Observational Astronomy

Problem Set 2: Solutions.

1. In an X-ray CCD, the number of electrons released when a photon is incident on the detector is given by N = E / w, where E is the energy of the incident photon, and w is a constant which depends upon the material. The variance on the number of electrons released is given by $\sigma_N^2 = F\,N$, where F is the Fano factor. Calculate the energy resolution in electron Volts of an X-ray CCD for photons of energy 2.2 keV, if *w* for silicon is 3.65 eV, and the Fano factor for silicon is 0.12. Calculate the resolving power R.

Solution:

$$R = \frac{\lambda}{\Delta\lambda} = \frac{E}{\Delta E}$$

FWHM=2.35 σ

E = N w $\quad \sigma_E = \frac{dE}{dN}\sigma_N = w\sigma_N = w\sqrt{FN} = \sqrt{wFE} = 31\text{eV}$

$\Delta E = 2.35\,\sigma_E = 72.9$ eV

$$R = \frac{E}{\Delta E} = 30.2$$

2. An instrument measures N_*=1 photon/s from an astronomical source and N_{sky}=2 photons/s from the background. If the dark current and readout noise are negligible, how long an exposure is required to achieve a SNR of 50?

Solution:

From lecture 7 (slide 329):

$$S/N = \frac{n_*}{\sigma_*} = \frac{n_*}{\sqrt{n_* + 2n_{sky}}} = \frac{N_*\sqrt{t}}{\sqrt{N_* + 2N_{sky}}} = \frac{\sqrt{t}}{\sqrt{5}} = 50$$

t=12500 s

3. The first CCDs consisted of 100 x 100 pixels, with an average quantum efficiency (QE) of 60%, while modern detectors contain 4000 x 4000 pixels (QE = 80%). Suppose that you were given a full night of observing time (8 hours) on a telescope with a mirror 6 m in diameter, equipped with such a modern CCD. How much longer would you have needed to cover the same area to the same depth with one of the early detectors? Assume that the pixel size is the same in both cases and that you use the same telescope.

Solution:

Area of the old CCD: $A_{old}=10^4$

Area of the new CCD: $A_{new}=1.6\cdot10^7$

To cover the same sky area with the old CCD one need to take $^{A_{new}}/_{A_{old}}$ times more shots.

To reach the same depth with the old CCD one need to observe $^{\eta_{new}}/_{\eta_{old}}$ longer.
Thus, the observer with the old CCD will have to observe

$$\frac{\eta_{new}\, A_{new}}{\eta_{old}\, A_{old}} = \frac{0.8\times1.6\times10^{7}}{0.6\times10^{4}} = 2133 \text{ nights } (\times 8 \text{ hours}) = 5.8 \text{ years}$$

4. After measuring total counts from each star on a CCD image by removing background noise, one can convert *instrumental counts to instrumental magnitudes*. Like the CCD counts, instrumental magnitudes only give relative brightness among stars on the same CCD frame.

$$m_{inst} = -2.5 \log c + \text{constant}$$

Here c is the CCD instrumental count and m_{inst} is the corresponding instrumental magnitude. The constant in the instrumental system is totally arbitrary. It is fine to set it as zero, with the only consequence that all your instrumental magnitudes will be large negative numbers. Or you can set it to a large enough positive value so all the instrumental magnitudes are positive numbers. It doesn't matter which you do – the only thing that matters is that the *difference in magnitudes corresponds to a ratio of counts*, which is preserved no matter what the constant is, since it subtracts out in the full formulation.
Find the flux (counts) you should get from a star to reach magnitude uncertainty σ_{mag} better than 0.1 mag and 0.01 mag.

Solution:

Calculate the error propagation:

$$\sigma_{mag} = \left|\frac{dm}{dc}\right| \sigma_c = \left|\frac{-2.5}{\ln(10)\cdot c}\right| \sigma_c = 1.09\,\frac{\sigma_c}{c} = \frac{1.09}{\sqrt{c}}$$

$$\sigma_c = \sqrt{c}$$

$$c = \frac{1.09^2}{\sigma_{mag}^2} = \frac{1.18}{\sigma_{mag}^2}$$

Thus, for $\sigma_{mag} < 0.1$ mag → $c > 118$; for $\sigma_{mag} < 0.01$ mag → $c > 11800$

5. Suppose that you were awarded observing time with the Hubble Space Telescope to observe the individual stars in a very compact multiple star consisting of five components. You have been able to measure magnitudes m of components to be 21.8±0.03, 19.8±0.01, 20.1±0.02, 19.4±0.01, 22.8±0.04. From the ground-based telescope, because of seeing-limited observing conditions, you will see just one brighter star instead of five components of a multiple star. Calculate its magnitude m and σ_m.

Solution:

m=-2.5 log F   F=2.512^-m^ $F_{sum} = \sum_i 2.512^{-m_i}$

$$m_{sum} = -2.5 \log \sum_i 2.512^{-m_i} = 18.463$$

$$\sigma^2_{m,sum} = \sum_i \left[-2.5 \frac{d\left(\log \sum_i 2.512^{-m_i}\right)}{dm_i}\right]^2 \sigma^2_{m_i} =$$

$$= \sum_i \left[-2.5 \frac{\ln 2.512 \times 2.512^{-m_i}}{\ln 10 \ \times \sum_i 2.512^{-m_i}}\right]^2 \sigma^2_{m_i} = \sum_i \left[\frac{2.512^{-m_i}}{F_{sum}}\right]^2 \sigma^2_{m_i} = 4.84 \times 10^{-5}$$

$$\sigma_{m,sum} = 0.007$$

$$m_{sum} = 18.463 \pm 0.007$$