

Observational Astronomy

Problem Set 2: Solutions.

1. In an X-ray CCD, the number of electrons released when a photon is incident on the detector is given by $N = E / w$, where E is the energy of the incident photon, and w is a constant which depends upon the material. The variance on the number of electrons released is given by $\sigma_N^2 = F N$, where F is the Fano factor. Calculate the energy resolution in electron Volts of an X-ray CCD for photons of energy 2.2 keV, if w for silicon is 3.65 eV, and the Fano factor for silicon is 0.12. Calculate the resolving power R .

Solution:

$$R = \frac{\lambda}{\Delta\lambda} = \frac{E}{\Delta E} \quad \text{FWHM} = 2.35 \sigma$$

$$E = N w \quad \sigma_E = \frac{dE}{dN} \sigma_N = w \sigma_N = w \sqrt{F N} = \sqrt{w F E} = 31 \text{ eV}$$

$$\Delta E = 2.35 \sigma_E = 72.9 \text{ eV}$$

$$R = \frac{E}{\Delta E} = 30.2$$

2. An instrument measures $N_* = 1$ photon/s from an astronomical source and $N_{sky} = 2$ photons/s from the background. If the dark current and readout noise are negligible, how long an exposure is required to achieve a SNR of 50?

Solution:

From lecture 7 (slide 335):

$$S/N = \frac{n_*}{\sigma_*} = \frac{n_*}{\sqrt{n_* + 2n_{sky}}} = \frac{N_* \sqrt{t}}{\sqrt{N_* + 2N_{sky}}} = \frac{\sqrt{t}}{\sqrt{5}} = 50 \quad t = 12500 \text{ s}$$

3. If the seeing at a good observatory site is 0.5 arc sec, at which wavelength do observations on a 20 cm (diameter) telescope become diffraction-limited instead of seeing limited?

Solution:

From lecture 2 (slide 72):

According to Rayleigh criterion, the angular resolution is given by $\vartheta \cong 1.22 \lambda / D$ (radians), where λ is wavelength and D is the diameter of the aperture.

We need to find at which wavelength ϑ reaches $0.5'' = 2.424 \cdot 10^{-6}$ rad: $\lambda \cong \vartheta \cdot D / 1.22 = 3974 \text{ \AA}$

Thus, at $\lambda > 4000 \text{ \AA}$ the angular resolution becomes larger than the seeing (so diffraction-limited rather than seeing limited).

4. The first CCDs consisted of 100 x 100 pixels, with an average quantum efficiency (QE) of 60%, while modern detectors contain 4000 x 4000 pixels (QE = 80%). Suppose that you were given a full night of observing time (8 hours) on a telescope with a mirror 6 m in diameter, equipped with such a modern CCD. How much longer would you have needed to cover the same area to the same depth with one of the early detectors? Assume that the pixel size is the same in both cases and that you use the same telescope.

Solution:

Area of the old CCD: $A_{old}=10^4$

Area of the new CCD: $A_{new}=1.6 \cdot 10^7$

To cover the same sky area with the old CCD one need to take A_{new}/A_{old} times more shots.

To reach the same depth with the old CCD one need to observe η_{new}/η_{old} longer.

Thus, the observer with the old CCD will have to observe

$$\frac{\eta_{new} A_{new}}{\eta_{old} A_{old}} = \frac{0.8 \times 1.6 \times 10^7}{0.6 \times 10^4} = \mathbf{2133 \text{ nights } (\times 8 \text{ hours})} = \mathbf{5.8 \text{ years}}$$

5. Suppose that you were awarded observing time with the Hubble Space Telescope to observe the individual stars in a very compact multiple star consisting of five components. You have been able to measure magnitudes m of components to be 21.8 ± 0.03 , 19.8 ± 0.01 , 20.1 ± 0.02 , 19.4 ± 0.01 , 22.8 ± 0.04 . From the ground-based telescope, because of seeing-limited observing conditions, you will see just one brighter star instead of five components of a multiple star. Calculate its magnitude m and σ_m .

Solution:

$$m = -2.5 \log F \quad F = 2.512^{-m} \quad F_{sum} = \sum_i 2.512^{-m_i}$$

$$m_{sum} = -2.5 \log \sum_i 2.512^{-m_i} = \mathbf{18.463}$$

$$\begin{aligned} \sigma_{m,sum}^2 &= \sum_i \left[-2.5 \frac{d(\log \sum_i 2.512^{-m_i})}{dm_i} \right]^2 \sigma_{m_i}^2 = \\ &= \sum_i \left[-2.5 \frac{\ln 2.512 \times 2.512^{-m_i}}{\ln 10 \times \sum_i 2.512^{-m_i}} \right]^2 \sigma_{m_i}^2 = \sum_i \left[\frac{2.512^{-m_i}}{F_{sum}} \right]^2 \sigma_{m_i}^2 = 4.84 \times 10^{-5} \end{aligned}$$

$$\sigma_{m,sum} = \mathbf{0.007}$$

$$m_{sum} = \mathbf{18.463 \pm 0.007}$$