ASTROPHYSICS OF INTERACTING BINARY STARS

Lecture 2

Vitaly Neustroev

From Lecture 1

- If only one radial velocity is known (SB1), a useful quantity is the mass function:

$$f(m_1, m_2) = \frac{{v_1}^3 P}{2\pi G} = \frac{(m_2 \sin i)^3}{(m_1 + m_2)^2}$$

Why ?

The mass function

- Why the mass function is useful?

$$f(m_1, m_2) = \frac{{v_1}^3 P}{2\pi G} = \frac{(m_2 \sin i)^3}{(m_1 + m_2)^2}$$

- If Star 1 is the visible, low-mass companion of the unseen high-mass Object 2, so $m_1 \ll m_2$, then

$$f \approx m_2 \sin^3 i$$

- Thus, the mass function f determined from the orbital perod P and the radial velocity semi-amplitude v of star 1gives a lower limit on the mass of the object 2.
- If f appears to be (much) larger than the maximum possible mass for a neutron star ($\sim$2.5 $M_\odot$), then star 2 is possibly a black hole.

How does mass transfer occur?

Roche lobe and Roche-Lobe overflow

How does mass transfer occur?

- Two possible mechanisms for mass transfer between stars in a binary system:
  - Stellar wind accretion:
    - If one component ejects mass in a stellar wind and a part of that material is gravitationally captured by the nearby companion.
  - Roche-lobe overflow:
    - If the binary orbit is sufficiently close, matter from the outer layers of one star can flow directly to the companion.
- Stellar winds from low mass and/or late type stars are not usually strong and mass transfer occurs mainly through Roche-lobe overflow.

Roche Model

- How large one of the stars can become before it starts to tranfer matter to its companion?
- For stars in close binary systems we need to consider the effects of rotational and tidal distortion.
- The stellar surface is determined by the shape of the potential surface Φ=constant. Thus, determining the shape of a star in a binary is equivalent to determining the shapes of the potential surfaces Φ=constant.
- A basic tool is the ”Roche model”: the total potential (gravitational and centrifugal forces) is approximated by the Roche potential Φ_R

Roche Model

- Assumptions:
  - The orbit is **circular**.
  - Stars **corotate** with the binary system.
  - The gravitational field generated by the two stars is approximated by that of two **point** masses.

Roche Model

- Cartesian coordinates (x,y,z) rotate with the binary, with origin at the primary;
- The x-axis lies along the line of centres;
- The y-axis is in the direction of orbital motion of the primary;
- The z-axis is perpendicular to the orbital plane;
- The total potential:

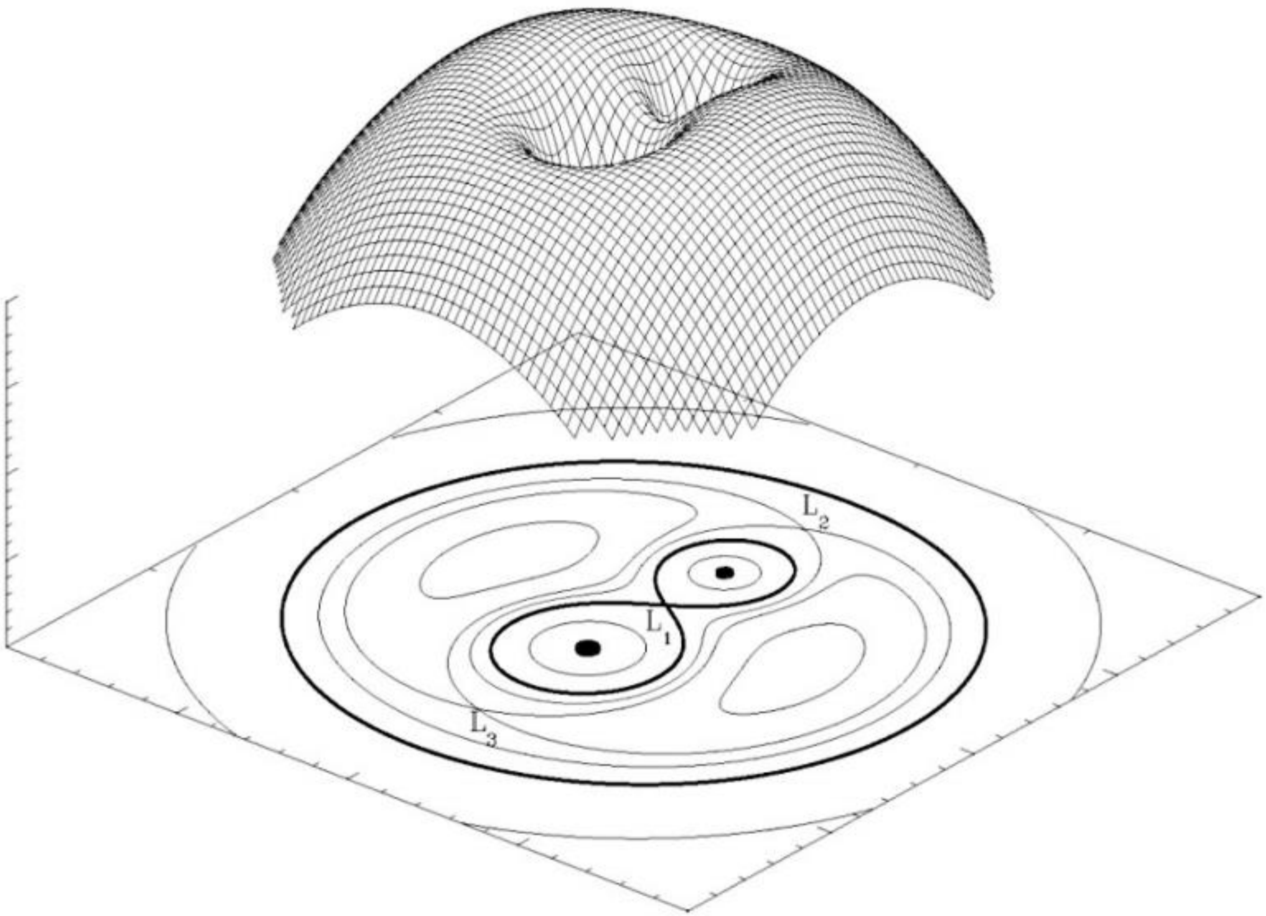
$$\Phi_R = \boxed{-\frac{GM(1)}{(x^2+y^2+z^2)^{1/2}} - \frac{GM(2)}{[(x-a)^2+y^2+z^2]^{1/2}}} \boxed{-\frac{1}{2}\Omega_{orb}^2[(x-\mu a)^2+y^2]}$$

The gravitational potential due to the two stars

The centrifugal potential

where $\mu = M(2)/[M(1)+M(2)]$ and $\Omega_{orb} = 2\pi/P_{orb}$.

Roche Model



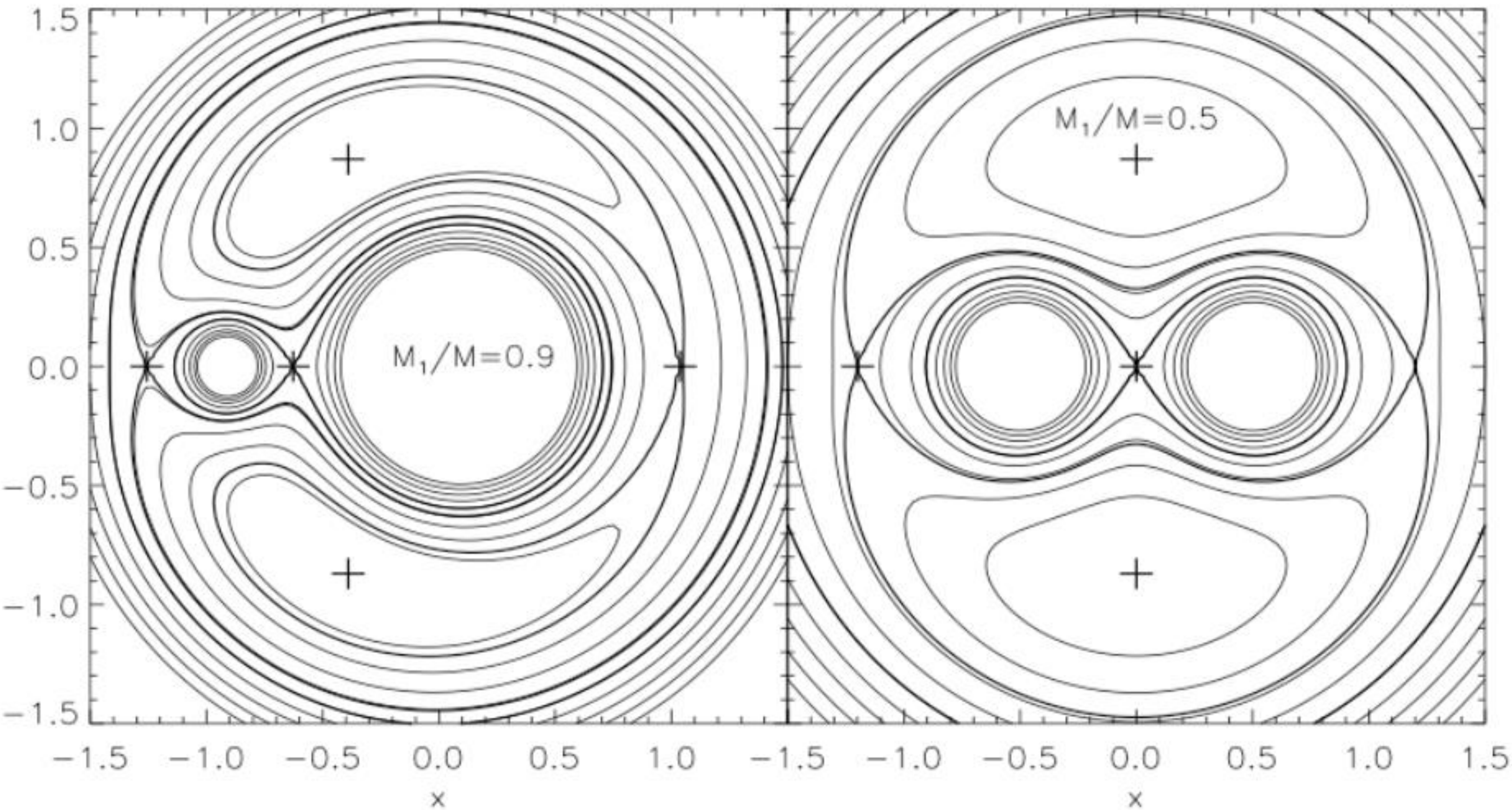
Roche Model

- Combining with Kepler's third law we can see that

$$\Phi_R = \frac{GM(1)}{a} F\left(\frac{x}{a}, \frac{y}{a}, \frac{z}{a}, q\right)$$

- Therefore *the shapes* of the Roche equipotentials, $\Phi_R = \text{const}$, are functions only of *q* and their scale is determined by *a*.

Roche Model

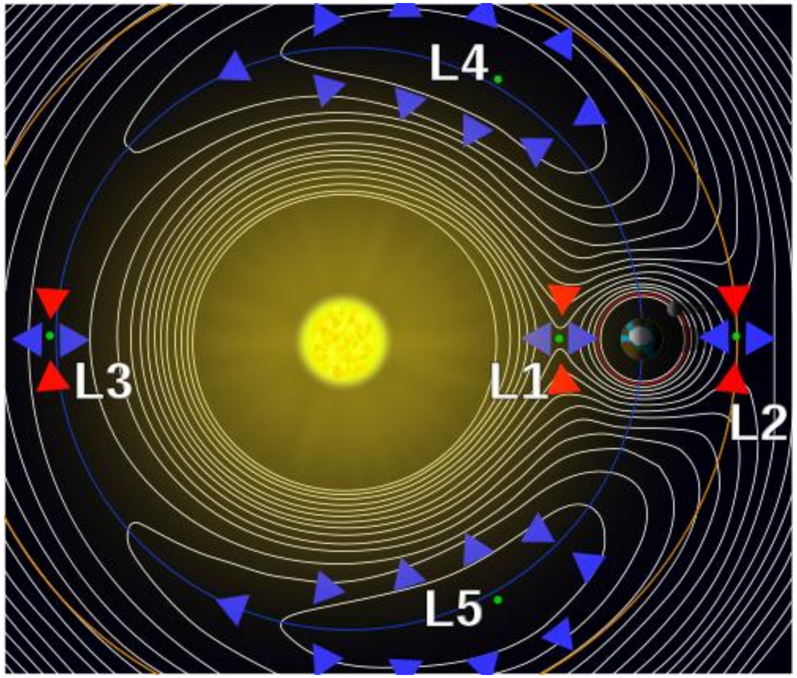


Roche Model

- Close to each object, the potential is dominated by the gravitational potential of the star, thus the surfaces are almost spherical.
- As one moves farther from a stellar centre, two effects start to became important:
  - the tidal effect, which causes an elongation in the direction of the companion
  - and flattening due to the centrifugal force.
- Consequently the surfaces are distorted in a way that their largest dimension is along the line of centres.
- The innermost equipotential surface which encloses both stars defines the critical "Roche lobe" of each star.

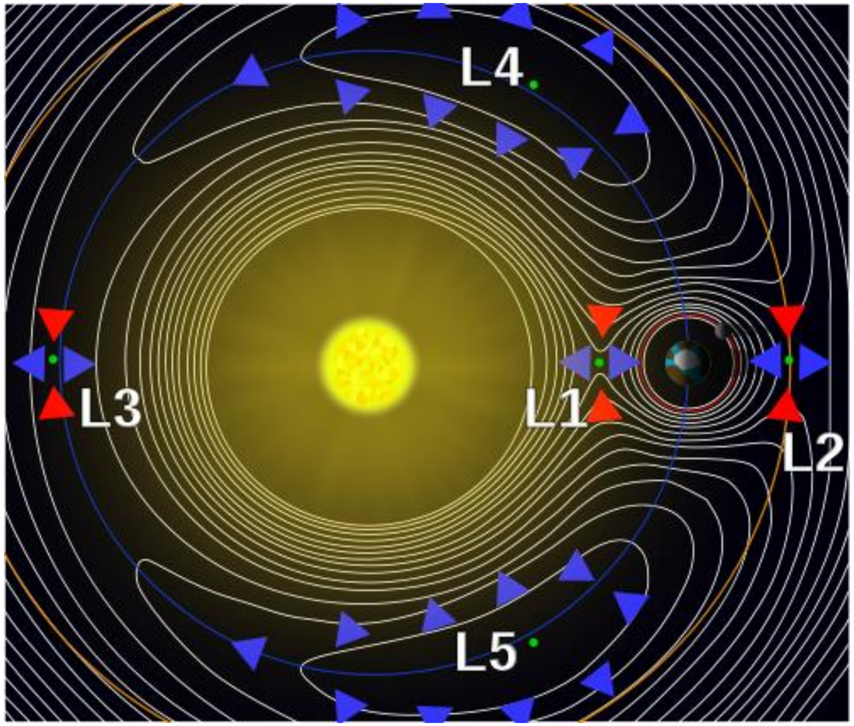
Roche-Lobe overflow

- The Roche lobe is the region of space around a star in a binary system within which orbiting material is gravitationally bound to that star.
- Inner Lagrangian point L_1 is the location where a particle, corotating with the binary, feels no net force - gravity from the two stars plus centrifugal force cancel.
- If one star fills its Roche lobe, gas can freely escape from the surface through L_1 and will be captured by the other star.



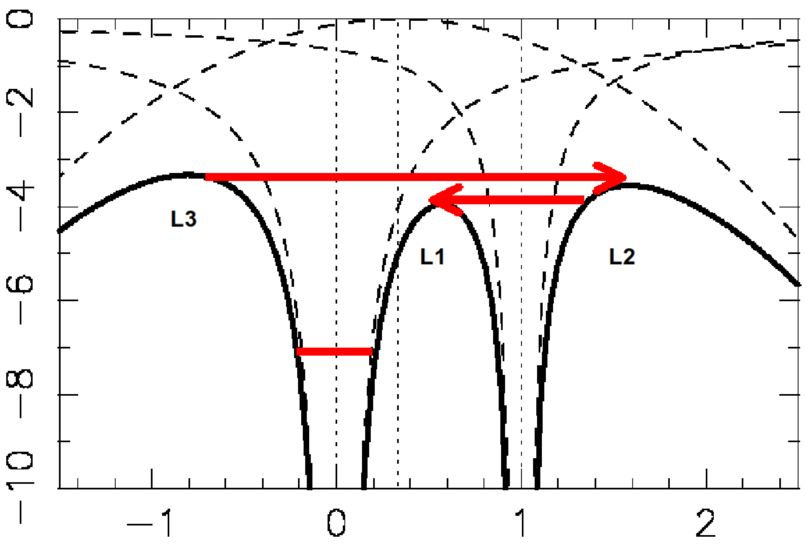
Mass transfer and loss

- L_1 – Inner Lagrange Point
  - matter can flow freely from one star to other
- L_2 – on opposite side of secondary
  - matter can most easily leave system
- L_3 - on opposite side of primary
- L_4, L_5 – in lobes perpendicular to line joining binary
  - form equilateral triangles with centres of two stars

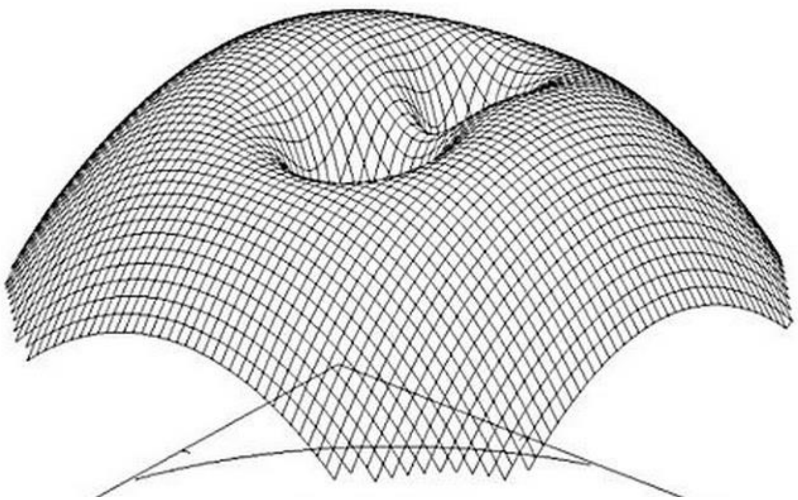


Mass transfer and loss

Slice along X axis



X-Y plane, 3D representation



Geometry of the Roche lobe

- The precise shape of the Roche lobe must be evaluated numerically. However, for many purposes it is useful to approximate the Roche lobe as a sphere of the same volume. An approximate formula for the radius of this sphere is

$$\frac{R_2}{a} = \frac{0.49\, q^{2/3}}{0.6\, q^{2/3} + \ln\left(1 + q^{1/3}\right)}$$

Eggleton (This formula gives results up to 1% accuracy over the entire range of q)

$$q = M_2/M_1$$

Geometry of the Roche lobe

- Distance R_{L1} from centre of primary to inner Lagrangian point:

$$\frac{R_{L_1}}{a} = 1 - w + \frac{1}{3}w^2 + \frac{1}{9}w^3$$

where

$w^3 = \dfrac{q}{3(1+q)}$	$q \leq 0.1$	Kopal (1959)
$\dfrac{R_{L_1}}{a} = 0.500 - 0.227 \log q$	$0.1 \leq q \leq 10$	Plavec & Kratochvil (1964)
$= (1.0015 + q^{0.4056})^{-1}$	$0.04 \leq q \leq 1$ error < 1%	Silber (1992)

Geometry of the Roche lobe

- Volume radius R_{L2} of the Roche lobe of the secondary:

$$\frac{R_L(2)}{a} = 0.38 + 0.20 \log q$$

$0.3 < q < 20$ accurate to 2% — Paczynski (1971)

$$\frac{R_L(2)}{a} = 0.462 \left(\frac{q}{1+q}\right)^{1/3}$$

$0 < q < 0.3$ accurate to 2% — Paczynski (1971)

$$\frac{R_L(2)}{a} = \frac{0.49 q^{2/3}}{0.6 q^{2/3} + \ln(1 + q^{1/3})}$$

$0 < q < \infty$ — Eggleton (1983)

accurate to better than 1%

- Equatorial Roche lobe radius (y direction) of the secondary

$$\frac{R_L(\mathrm{eq})}{a} = 0.378 q^{-0.2084}$$

$0.1 < q < 1$ — Plavec & Kratochvil (1964)

accurate to 1% over $0.2 \le q \le 1$

Reaction to mass loss

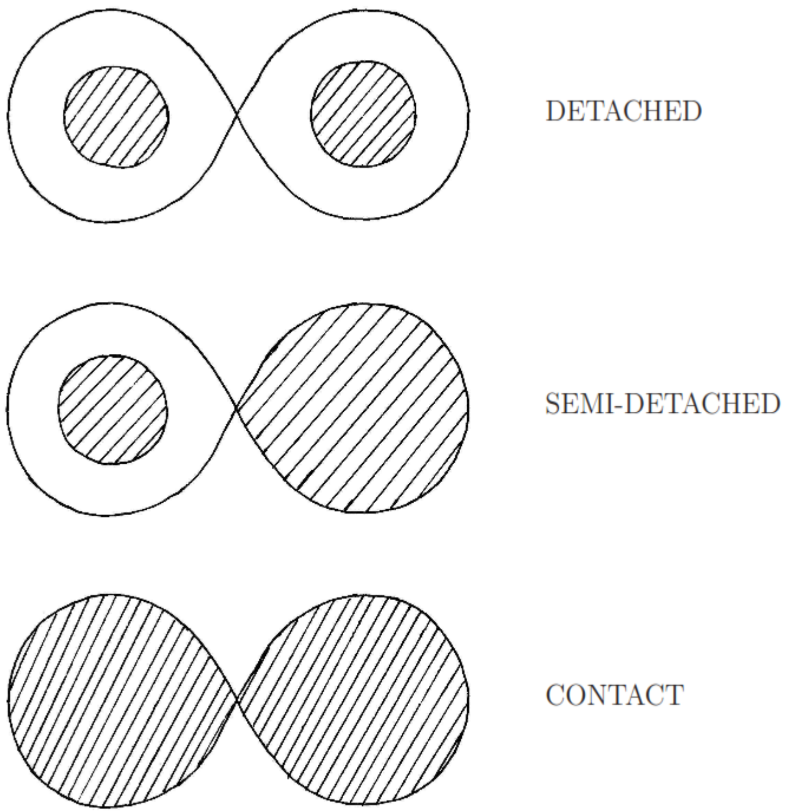
- Star reacts to mass loss:
  - expands / contracts
  - Roche-lobe also expands or contracts
- $q > 5/6$ - unstable mass transfer
  - Roche lobe shrinks down around the star, stripping it down
  - Rapid (dynamical)
  - violent
  - rare because very fast
  - must occur (more massive stars evolve first)
- $q < 5/6$ - stable mass transfer
  - conservative mass transfer makes Roche lobe expand.
    - – cuts off mass transfer

Mass transfer in binary systems

- A star can fill its Roche lobe either:
  - Due to expansion - e.g. the star swells to become a giant on leaving the main sequence.
  - Because the Roche lobe shrinks: binary loses angular momentum, stars spiral together, Roche lobe closes in on one or both stars.
- Four cases:
  - Case A: mass transfer while donor is on main sequence
  - Case B: donor star is in (or evolving to) Red Giant phase
  - Case C: donor star is in Super-Giant phase
  - Case D: donor star is a white dwarf
- Mass transfer changes mass ratio:
  - changes Roche-lobe sizes
  - can drive further mass transfer

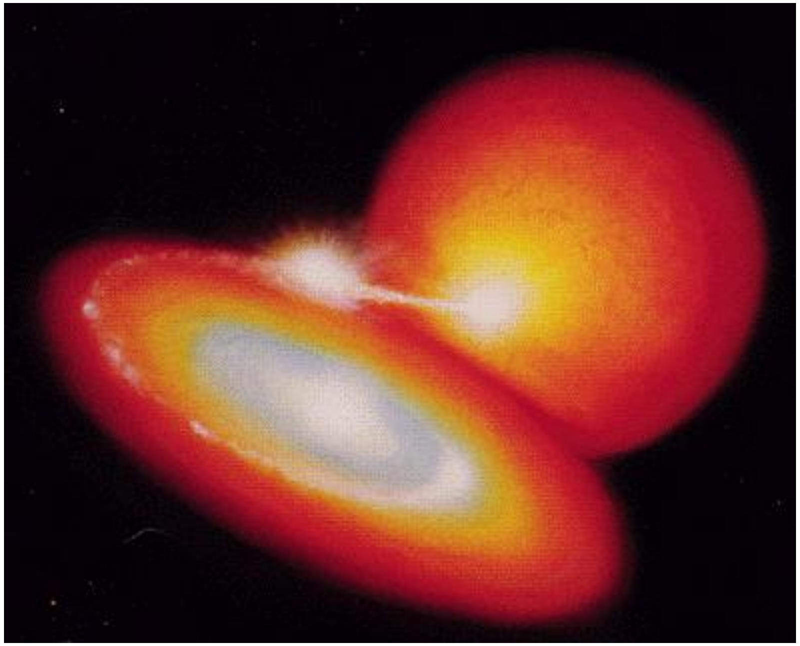
Mass transfer in binary systems

- Describe a binary system as:
  - **Detached**: neither star fills its Roche lobe, both are roughly spherical.
  - **Semi-detached**: one star fills its Roche lobe, and is highly distorted. Mass flows onto the other star in the binary.
  - **Contact**: both stars fill their Roche lobes - touch at the L_1 point.



Transfer of matter

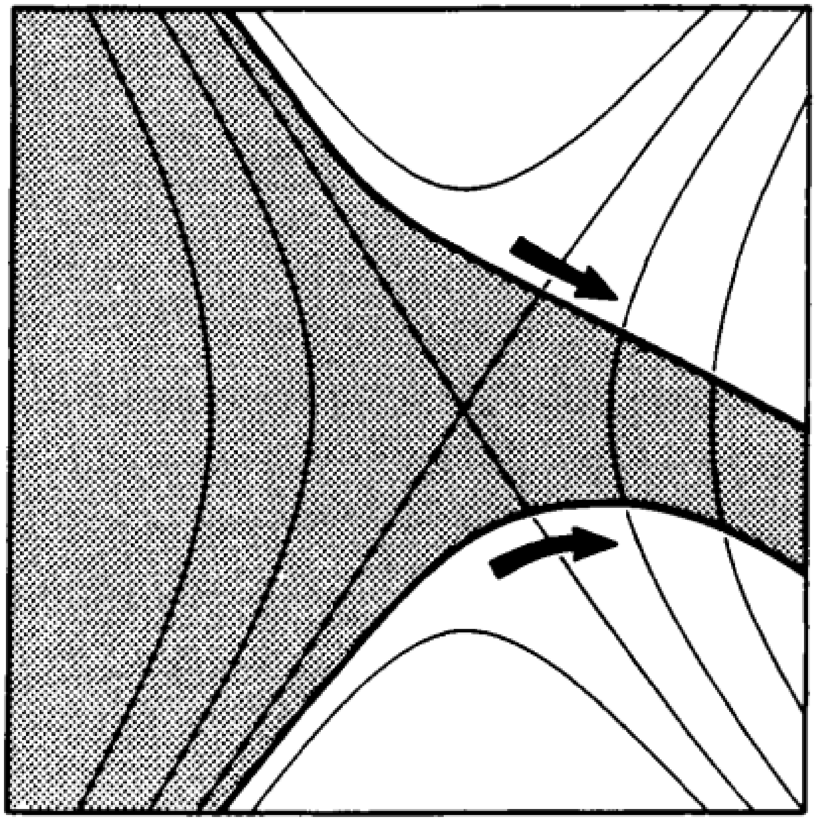
- In a semi-detached system, gas flowing through L_1 has too much angular momentum to fall directly onto the surface of the other star.
- Gas forms an accretion disk around the mass gaining star, through which the gas slowly spirals in before being accreted.
- This occurs if the accreting star does not have a strong magnetic field.



Artist's Conception: Dana Berry, STScI

Transfer of matter

- At L_1 gas can escape from the atmosphere of the secondary into the Roche lobe of the primary. The flow resembles the escape of gas through a nozzle into a vacuum
- The flow velocity is approximately the thermal velocity of the atoms in the gas.
- The stream leaving L_1 has a core that is denser than the outer regions; the density profile should be approximately Gaussian.
- Details of the stream lines in the vicinity of L_1 are given by Lubow & Shu (1975).



Transfer of matter

- **What happens to the gas after it has flowed through the L_1 point and when it starts to be accelerated towards star 2?**

- The flow velocity through L_1 is roughly equal to the sound speed in the atmosphere of the mass-losing star:

$$c_s \cong 15\sqrt{T_4}\, km\, s^{-1}$$

Rough estimate

where T_4 is in the units of 10^4 K.

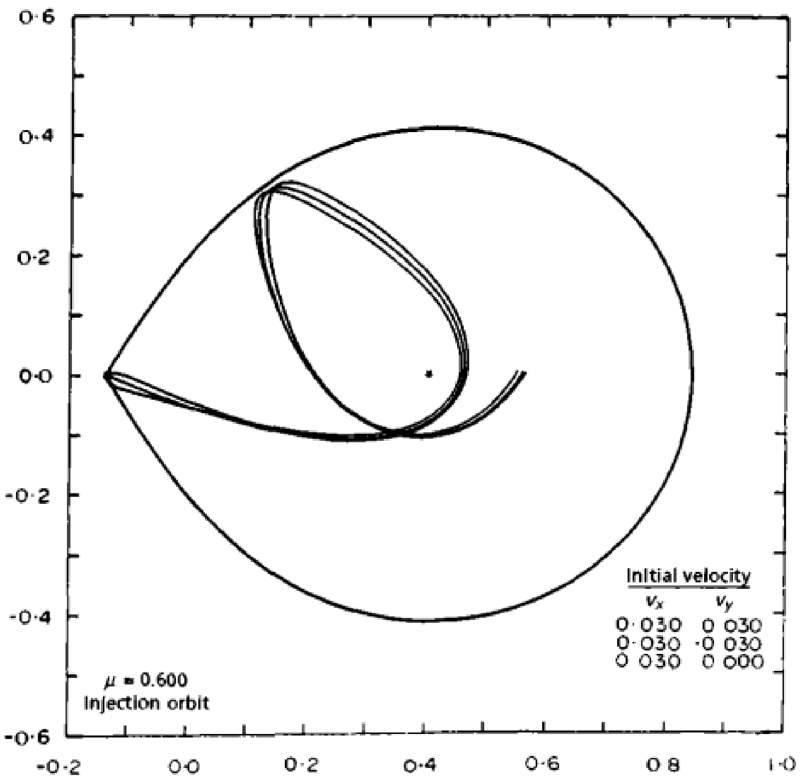
- T varies from ~3000K to ~30000K
- Thus, $c_S \approx 10\text{-}30$ km/s

Transfer of matter

- As the stream of gas flows away from L_1 the stream lines are deflected by the Coriolis effect and make an angle with the x-axis that is a function only of q.
- After leaving L_1 the gas particles fall towards the primary, which increases their original sonic velocities to highly supersonic:
  compare with the dynamical velocity of the binary system (the orbital velocities of the stars).
- In general, the sound speed of the gas is much less than the dynamical velocities of the stars: $c_S \ll V_{orb}$

Transfer of matter

- The stream expands transversely at the velocity of sound, so pressure forces are soon negligible and the stream trajectory is well described by following the orbits of single particles ejected from L_1 in all directions at sonic velocities.



Transfer of matter

- The stream trajectory can be found from integrating the equations of motion for a particle in the rotating binary frame.
- In conserving energy along the trajectory, a particle obeys the equation

$$\frac{1}{2}\dot{r}^2 + \Phi_R = const$$

- If a particle starts with almost zero velocity ($\dot{r} = 0$) on the Roche lobe, it does not have sufficient energy to cross the lobe at any other point.
- Thus the trajectory lies entirely within the Roche lobe of the primary and whenever the particle approaches the lobe it does so with low velocity.
- The Roche lobes are therefore also known as zero velocity surfaces.

Transfer of matter

- Flannery (1975, MNRAS, 170, 325):

The equations of motion are formulated in the conventional RTB problem frame of reference which corotates with the binary, and the physical variables are reduced to the standard dimensionless variables. Scale factors for relevant physical quantities are the following: length, the binary separation A; mass, the binary total mass M; time, the inverse of the orbital circular frequency ω^{-1}; velocity, ωA; energy, GM/A; and the angular momentum, ωA^2. The blue star is designated as star 1 and has mass fraction μ, i.e. $M_1 = \mu M$. With the X axis along the line of centres and the coordinate origin at the centre of mass, the equations of motion are

$$\ddot{x} = 2\dot{y} + x - \mu\frac{(x-x_1)}{r_1^3} - (1-\mu)\frac{(x-x_2)}{r_2^3}, \quad (1)$$

$$\ddot{y} = -2\dot{x} + y - \frac{\mu y}{r_1^3} - (1-\mu)\frac{y}{r_2^3}, \quad (2)$$

where $r_1(r_2)$ is the particle's distance from star 1(2), and dots represent time derivatives. The quantities j_t and r_t defined above are evaluated as

RTB – restricted three-body approximation

Transfer of matter

- The stream has a distance of closest approach r_{min} from the centre of the primary, obtainable from trajectory computations, and approximated to 1% accuracy by

$$\frac{r_{\min}}{a} = 0.0488q^{-0.464} \qquad 0.05 < q < 1$$

- If $R_* > r_{min}$ then the stream strikes the surface of the star directly.
- However, if this is a compact star (a white dwarf, neutron star or black hole), then the stream fails to strike the star directly.
- Instead, the gas flow undergoes self interaction as a consequence of which it **circularizes**.

Accretion Disks

Formation of an Accretion Disk,

Tidal limitations,

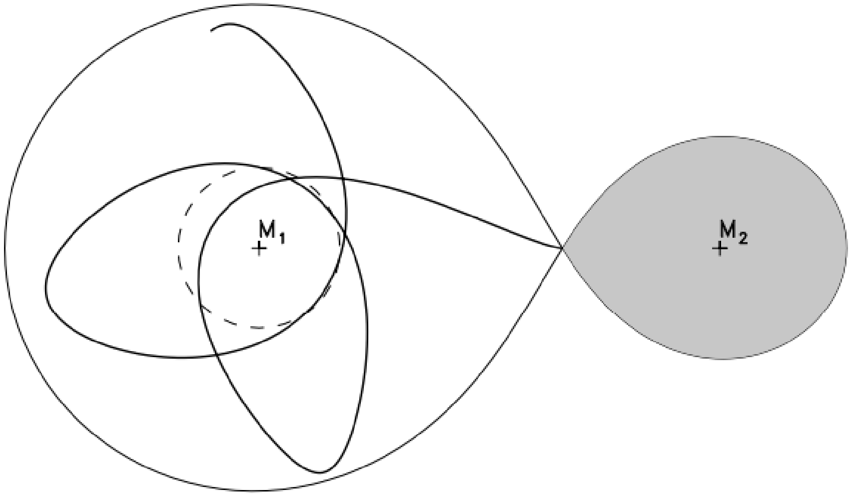
Boundary layer,

Bright spot,

Stream-Disk overflow

Formation of a Ring

- Angular momentum is conserved.
- A circular orbit has the least energy for a given angular momentum → the dissipation will tend to produce a ring of gas.
- Circularization Radius ≈ same angular momentum as L_1.



Formation of a Ring

- A circular orbit at a distance R from the primary has a Keplerian velocity

$$V_K(R) = \sqrt{\frac{GM}{R}}$$

- If the gas continues to conserve angular momentum while dissipating energy, the radius of the circularization radius r_{circ} is determined from

$$r_{circ} V_K(r_{circ}) \approx \frac{2\pi}{P_{Orb}} {R_{L1}}^2$$

Formation of a Ring

- Then

$$\frac{r_{circ}}{a} = (1+q)\left(\frac{R_{L1}}{a}\right)^4 \approx (1+q)(0.5 - 0.227\log q)^4$$

- A more accurate value (to 1%):

$$\frac{r_{circ}}{a} = 0.0859\, q^{-0.426} \qquad\qquad 0.05 \le q < 1$$

$$\mathrm{r_{circ}} \sim 1.75\ \mathrm{r_{min}}$$

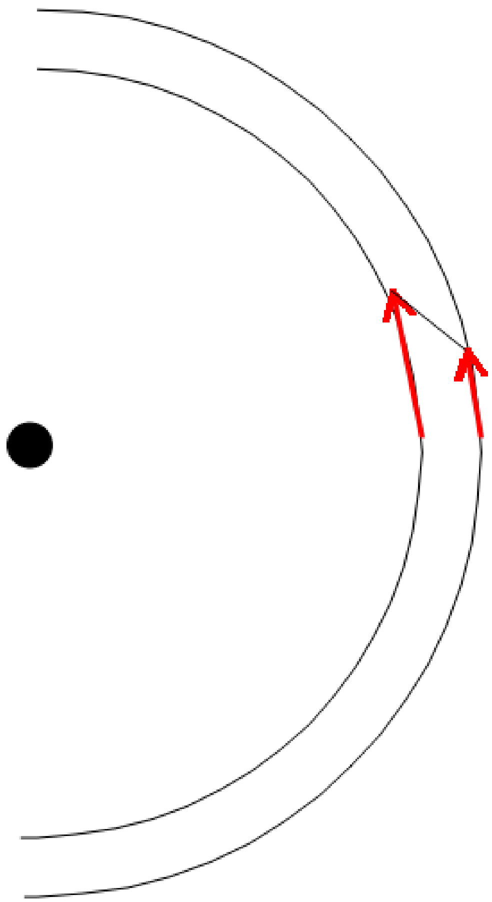
- If angular momentum was conserved, material would stay in a ring without expanding or accreting onto the primary.

Formation of an Accretion Disk

- The ring that is formed has a finite radial extent and rotates differentially.
- Keppler Velocity: $V = \sqrt{\frac{GM}{R}}$
- Differential rotation (Shear):

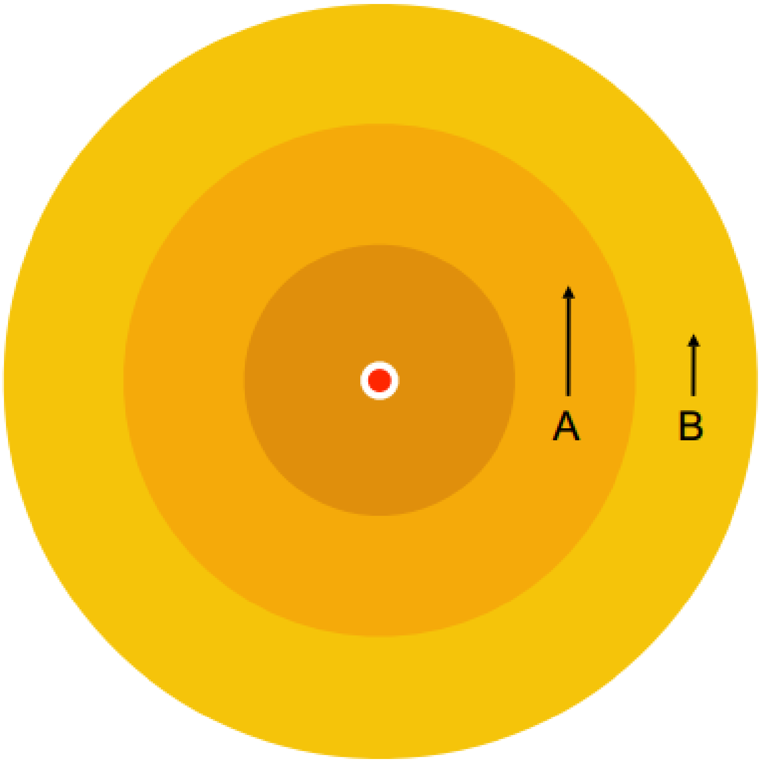
$$\Delta V = \frac{d}{dR}\left(\sqrt{\frac{GM}{R}}\right)\Delta R = \frac{V}{2}\frac{\Delta R}{R}$$

- Friction:
  - opposes shear
  - causes ring to spread inward + outward
  - Diffusion



Formation of an Accretion Disk

- Ring A moves faster than ring B. Friction between the two will try to slow down A and speed up B.
- Angular momentum is transferred from A to B.
- If ring A looses angular momentum, but is forced to remain on a Kepler orbit, it must move inward! Ring B moves outward (which has friction with a ring C, which has friction with D, etc.).
- Disk is formed!



Formation of an Accretion Disk

FITdisk

(https://vitaly.neustroev.net/teaching/tools/fitdisk.zip)