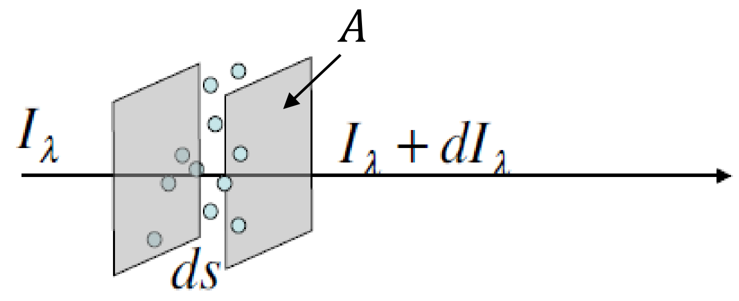
Interaction radiation – matter

- As noticed above, specific intensity I_λ is ***independent of distance*** from the source and remains the same as radiation propagates through **free** space.
- However, space is **not** always **free** (consider, for instance, stellar interiors).
- Thus, energy can be removed from, or delivered to, the radiation field.
- In this case, the intensity of light will change.

Absorption coefficient (1)

- Consider radiation shining through a medium



- The intensity of light is found experimentally to decrease by an amount dI_λ where

$$dI_\lambda = -\kappa_\lambda \rho I_\lambda ds$$

Here κ_λ is the so-called mass absorption coefficient (alias opacity) [cm² g⁻¹], ρ is the density (in mass per unit volume), and ds is a length.

It can also be represented as $\alpha_\lambda = \kappa_\lambda \rho$, where α_λ is the absorption coefficient [cm⁻¹].

Absorption coefficient (2)

Number density of absorbers (particles per unit volume) = n
Each absorber has cross-sectional area = σ_λ (units cm^2)

If a light beam travels through ds, total area of absorbers is:

$$\text{number of absorbers} \times \text{cross-section} = n\, A\, \text{ds} \times \sigma_\lambda$$

Fraction of radiation absorbed = fraction of area blocked:

$$\frac{dI_\lambda}{I_\lambda} = -\frac{n\ A\ ds\ \sigma_\lambda}{A} = -n\sigma_\lambda ds$$

$$dI_\lambda = -n\sigma_\lambda I_\lambda ds = -\alpha_\lambda I_\lambda ds$$

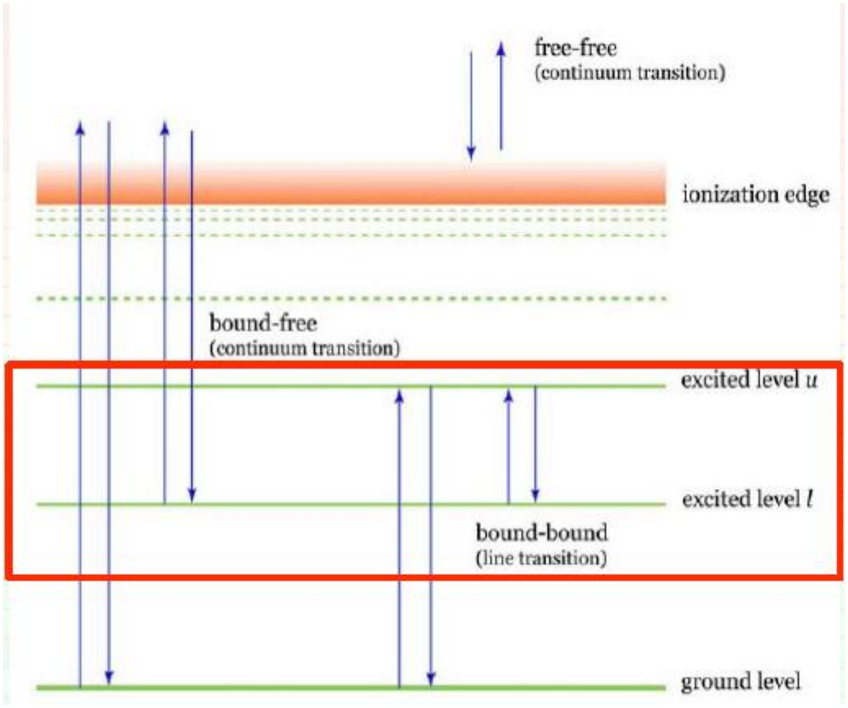
Thus, $\alpha_\lambda = \kappa_\lambda \rho = n\sigma_\lambda$, where α_λ is the absorption coefficient [cm^{-1}].

The photon mean free path is inversely proportional to α_λ.

Opacity

Opacity is the resistance of material to the flow of radiation through it. In most stellar interiors it is determined by all the processes which scatter and absorb photons:

- bound-bound absorption
- bound-free absorption
- free-free absorption
- scattering



The first three are known as true absorption processes because they involve the disappearance of a photon, whereas the fourth process only alters the direction of a photon.

Bound-bound absorption

- Bound-bound absorptions occur when an electron is moved from one orbit in an atom or ion into another orbit of higher energy due to the absorption of a photon. If the energy of the two orbits is E_1 and E_2, a photon of frequency ν_{bb} will produce a transition if

$$E_2 - E_1 = h\nu_{bb}$$

- Bound-bound processes are responsible for the spectral lines visible in stellar spectra, which are formed in the atmospheres of stars.

- In stellar interiors, however, bound-bound processes are **not** of great importance as most of the atoms are highly ionised and only a small fraction contain electrons in bound orbits.
- In addition, most of the photons in stellar interiors are so energetic that they are more likely to cause bound-free absorptions, as described below.

Bound-free absorption

- Bound-free absorptions involve the ejection of an electron from a bound orbit around an atom or ion into a free hyperbolic orbit due to the absorption of a photon. A photon of frequency ν_{bf} will convert a bound electron of energy E_1 into a free electron of energy E_3 if

$$E_3 - E_1 = h\nu_{bf}$$

- Provided the photon has sufficient energy to remove the electron from the atom or ion, any value of energy can lead to a bound-free process.
- Bound-free processes hence lead to continuous absorption in stellar atmospheres.
- In stellar interiors, however, the importance of bound-free processes is **reduced** due to the rarity of bound electrons.

Free-free absorption & emission

- Free-free absorption occurs when a free electron of energy E_3 absorbs a photon of frequency ν_{ff} and moves to a state with energy E_4, where

$$E_4 - E_3 = h\nu_{ff}$$

- There is no restriction on the energy of a photon which can induce a free-free transition and hence free-free absorption is a **continuous** absorption process which operates in **both** stellar atmospheres and stellar interiors.

- Note that, in both free-free and bound-free absorption, low energy photons are more likely to be absorbed than high energy photons.

Scattering

- In addition to the above absorption processes, it is also possible for a photon to be scattered by an electron or an atom. One can think of scattering as a collision between two particles which bounce of one another.

- For example, **electron scattering** – deflection of a photon from its original path by a free electron, without changing its wavelength.

- There are a lot of free electrons in stellar interiors, so this is an important process which operates in stellar interiors.
- Although this process does not lead to the true absorption of radiation, it does slow the rate at which energy escapes from a star because it continually changes the direction of the photons.

Example: Thomson scattering

A free electron has a cross section to radiation given by the Thomson value:

$$\sigma_T = 6.7\times10^{-25}\ \text{cm}^2$$

…independent of frequency. The opacity is therefore:

Avogadro constant

$$\kappa_\lambda = \frac{n}{\rho}\sigma_T = N_A\sigma_T = \underbrace{0.4\ \text{cm}^2\text{g}^{-1}}_{\text{If the gas is pure hydrogen (protons and electrons only)}}$$

(note: we should distinguish between absorption and scattering, but don't need to worry about that here…)

Optical Depth

- Two physical processes contribute to the opacity κ_λ (note that subscript λ just means that absorption is photon-wavelength dependent);
  - (i) true absorption where the photon is destroyed and the energy thermalized;
  - (ii) scattering where the photon is shifted in direction and removed from the solid angle under consideration.

$$dI_\lambda = -\kappa_\lambda \rho I_\lambda ds \Rightarrow \frac{dI_\lambda}{I_\lambda} = -\kappa_\lambda \rho ds \Rightarrow \ln I_\lambda = -\int_0^s \kappa_\lambda \rho ds + \ln C$$

$$I_\lambda = Ce^{-\int_0^s \kappa_\lambda \rho ds} = Ce^{-\tau_\lambda}$$

If s=0, then $C=I_\lambda{}^0$ $$= I_\lambda{}^0 e^{-\tau_\lambda}$$ the usual simple extinction law

$$\boldsymbol{\tau_\lambda = \int_0^s \kappa_\lambda \rho ds}$$ the "optical depth"

Importance of optical depth

- We can write the change in specific intensity over a path length as

$$dI_\lambda = -I_\lambda d\tau_\lambda$$

This is a "passive" situation where no emission occurs and is the simplest example of the radiative transfer equation.

- An optical depth of $\tau=0$ corresponds to no reduction in intensity (i.e. the top of photosphere for a star).
- An optical depth of $\tau=1$ corresponds to a reduction in intensity by a factor of $e=2.7$.
- If the optical depth is large ($\tau \gg 1$) negligible intensity reaches the observer.
- In stellar atmospheres, typical photons originate from $\tau=2/3$ (the proof will follow later on).

Emission coefficient and Source function

- We can also treat emission processes in the same way as absorption via a (volume) emission coefficient ε_λ [erg/s/cm³/str/Å] , or a (mass) emission coefficient j_λ [erg/s/g/str/Å]

$$dI_\lambda = \varepsilon_\lambda ds = j_\lambda \rho ds$$

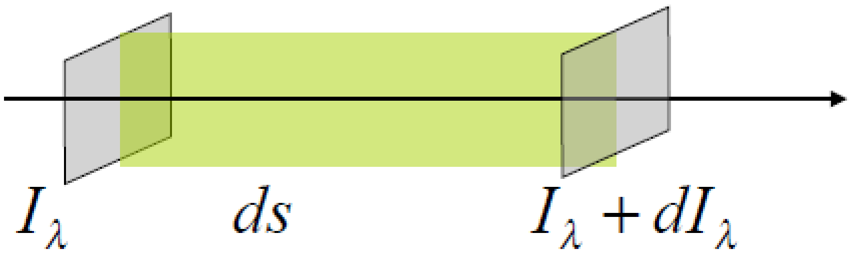
- Physical processes contributing to ε_λ , are
  (i) True (real) emission – the creation of photons;
  (ii) scattering of photons into a given direction from other directions.

- The ratio of emission to absorption coefficients is called the Source function

$$S_\lambda = j_\lambda / \kappa_\lambda$$

Radiative transfer equation (1)

- The primary mode of energy transport through the surface layers of a star is by radiation.
- The radiative transfer equation describes how the physical properties of the material are coupled to the spectrum we ultimately measure.
- Recall, energy can be removed from (true absorption or scattered), or delivered to (true emission or scattered) a ray of radiation:



- The rate of change of (specific) intensity is:

$$dI_\lambda = -\kappa_\lambda \rho I_\lambda ds + j_\lambda \rho ds$$

Radiative transfer equation (2)

$$dI_\lambda = -\kappa_\lambda \rho I_\lambda ds + j_\lambda \rho ds$$

$$d\tau_\lambda = \kappa_\lambda \rho ds$$
$$S_\lambda = j_\lambda / \kappa_\lambda$$

- We can re-write this equation in terms of the optical depth τ_λ and the source function S_λ

$$dI_\lambda / d\tau_\lambda = -I_\lambda + S_\lambda$$

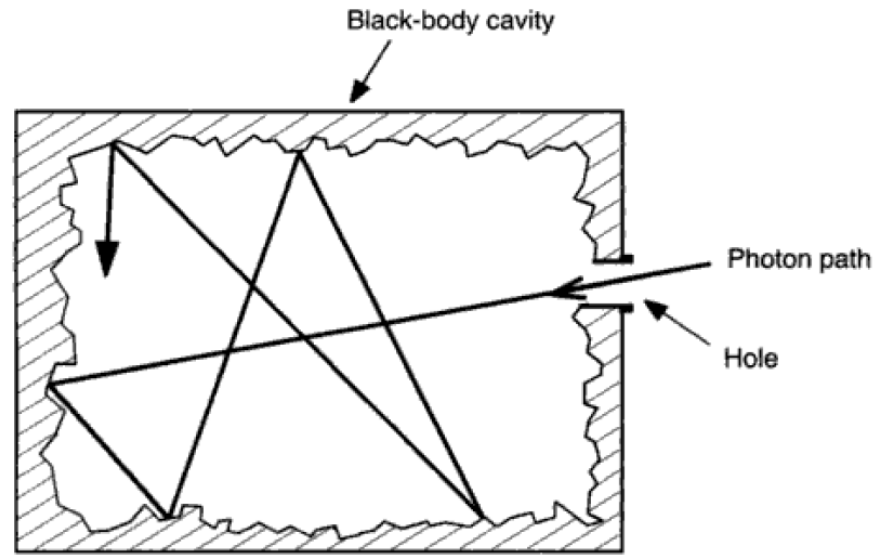
- **This is the (parallel-ray) equation of radiative transfer (RTE)**. It will need a small modification before it is applicable to stars, but we can already gain some insight from its solution.
- If $S_\lambda < I_\lambda$, the intensity will decrease with increasing τ_λ, it will stay constant if $S_\lambda = I_\lambda$ and increase if $S_\lambda > I_\lambda$. *When* $\tau_\lambda \rightarrow \infty$, $I_\lambda \rightarrow S_\lambda$

Thermodynamic Equilibrium (TE)

The Black Body

Imagine a box which is completely closed except for a small hole. Any light entering the box will have a very small likely hood of escaping & will eventually be absorbed by the gas or walls. For constant temperature walls, this is in **thermodynamic equilibrium**.

If this box is heated the walls will emit photons, filling the inside with radiation. A small fraction of the radiation will leak out of the hole, but so little that the gas within it remains in equilibrium. **The emitted radiation is that of a black-body.**



The black body intensity is defined (following discovery by Max Planck in 1900) as either

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \quad \text{or} \quad B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT} - 1}$$

where c=2.99x10^{10} cm, h=6.57x20^{-27} erg s, k=1.38x10^{-16} erg/s.

Physical interpretation of S_λ

$$dI_\lambda/d\tau_\lambda = -I_\lambda + S_\lambda$$

- If $S_\lambda<I_\lambda$, the intensity will decrease with increasing τ_λ
- The intensity will increase if $S_\lambda>I_\lambda$
- It will stay constant if $S_\lambda=I_\lambda$

- Thermodynamic Equilibrium (TE) → nothing changes with time
- A beam of light passing through such a gas volume will not change either

$$dI_\lambda/d\tau_\lambda = 0 \longrightarrow S_\lambda = I_\lambda = B_\lambda$$

in TE, the source function equals the Planck function

$$\boxed{\kappa_\lambda B_\lambda = j_\lambda \quad \text{or} \quad \alpha_\lambda B_\lambda = \varepsilon_\lambda}$$

The law of Kirchhoff

Radiative transfer equation (3)

$$dI_\lambda/d\tau_\lambda = -I_\lambda + S_\lambda$$

One can formally solve this form of the RTE, assuming that S_λ is constant along the path. **Class task: solve it** using an integrating factor e^{τ_λ}

i.e.

$$e^{\tau_\lambda}\frac{dI_\lambda}{d\tau_\lambda} = -e^{\tau_\lambda}I_\lambda + e^{\tau_\lambda}S \quad \text{so} \quad e^{\tau_\lambda}\frac{dI_\lambda}{d\tau_\lambda} + e^{\tau_\lambda}I_\lambda = e^{\tau_\lambda}\frac{dI_\lambda}{d\tau_\lambda} + I_\lambda\frac{de^{\tau_\lambda}}{d\tau_\lambda} = e^{\tau_\lambda}S_\lambda$$

$$e^{\tau_\lambda}\frac{dI_\lambda}{d\tau_\lambda} + I_\lambda\frac{de^{\tau_\lambda}}{d\tau_\lambda} = \frac{d}{d\tau_\lambda}(e^{\tau_\lambda}I_\lambda) \quad \text{so} \quad \frac{d}{d\tau_\lambda}(e^{\tau_\lambda}I_\lambda) = e^{\tau_\lambda}S_\lambda$$

Now integrate:

constant

$$\int_0^{\tau_\lambda}\frac{d}{d\tau_\lambda}(e^{\tau_\lambda}I_\lambda)\,d\tau_\lambda = [e^{\tau_\lambda}I_\lambda]_0^{\tau_\lambda} = \int_0^{\tau_\lambda}e^{\tau_\lambda}S_\lambda\,d\tau_\lambda = [e^{\tau_\lambda}S_\lambda]_0^{\tau_\lambda}$$

Radiative transfer equation (4)

Inserting boundary conditions:

$$I_\lambda e^{\tau_\lambda} - I_{\lambda 0} = S_\lambda (e^{\tau_\lambda} - 1)$$

$I_\lambda(\tau_\lambda = 0) = I_{\lambda 0}$

Rearrange:

$$I_\lambda = S_\lambda (1 - e^{-\tau_\lambda}) + I_{\lambda 0} e^{-\tau_\lambda}$$

The second term of the RHS describes the amount of radiation left over from the intensity entering the box, after it has passed through an optical depth τ, the first term gives the contribution of the intensity from the radiation emitted along the path.

Constant S_λ along the path is **a very rude assumption**!
See D. Gray (pp. 127-129) for more accurate integration.

$$I_\lambda(\tau_\lambda) = \int_0^{\tau_\lambda} S_\lambda(t_\lambda) e^{-(\tau_\lambda - t_\lambda)}\, dt_\lambda + I_{\lambda 0} e^{-\tau_\lambda}$$

This is the formal solution of the RTE which assumes that the source function is known (D. Gray)