Astrophysics

Compulsory Home Exercises. Problem Set 2.

Return by Tuesday, March 1, 2022.

Please, write down **every step in your line of thinking** and state assumptions etc.
A sole answer is not enough.

Problem 2.1

What is the energy released by the nuclear reactions of carbon burning (fusion of 2 carbon nuclei)? Give the answer in MeV and ergs per gram.

Problem 2.2

Calculate the mean molecular weight μ for

1) the completely ionized stellar interior, where we have 45% hydrogen, 52% helium, and 3% heavy elements by mass,
2) completely ionized hydrogen,
3) completely ionized helium,
4) neutral gas at the solar interior abundance, 73% hydrogen, 25% helium, and 2% heavy elements by mass.

Problem 2.3

Prove that for the case when Z is negligible, the mean molecular weight per electron, $\mu_e = \frac{\rho}{n_e m_H}$, can be approximately expressed as

$$\mu_e \approx \frac{2}{1+X}$$

Problem 2.4

Does a lower Gamow energy E_G increase or decrease the probability of penetration?

Problem 2.5

a) Calculate the Gamow energy E_G (in electronvolts) for the collision of two α-particles (helium-4 nuclei, $^2_2\mathrm{He}$) and find the penetration probability P_{pen} for the typical kinetic energy of particles in the Sun's core, E ~ 1 keV. Compare the results with the case of two protons. Explain the result.
b) What temperature is required to have the probability of penetration of two α-particles similar to that of two protons in the Sun's core?

Problem 2.6

We have seen that a polytropic model of the Sun shows quite good agreement with the results of a detailed solution of the equations of stellar structure. Using the n=3 polytrope and the solar mass and radius, find the central pressure P_c, central density ρ_c, and temperature T_c at the centre of the Sun.

Problem 2.7

Prove that according to the virial theorem, the mean temperature of a star can be expressed as

$$\bar{T} \propto M^{2/3}\rho^{1/3}$$