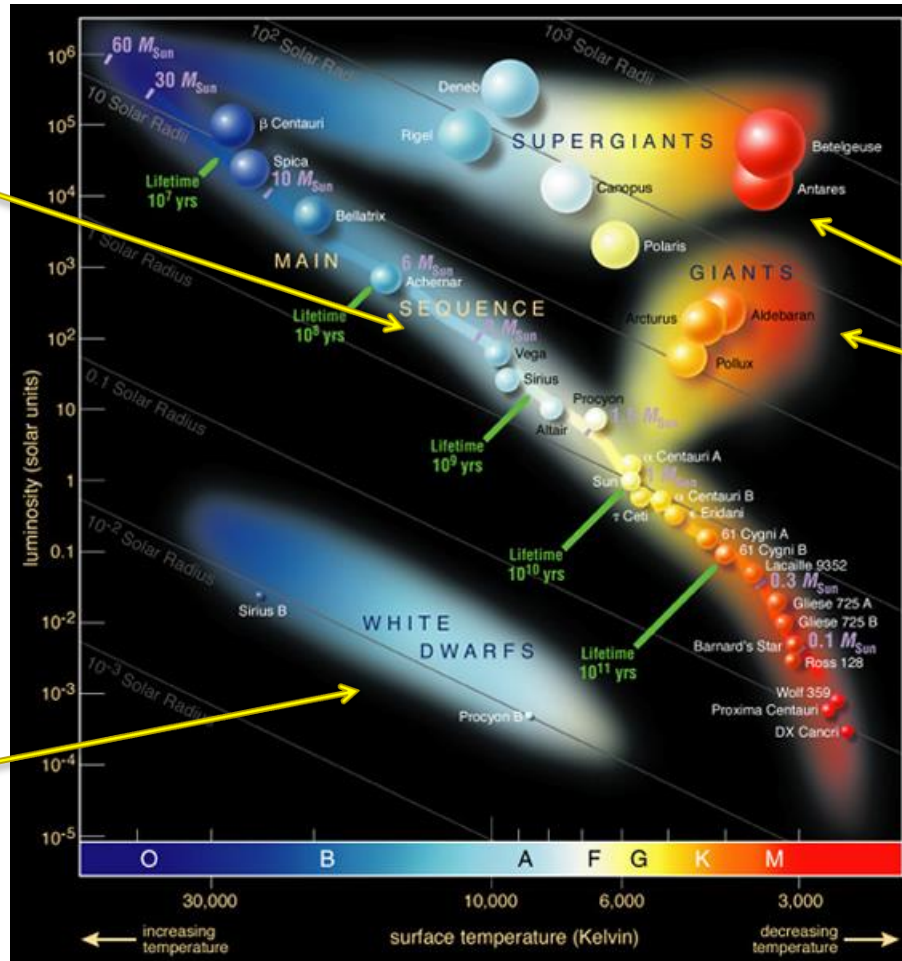
90% of a lifetime a star spends at the Main Sequence, but before it, a star must be formed and have arrived there.

The HR diagram

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Most of the stars lie on the **Main Sequence**, with increasing L as T increases

A relatively hot star can have very low luminosity, if its radius is very small ($0.01 R_{\odot}$): **White Dwarfs**



A relatively cool star can be quite luminous if it has a large enough radius (10 - $100 R_{\odot}$): **Red Giants** and **Supergiants**

Star formation

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- Observations indicate that stars are formed in giant molecular clouds with masses of order $10^2 - 10^5 M_{\odot}$. These clouds have typical dimensions of ~ 100 parsec, temperatures of $10-100$ K and densities of $10^2-10^4 \text{ cm}^{-3}$ (where the lowest temperatures pertain to the densest parts of the cloud).
- A certain fraction, about 1%, of the cloud material is in the form of dust which makes the clouds very opaque to visual wavelengths.
- While the densities of molecular clouds are among the highest encountered in the Interstellar Medium (ISM), even this gas is extremely rarified compared to gas at an atmospheric density of $\sim 10^{19} \text{ cm}^{-3}$. In fact, the densities of molecular clouds are many orders of magnitude lower than the density of the best vacuum achievable in the laboratory.
- As we have seen, the mean mass densities inside stars are $\sim 1 \text{ g cm}^{-3}$, i.e., particle densities of $\sim 10^{24} \text{ cm}^{-3}$. Thus, to form new stars, some regions of a molecular cloud must be compressed by many orders of magnitude.

The nearest giant molecular clouds are in the Orion star-forming region, at a distance of about 500 pc.





In spiral galaxies, star formation is concentrated along spiral arms.

Spiral arms are places where gas is compressed, probably the first step toward star formation.

What we know from observations?

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The details of the process of star formation are not understood yet. We can outline, however, some of the general criteria under which gravitational contraction of a gas cloud can proceed, and potentially lead to the conditions required for star formation. This is what we know from observations:

- Stars form out of **molecular gas** which is assembled into **dense molecular clouds** in spiral arms.
- Molecular clouds have a complex, often filamentary structure. Individual stars, or small groups, form from the smallest scale structures, cloud cores of size ~ 0.1 pc.
- Molecular clouds probably have lifetimes of 10^6 to 10^7 yr, which is only a few dynamical times. Star formation is a fairly rapid process once molecular clouds have formed.
- If massive stars form within a young cluster, their ionizing radiation / stellar winds / supernovae destroy the molecular cloud on a short time scale.
- Most stars ($\sim 80\%$) form in clusters at least as rich as Orion.