Astrophysics

Compulsory Home Exercises. Problem Set 1.

Return by Wednesday, February 9, 2022 (before the lecture).

Please, write down every step in your line of thinking and state assumptions etc.
A sole answer is not enough.

Problem 1.1

In a star of mass M, the density decreases from the centre to the surface as a function of radial distance r, according to

$$\rho = \rho_0 \left[1 - \left(\frac{r}{R}\right)^2\right]$$

where ρ_0 is a given constant and R is the star's radius,

(a) Find $m(r)$.
(b) Derive the relation between M and R.
(c) Show that the average density of the star (total mass divided by total volume) is $0.4\rho_0$.

Problem 1.2

For a star of mass M and radius R, find the central pressure and check the validity of inequality $P_c \geq \frac{3}{8\pi}\frac{GM^2}{R^4}$ for the following cases:

(a) a uniform density
(b) a density profile as in Exercise 1.1.

Problem 1.3

In Lecture 4 we obtained the mean temperature of the star to be

$$\bar{T} = \frac{e_G}{3}\frac{\mu m_p}{k}\frac{GM}{R}$$

For a star of mass M and radius R, find the value of e_G for two cases:

(a) a uniform density
(b) a density profile as in Exercise 1.1.

Problem 1.4 (from lecture 5):

Assume that the whole mass of a star is concentrated in the centre. Please show that the time for collapse from radius *R* to 0 is

$$t_{dyn} = \frac{\pi}{2\sqrt{2}}\left(\frac{R^3}{GM}\right)^{1/2}$$

Problem 1.5 (from lecture 5):

How much mass per time must the Sun accrete in order for its luminosity to equal that of observed? Show the answer in Solar masses. Discuss how this process of accretion will affect Earth?

Problem 1.6 (Expanded problem from lecture 7):

A B5V star in the LMC – distance 50 kpc – has V=13.5 mag, B-V=-0.07 mag.

(a) What is its bolometric luminosity, relative to the Sun?
(b) What is its stellar radius?
(c) At what wavelength does its radiation peak (assuming Wien's law)?