Observational Astronomy

Problem Set 1: Solutions

1. A set of 13 measurements are made on a physical quantity. The following values are obtained: 0, 1, 2, 3, …, 11, 12. Estimate the mean value $\langle x \rangle$, the RMS spread σ_x and the accuracy of the mean $\sigma_{\langle x \rangle}$.

Answer: $\langle x \rangle=6$; $\sigma_x = 3.89$; $\sigma_{\langle x \rangle}=1.08$

2. A new set of 36 measurements are made with the result that the values

0, 1, 2, …, 5, 6, 7, …, 11, 12

occur 0, 1, 2, …, 5, 6, 5, …, 1, 0 times respectively.
Estimate $\langle x \rangle$, σ_x , $\sigma_{\langle x \rangle}$, *median* and *mode*.

Answer: $\langle x \rangle=6$; $\sigma_x = 2.45$; $\sigma_{\langle x \rangle}=0.41$; median=mode=6

3. Four separate groups of astronomers obtained the following estimates of the temperature of a white dwarf: 15000 ± 1000 K, 14000 ± 500 K, 14400 ± 800 K and 20000 ± 5000 K. What is the best estimate of the temperature? What would have been the best estimate if you had neglected the accuracies of the individual measurements?

Answer: weighted mean = 14282 ± 389 K; mean = 15850 K
Then the best measurement, closest to the weighted mean and with smallest error is 14000 ± 500 K (14400 ± 800 K is also ok). Neglecting the accuracies - 15000 ± 1000 K

4. If an object is placed at a distance ***p*** from a lens and an image is formed at a distance ***q*** from the lens, the lens's focal length ***f*** can be found as

$$f = \frac{pq}{p+q}$$

Suppose that p and q are measured as ***p*** = 1450 ± 0.5 and ***q*** = 652.5 ± 2, both in centimetres. Find ***f*** and the uncertainty $\boldsymbol{\sigma_f}$.

Answer: f = 450 ± 0.95

5. After measuring total counts from each star on a CCD image by removing background noise, one can convert *instrumental counts to instrumental magnitudes*. Like the CCD counts, instrumental magnitudes only give relative brightness among stars on the same CCD frame.

$$m_{inst} = -2.5 \log c + \text{constant}$$

Here c is the CCD instrumental count and m_{inst} is the corresponding instrumental magnitude. The constant in the instrumental system is totally arbitrary. It is fine to set it as zero, with the only consequence that all your instrumental magnitudes will be large negative numbers. Or you can set it to a large enough positive value so all the instrumental magnitudes are positive numbers. It doesn't matter which you do – the only thing that matters is that the *difference in magnitudes corresponds to a ratio of counts*, which is preserved no matter what the constant is, since it subtracts out in the full formulation.

Find the flux (counts) you should get from a star to reach magnitude uncertainty σ_{mag} better than 0.1 mag and 0.01 mag.

Answer:

$$\sigma_m = \left|\frac{dm}{dc}\right| \sigma_c = \left|\frac{-2.5}{\ln(10) \cdot c}\right| \sigma_c = 1.09 \frac{\sigma_c}{c} = \frac{1.09}{\sqrt{c}}$$

$$\sigma_c = \sqrt{c}$$

$$c = \frac{1.09^2}{\sigma_m^2} = \frac{1.18}{\sigma_m^2}$$

Thus, for $\sigma_m < 0.1$ mag → c > 118. for $\sigma_m < 0.01$ mag → c > 11800