Observational Astronomy

Problems Set 3: Solutions.

1. What is the expected value of the Fried parameter at a wavelength of 5500 Å if the observed seeing is 0.6 arcsec? What is the corresponding value of r_0 at 1.6 microns in the infrared assuming Kolmogorov turbulence?
   Solution:
   From Lecture 4, slide 188: Full Width Half Maximum of the point spread function due to atmospheric turbulence (the seeing) is given by $\beta = 0.98\ \lambda / r_0$.
   Then r_0 = 18.5 cm for 5500 Å and 53.9 cm for 1.6 microns.

2. Which has a greater energy flux, 10 photons $cm^{-2}\ s^{-1}$ at 10 Å or 10^5 photons $cm^{-2}\ s^{-1}$ at 5000 Å?
   $F_\lambda = N \times h\nu = N \times hc / \lambda$: $F_{10Å} = 1.99\times10^{-8}$ erg $cm^{-2}\ s^{-1}$ $F_{5000Å} = 3.97\times10^{-7}$ erg $cm^{-2}\ s^{-1}$
   Answer: 10^5 photons $cm^{-2}\ s^{-1}$ at 5000 Å is larger ($3.97\times10^{-7} > 1.99\times10^{-8}$ erg $cm^{-2}\ s^{-1}$)

3. It is often claimed that stellar magnitude errors can be taken as fractional errors of photometric accuracy. Although this is not quite correct but close to it. Prove it.

   From Statistics: $\sigma_q = \left|\frac{dq}{dx}\right| \sigma_x$ $\quad F \sim 2.512^{-m}$ $\rightarrow$

$$\sigma_F = \left|\frac{2.512^{-m}}{dm}\right| \sigma_m = 2.512^{-m} \ln 2.512\, \sigma_m = 0.92 \times 2.512^{-m}\, \sigma_m$$

$$\sigma_F \,/\, 2.512^{-m} = \sigma_F \,/\, F = 0.92\, \sigma_m$$

   Thus, $\frac{\sigma_F}{F}\ \% = 92\, \sigma_m \approx 100\, \sigma_m$
   For example, if V=11.23±0.03 then $\sigma_m = 0.03$ and one can claim that the photometric accuracy is ~3%.

4. A star has a measured I-band magnitude of 22.0. How many photons per second are detected from this star by the William Herschel Telescope on La Palma (4.2 m diameter), assuming that the telescope and imaging optics have a throughput of 60%, the detector has a quantum efficiency of 80%, the sky has a brightness of 20 magnitudes per square arcsec, and the seeing is 1 arcsec . Estimate the exposure time required to detect the star at a signal-to-noise ratio of 20.

   Solution:
   How many photons per second are detected from this star by the William Herschel Telescope:
   From Lecture 11, slide 408:
   $N_{star} = \eta\ \varepsilon_{atm}\ \varepsilon_{tel}\ \varepsilon_{filt}\ \varepsilon_{win}\ \varepsilon_{geom}\ \phi\ \Delta\lambda\ A\ t = \eta\ \varepsilon\ \phi_{star}\ \Delta\lambda\ A\ t$
   η=0.8
   $\varepsilon = \varepsilon_{atm}\ \varepsilon_{tel}\ \varepsilon_{filt}\ \varepsilon_{win}$ =0.6
   $\Delta\lambda$=1500 Å
   $A=\pi D^2/4=138544\ cm^2$
   For simplicity, we can assume that ε_{geom} =1.0
   However, the WHT telescope is of a Ritchey Chretien Cassegrain system, it has a secondary mirror with the diameter 1.0 m (e.g, https://www.ing.iac.es/PR/wht_info/whtoptics.html). Then from Lecture 10, slide 407, ε_{geom} =0.94
   $\phi_{star}=F/h\nu=F\lambda/hc= F_0\lambda/hc\times2.512^{-m}=7.18\times10^{-7}$ photons $s^{-1}\ cm^{-2}\ Å^{-1}$

Thus, N_{star} = 0.8×0.6×0.94× 7.18×10^{-7} × 1500×10^{-8} × 138544

Answer:

from the star ~67 phot/sec (~72 if ε_{geom} =1.0)

from the sky ~425 phot/sec from square arcsec (~452 if ε_{geom} =1.0)

Estimate the exposure time required to detect the star at a signal-to-noise ratio of 20.

S/N = $\frac{N_* t}{\sqrt{N_* t + 2 n_{sky} t}} \rightarrow \frac{N_* \sqrt{t}}{\sqrt{N_* + 2 n_{sky}}}$ → t=81 sec (~76 sec if ε_{geom} =1.0)

5. Calculate the flux F_λ of a star (in erg s^{-1} cm^{-2} Å^{-1}) having Vega magnitude R=15 and AB magnitude r=15 (λ_c = 6156 Å).

   Answer:

   Vega: $F = F_0 * 2.512^{-15}$ =2.18×10^{-15} erg s^{-1}cm^{-2} Å^{-1}

   F_0= 2.177×10^{-9} erg s^{-1}cm^{-2} Å^{-1} (from Table in slide 394, Lecture 10)

   AB (method 1):

   $m = -2.5 \log F_\nu - 48.6$; F_ν [ergs s^{-1} cm^{-2} Hz^{-1}]=10^{-8} $\frac{\lambda[\text{Å}]^2}{c[\text{cm s}^{-1}]}$ F_λ [ergs s^{-1} cm^{-2} Å^{-1}] (slide 382, Lec. 9)

   F_ν = 3.63×10^{-26} ergs s^{-1} cm^{-2} Hz^{-1}

   F_λ =2.87×10^{-15} erg s^{-1}cm^{-2} Å^{-1}

   AB (method 2):

   In AB magnitudes, mag 0 has a flux of 3631 Jy (slide 382, Lecture 9)

   Then F_ν [Jy] = 3631*2.512^{-15} = 3.63×10^{-3} Jy

   slide 396, Lecture 9: F_λ [erg s^{-1} cm^{-2} Å^{-1}]= 3.00×10^{-5} λ^{-2} F_ν [Jy] = 2.87×10^{-15} erg s^{-1}cm^{-2} Å^{-1}

6. What fraction of the photons in the V band of a bright star would be absorbed by the atmosphere if one were to observe the star at an airmass of 2.5, and at the zenith (airmass = 1)? Assume that the atmospheric extinction k(λ) in the V band is 0.15 mag airmass^{-1}.

   Answer: 13% absorbed at the zenith and 29% at the airmass of 2.5 (Lecture 12, Slide 440).

   (1-2.512$^{-0.15*1}$)*100% and (1-2.512$^{-0.15*2.5}$)*100%

7. In making differential observations, explain why you should know the colours of the variable and comparison stars.

   Short answer (but you had to elaborate it!): There is a colour term in the accurate formula, caused by the variation in spectral profile of the stars and the filter response over the passband (Slides 443-443, Lecture 12).