ASTROPHYSICS OF INTERACTING BINARY STARS

Lecture 4

Vitaly Neustroev

More about Accretion Disks

The angular momentum problem

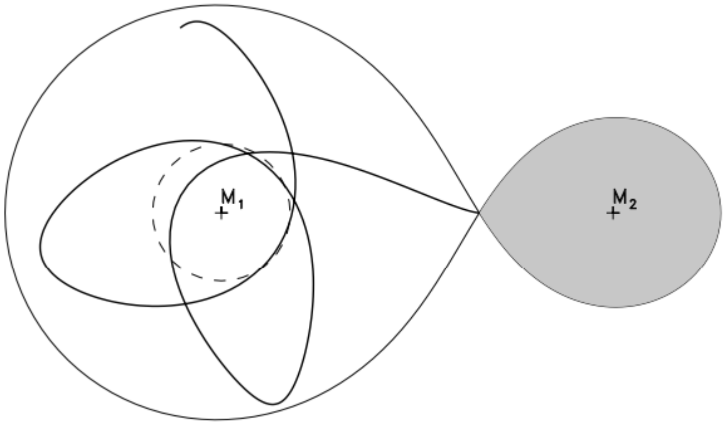
Basic theory of geometrically thin disks

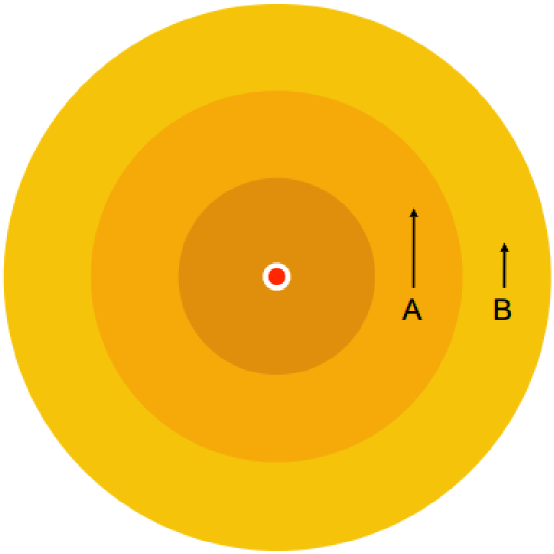
Formation of a Ring. What next?

- Angular momentum is conserved. A circular orbit has the least energy for a given angular momentum → the dissipation will tend to produce a ring of gas.
- The ring that is formed has a finite radial extent and rotates differentially.
- Differential rotation (Shear):

$$\Delta V = \frac{d}{dR}\left(\sqrt{\frac{GM}{R}}\right)\Delta R = \frac{V}{2}\frac{\Delta R}{R}$$

- Friction causes ring to spread inward & outward.
- Disk is formed!



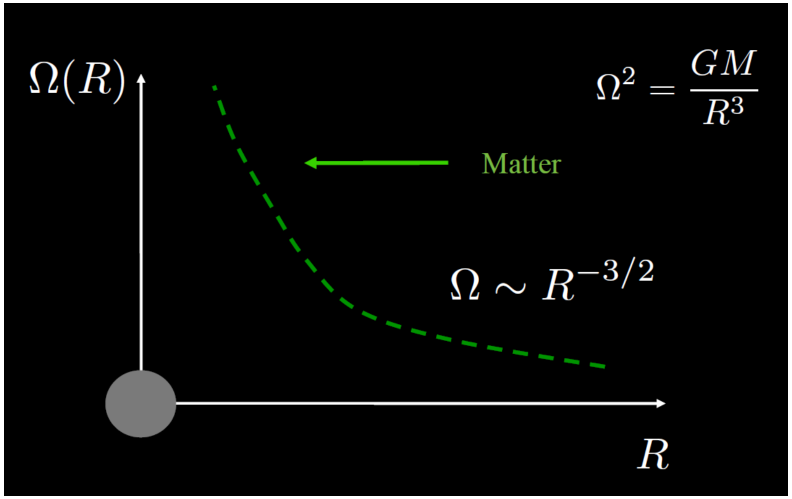


Formation of an Accretion Disk

Angular velocity in Keplerian disk
("differential rotation"):

$$\Omega_K = V_K/R = \sqrt{GM/R^3}$$

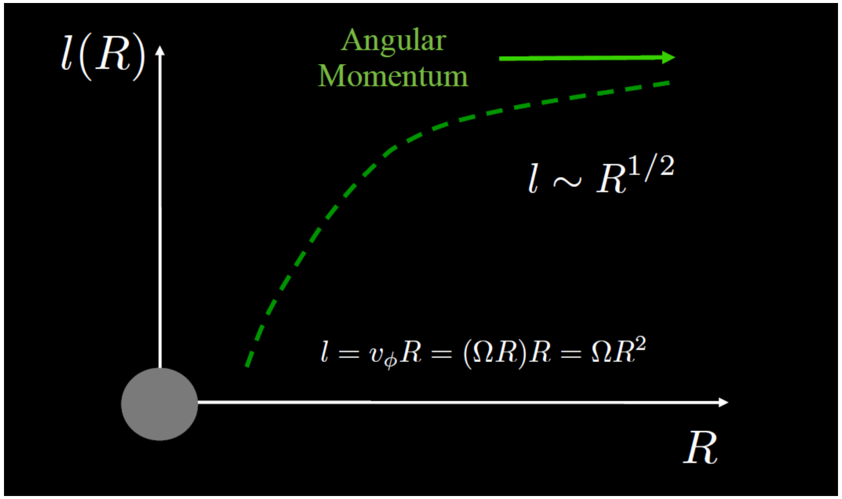
Increases with decreasing R!



Angular momentum per unit mass
("specific angular momentum"):

$$l(R) = RV_K = R^2\Omega_K = \sqrt{GMR}$$

Decreases with decreasing R!



If the disk were a collection of non-interacting particles there would be no accretion.

Gas in the disk must lose angular momentum!

The angular momentum problem

How does the accreting matter lose its angular momentum?

- Angular momentum is strictly conserved!
- Gas must shed its angular momentum for it to be actually accreted. Total angular momentum lost when mass moves in unit time from R + dR to R:

$$\frac{dl}{dR} = \dot{M} \cdot \frac{d(R^2\Omega_K)}{dR}$$

- Suppose that there is some kind of "viscosity" in the disk
  - Different annuli of the disk rub against each other and exchange angular momentum
  - Results in most of the matter moving inwards and eventually accreting
  - Angular momentum carried outwards by a small amount of material

Thin Accretion Disks (1)

- To start our study of the dynamics of disks, let's first take symmetry considerations and vertical integration of variables, to reduce the dimension of the problem and the number of independent variables:
  - We will assume that the disk is physically ***thin***: the height h of the disk in the z direction is much smaller than the extent of the disk in the R direction. It requires that radiation pressure is negligable.
  - We will assume an axisymmetric disk, which means that all quantities are independent of the φ coordinate.
- The idea is to get rid of all z dependencies by integrating the equations through the depth of the disk, so we assumed that the flow is symmetric with respect to the equatorial plane (mirror symmetry about this plane).
  This procedure allows us to ***decouple*** the vertical and radial directions.

Thin Accretion Disks (2)

- This implies that rather than dealing with quantities per unit volume, we will deal instead with quantities per unit surface.
- Integrating the density ρ along the z-axis we obtain the surface density, defined as

$$\Sigma(R) \equiv \int_{-\infty}^{+\infty} \rho(R)\, dz = \int_{-H(R)}^{+H(R)} \rho(R)\, dz = h\rho$$

- For example, we can now calculate the amount of mass crossing radius R:

$$\dot{M} = 2\pi R \cdot \Sigma \cdot V_R$$

Steady Accretion Disks

- We also assume a stationary (**steady**) disk - the physical quantities in a disk do not change with time.
- **Important**, stationarity does not mean that there is no flow, for instance in radial direction in a disk. The only requirement is that this flow proceeds in such a manner that the physical quantities, like the surface density Σ and the radial velocity V_R remain unchanged. In other words: stationarity means that the time derivatives in the equations vanish.

- The disc is assumed to be optically ***thick***. This allows the maximum amount of heat to radiate away from the surface of the disk before matter falls into the accreting star.
- Disk self-gravitation is **negligible** so material in differential or Keplerian rotation with angular velocity $\Omega_K(R)$.

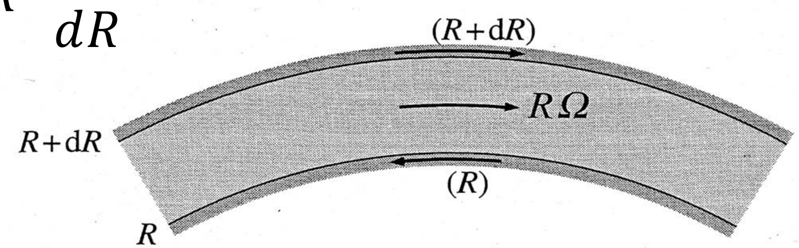
Viscous accretion disks (1)

- Consider two consecutive rings on either side of some surface of constant R in the accretion disk and with vertical thickness h, then the outer annulus exerts a viscous force (μ – coefficient of dynamic viscosity):

$$F_{visc} = 2\pi Rh\,\mu\,R\frac{d\Omega}{dR}$$

- In terms of the kinematic viscosity $\nu = \mu/\rho$

$$F_{visc} = 2\pi\nu\Sigma R^2\frac{d\Omega}{dR}$$

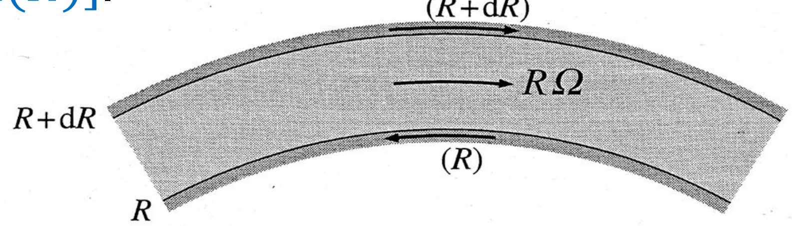


- This force is acting at a distance R from the centre of rotation, so it will exert a torque by the outer ring on the inner ring

$$Q(R) = F_{visc}R = 2\pi\nu\Sigma R^3\,\frac{d\Omega}{dR}$$

Viscous accretion disks (2)

- The torque by the outer ring on the inner ring: $Q(R) = 2\pi\nu\Sigma R^3 \frac{d\Omega}{dR}$
- The direction of the torque is such that the fluid at a radius less than *R* (which is rotating more rapidly) feels a backward torque and looses the angular momentum whereas the fluid at a radius larger than *R* gains the angular momentum.
- To determine the radial structure of the disk we have to equate this torque to the rate of loss of specific angular momentum.
- Consider a ring located between R and $(R + dR)$.
  In unit time, a mass $\dot{M}$ enters the ring at $(R + dR)$ with specific angular momentum $(R + dR)^2[\Omega(R + dR)]$ and leaves at R with specific angular momentum $R^2[\Omega(R)]$.



Viscous accretion disks (3)

$$Q(R) = 2\pi\nu\Sigma R^3 \frac{d\Omega}{dR}$$

- Thus, the net angular momentum lost by the fluid per unit time in this ring is $$\dot{M}[d(R^2\Omega)/\,dR]\,dR$$
- This angular momentum is lost because the torque is acting at both R and $(R + dR)$ whose net effect is $(dQ/\,dR)\,dR$. This lead to

$$\dot{M}\left[\frac{d(R^2\Omega)}{dR}\right]dR = -\frac{d}{dR}\left[2\pi\nu\Sigma R^3 \frac{d\Omega}{dR}\right]dR$$

- Using $\Omega_K = \sqrt{GM/R^3}$ and integrating, we get

$$\nu\Sigma\sqrt{R} = \frac{\dot{M}}{3\pi}\sqrt{R} + constant$$

Viscous accretion disks (4)

$$\nu\Sigma\sqrt{R} = \frac{\dot{M}}{3\pi}\sqrt{R} + constant$$

Constant can be obtained from no torque boundary condition at inner edge of disk at $R = R_*$ at which $\frac{dQ}{dR(R_*)} = 0$

Thus, we get

$$\nu\Sigma = \frac{\dot{M}}{3\pi}\left[1 - \left(\frac{R_*}{R}\right)^{1/2}\right]$$

Viscous accretion disks (6)

- All these rings rubbing against one another not only transfer angular momentum, they also make the disk hot. The power that is generated on an annulus takes the schematic form

  Power = (torque) ×(relative angular velocity of neighboring annuli)

- Each ring has two plane faces of area $4\pi R dR$, so the radiative dissipation from the disk per unit area is

$$D(R) = \frac{Q}{4\pi R}\,\frac{d\Omega}{dR} = \frac{1}{2}\nu\Sigma\left(R\,\frac{d\Omega}{dR}\right)^2$$

- Evaluating for circular Keplerian orbits ($\Omega_K = \sqrt{GM/R^3}$):

$$D(R) = \frac{9}{8}\nu\Sigma\frac{QM}{R^3}$$

Viscous accretion disks (7)

- We then have:

$$\nu\Sigma = \frac{\dot{M}}{3\pi}\left[1 - \left(\frac{R_*}{R}\right)^{1/2}\right]$$

$$D(R) = \frac{3GM\dot{M}}{8\pi R^3}\left[1 - \left(\frac{R_*}{R}\right)^{1/2}\right]$$

and hence the radiation energy flux through the disk faces is **independent of viscosity**

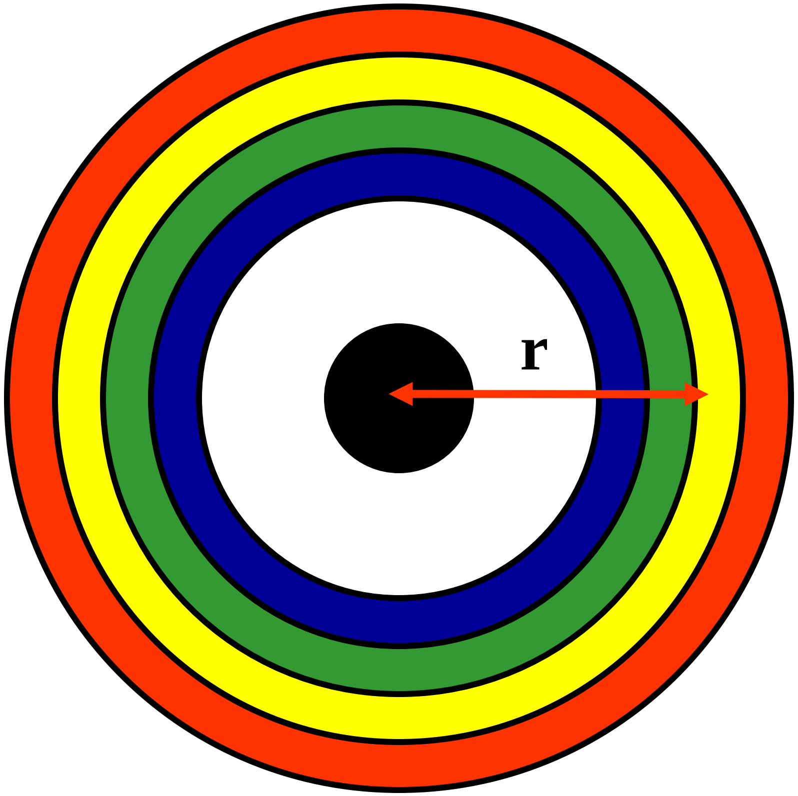
- The total disk luminosity is

$$L_{disk} = 2 \times \int_{R_*}^{\infty} D(R) \times 2\pi R dR = \frac{GM\dot{M}}{2R_*}$$

2 sides of a disk

i.e., half the gravitational energy released in accreting the gas to radius R_*. The remaining gravitational energy goes into rotational energy, which may be either dissipated in a boundary layer or sucked into a black hole.

Accretion Disk Temperature Structure



- If the accretion disk is optically thick, it can be considered as rings or annuli of blackbody emission.
- Dissipation rate, *D*(*R*) is

$$D(R) = \frac{3GM\dot{M}}{8\pi R^3}\left[1 - \left(\frac{R_*}{R}\right)^{1/2}\right]$$

$=$ blackbody flux

$= \sigma T(R)^4$

Accretion Disk Temperature Structure

- Thus temperature as a function of radius *T(R)*:

$$T(R) = \left\{ \frac{3GM\dot{M}}{8\pi R^3 \sigma} \left[1 - \left(\frac{R_*}{R} \right)^{1/2} \right] \right\}^{1/4}$$

- and if $$T_* = \left(\frac{3GM\dot{M}}{8\pi R_*^3 \sigma} \right)^{1/4}$$

- Then for $R \gg R_*$

$$T(R) = T_* \left(R / R_* \right)^{-3/4}$$

Accretion Disk Temperature Structure

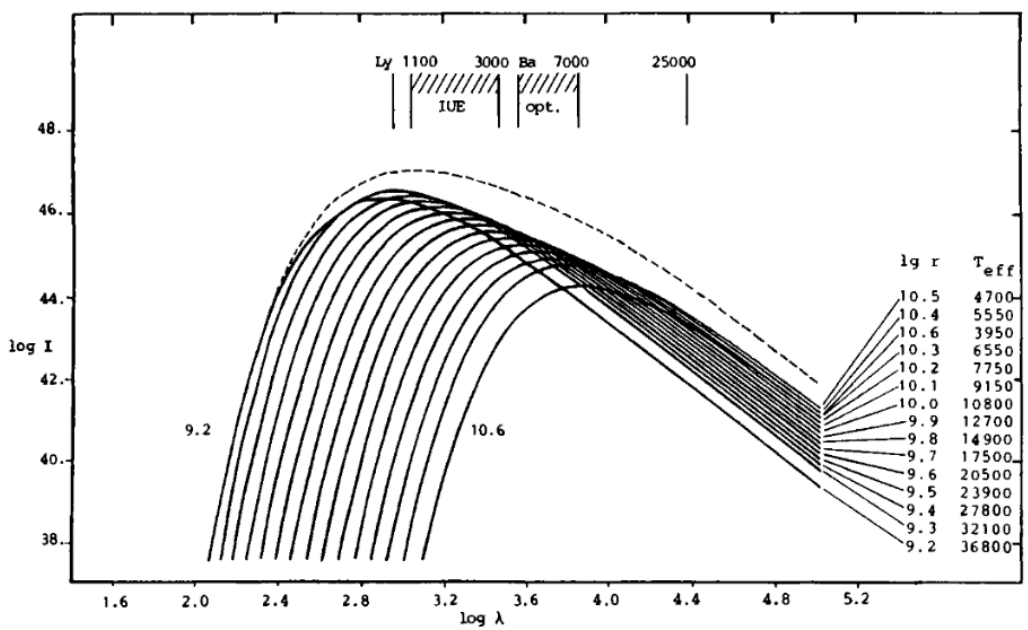
- In dwarf novae in outburst and long-period novalikes, this simple $R^{-3/4}$ radial temperature profile is indeed observed.

- In quiescent dwarf novae a much flatter profile is observed. This is thought to be because the disk does not achieve a steady state in quiescence.

Accretion Disk Spectrum

- Integrating the blackbody spectrum over radius gives the predicted spectrum of an optically thick, geometrically thin, steady-state accretion disk

$$S_\lambda \propto \int_{R_{in}}^{R_{out}} B_\lambda[T(R)] 2\pi R dR$$

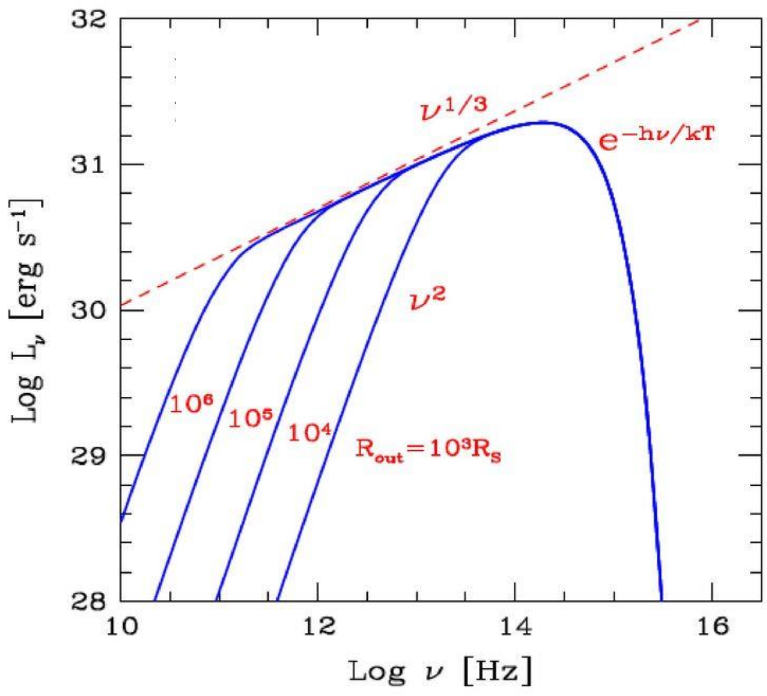


Contributions of BB annuli to the total intensity distribution of an accretion disk.

Accretion Disk Spectrum

- The continuum spectrum S_ν (in frequency units) of a disk with different ratios R_{out}/R_{in}: $$S_\nu \propto \int_{R_{in}}^{R_{out}} B_\nu[T(R)]2\pi R dR,$$

Spectrum produced by standard geometrically thin, optically thick accretion disk



The spectrum can be divided into three regions:

From the outer edge of the disc we will see the Rayleigh-Jeans tail of T_{outer}

$$S_\nu \propto \nu^2$$

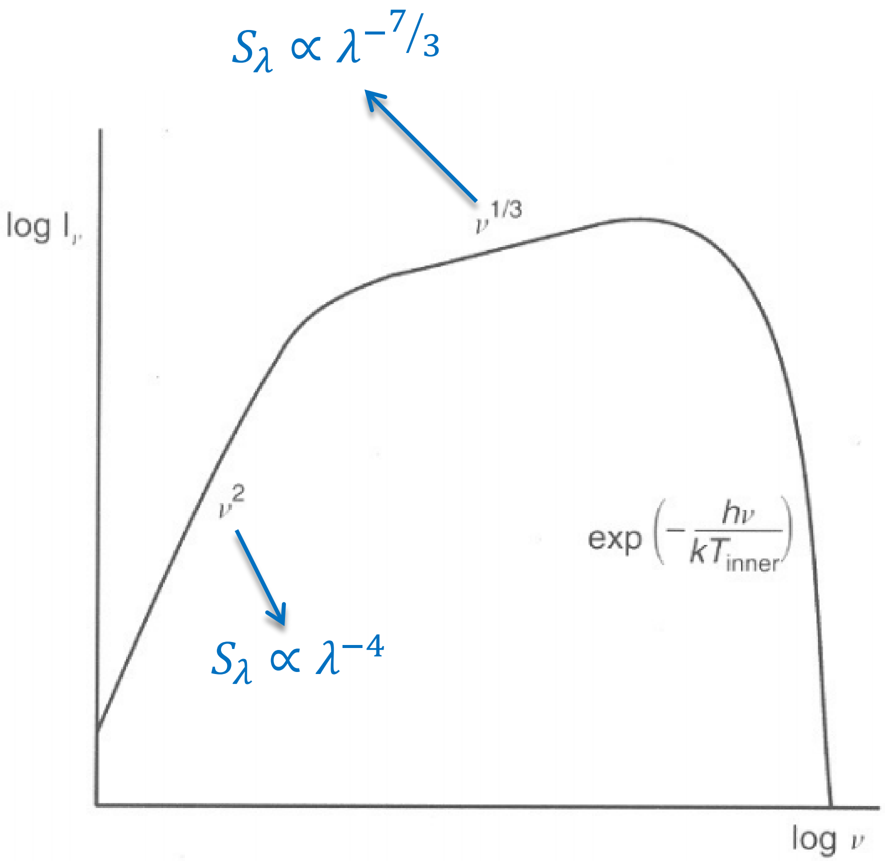
From the inner edge, an exponential cut-off

$$S_\nu \propto e^{-h\nu/kT_{inner}}$$

And the flat part which is sometimes considered a characteristic disc spectrum:

$$S_\nu \propto \nu^{1/3}$$

Accretion Disk Spectrum



The flat part is considered a characteristic disc spectrum:

$$S_\nu \propto \nu^{1/3}$$

However! This part of the curve may be **quite short** and the spectrum is not very different from a blackbody, unless T_{out} is appreciably smaller than T_{in}.