Observational Astronomy

Problems Set 3: Solutions.

1. Which has a greater energy flux, 10 photons cm^{-2} s^{-1} at 10 Å or 10^5 photons cm^{-2} s^{-1} at 5000 Å?
   Answer:
   $F_\lambda = N \times h\nu = N \times hc / \lambda$: $F_{10Å} = 1.99\times10^{-8}$ erg cm^{-2} s^{-1} $F_{5000Å} = 3.97\times10^{-7}$ erg cm^{-2} s^{-1}
   Answer: 10^5 photons cm^{-2} s^{-1} at 5000 Å are larger ($3.97\times10^{-7} > 1.99\times10^{-8}$ erg cm^{-2} s^{-1})

   Or $F_{5000Å} / F_{10Å} = N_{5000Å} / N_{10Å} \times 10Å / 5000Å = 10^5 / 10 \times 10 / 5000 = 20$

2. It is often claimed that stellar magnitude errors can be taken as fractional errors of photometric accuracy. Although this is not quite correct but close to it. Prove it.

   Solution:

$$F \sim 2.512^{-m} = 10^{-0.4m}$$

   From Statistics: $\sigma_q = \left|\frac{dq}{dx}\right| \sigma_x$ →

$$\sigma_F = \left|\frac{dF}{dm}\right| \sigma_m = \left|\frac{2.512^{-m}}{dm}\right| \sigma_m = \ln 2.512 \cdot\ 2.512^{-m}\ \sigma_m = 0.921\ F\ \sigma_m$$

   or

$$\sigma_F = \left|\frac{10^{-0.4m}}{dm}\right| \sigma_m = 10^{-0.4m} \cdot \ln 10 \cdot 0.4 = 0.921 \cdot 10^{-0.4m}\ \sigma_m = 0.921\ F\ \sigma_m$$

$$\frac{\sigma_F}{F} = 0.921\sigma_m$$

3. A star has a measured I-band magnitude of 22.0. How many photons per second are detected from this star by the William Herschel Telescope on La Palma (4.2 m diameter), assuming that the telescope and imaging optics have a throughput of 60%, the detector has a quantum efficiency of 80%, the sky has a brightness of 20 magnitudes per square arcsec, and the seeing is 1 arcsec . Estimate the exposure time required to detect the star at a signal-to-noise ratio of 20.

   Solution:
   How many photons per second are detected from this star by the William Herschel Telescope:
   From Lecture 10, slide 411:
   $N_{star} = \eta\ \varepsilon_{atm}\ \varepsilon_{tel}\ \varepsilon_{filt}\ \varepsilon_{win}\ \varepsilon_{geom}\ \phi\ \Delta\lambda\ A\ t = \eta\ \varepsilon\ \phi_{star}\ \Delta\lambda\ A\ t$
   $\eta=0.8$
   $\varepsilon = \varepsilon_{atm}\ \varepsilon_{tel}\ \varepsilon_{filt}\ \varepsilon_{win} = 0.6$
   $\Delta\lambda = 1500$ Å
   $A = \pi D^2/4 = 138544$ cm^2
   For simplicity, we can assume that $\varepsilon_{geom} = 1.0$

However, the WHT telescope is of a Ritchey Chretien Cassegrain system, it has a secondary mirror with the diameter 1.0 m (e.g, https://www.ing.iac.es/PR/wht_info/whtoptics.html). Then from Lecture 10, slide 410, ε_{geom} =0.94

$\phi_{star}=F/h\nu=F\lambda/hc= F_0\lambda/hc\times2.512^{-m}=7.18\times10^{-7}$ photons s^{-1} cm^{-2} $Å^{-1}$

Thus, $N_{star} = 0.8\times0.6\times0.94\times 7.18\times10^{-7} \times 1500\times10^{-8} \times 138544$

Answer:

from the star N_{star}~67 phot/sec (~72 if ε_{geom} =1.0)

from the sky N_{sky}~425 phot/sec from square arcsec (~452 if ε_{geom} =1.0)

Estimate the exposure time required to detect the star at a signal-to-noise ratio of 20.

From Lecture 8, slide 360:

S/N = $\frac{N_* t}{\sqrt{N_* t + 2N_{sky} t}} \rightarrow \frac{N_*\sqrt{t}}{\sqrt{N_* + 2N_{sky}}}$ → t=81 sec (~76 sec if ε_{geom} =1.0)

4. Calculate the flux F_λ of a star (in erg s^{-1} cm^{-2} $Å^{-1}$) having Vega magnitude R=15 and AB magnitude r=15 (λ_c = 6156 Å).

Answer:

Vega: $F=F_0*2.512^{-15}=2.18\times10^{-15}$ erg $s^{-1}cm^{-2}$ $Å^{-1}$

F_0= 2.177×10^{-9} erg $s^{-1}cm^{-2}$ $Å^{-1}$ (from Table in slide 397, Lecture 09)

AB (method 1):

$m = -2.5\,log\,F_\nu - 48.6$; F_ν [ergs s^{-1} cm^{-2} Hz^{-1}]=$10^{-8}\frac{\lambda[Å]^2}{c[\text{cm s}^{-1}]}$ F_λ [ergs s^{-1} cm^{-2} $Å^{-1}$] (slide 386, Lec. 9)

$F_\nu = 3.63\times10^{-26}$ ergs s^{-1} cm^{-2} Hz^{-1}

$F_\lambda = 2.87\times10^{-15}$ erg $s^{-1}cm^{-2}$ $Å^{-1}$

AB (method 2):

In AB magnitudes, mag 0 has a flux of 3631 Jy (slide 386, Lecture 9)

Then F_ν [Jy] = $3631*2.512^{-15} = 3.63\times10^{-3}$ Jy

slide 399, Lecture 9: F_λ [erg s^{-1} cm^{-2} $Å^{-1}$]= 3.00×10^{-5} λ^{-2} F_ν [Jy] = 2.87×10^{-15} erg $s^{-1}cm^{-2}$ $Å^{-1}$

5. What fraction of the photons in the V band of a bright star would be absorbed by the atmosphere if one were to observe the star at an airmass of 2.5, and at the zenith (airmass = 1)? Assume that the atmospheric extinction k(λ) in the V band is 0.15 mag $airmass^{-1}$.

Answer: 13% absorbed at the zenith and 29% at the airmass of 2.5

(Lecture 11, Slide 443): $m_{obs} - m_{true} = k(\lambda)\,X$

$(1-2.512^{-0.15*1})*100\%$ and $(1-2.512^{-0.15*2.5})*100\%$

6. In making differential observations, explain why you should know the colours of the variable and comparison stars.

Short answer (but you had to elaborate it!): There is a colour term in the accurate formula, caused by the variation in spectral profile of the stars and the filter response over the passband (Slides 446-447, Lecture 11).