

# Astrophysics

## Compulsory Home Exercises. Problem Set 4.

Return by Wednesday, March 30, 2022 (before the lecture).

Please, write down every step in your line of thinking and state assumptions etc.  
A sole answer is not enough.

1. Let's approximate the directional ( $\mu$ ) dependence of the specific intensity using the first-order Taylor expansion,

$$I(\tau, \mu) = I_0(\tau) + I_1(\tau) \mu$$

where  $I_0$  and  $I_1$  depend on the vertical optical depth  $\tau$  but not on  $\mu$ .  
Assuming a gray atmosphere,

- a. Derive expressions for the flux  $F$ , mean intensity  $J$ , and radiation pressure  $P_{\text{rad}}$ , in terms of  $I_0$  and  $I_1$ .
- b. Show that the radiation field obeys the Eddington approximation and

$$P_{\text{rad}} = \frac{4\pi}{3c} J$$

- c. Within the Eddington approximation, the solution of the radiative transfer equation is

$$S = \frac{3}{4\pi} \left( \tau + \frac{2}{3} \right) F$$

Use this solution to find an expression for  $I_0$  as a function of  $I_1$  and  $\tau$ .

- d. Show that if  $\tau \gg 1$  then  $I_0 \gg I_1$  (this result justifies the use of the first-order Taylor expansion at large optical depths, showing that the radiation field becomes isotropic).
2. Assuming a grey atmosphere and using the Eddington approximation and an expression for the source function  $S$  as a function of vertical optical depth  $\tau$  and flux  $F$  (Lecture 18),

- a. Calculate the upward specific intensity  $I(\tau, \mu)$  as a function of  $F$ ,  $\tau_v$ , and direction  $\mu > 0$ , using the formal solution to RTE (Lecture 18)

$$I_\lambda(\tau_{\lambda,v}, \mu > 0) = \int_{\tau_{\lambda,v}}^{\infty} S_\lambda e^{(\tau_{\lambda,v}-t)/\mu} \frac{dt}{\mu},$$

- b. Using your expression for the upward specific intensity  $I(\tau, \mu)$ , evaluate the upward component of the radiative flux,

$$F^+ = 2\pi \int_0^1 I(\mu) \mu d\mu$$

- c. Find the corresponding downward component of the flux,  $F^-$ .