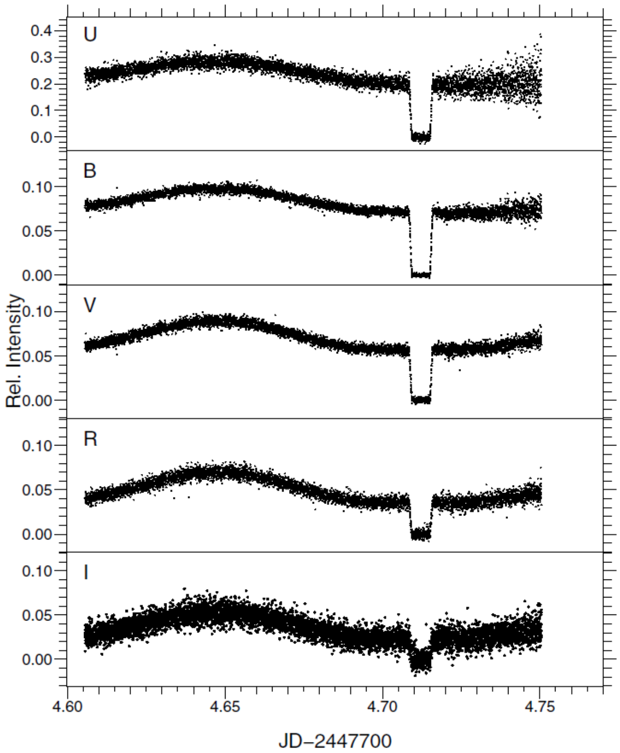
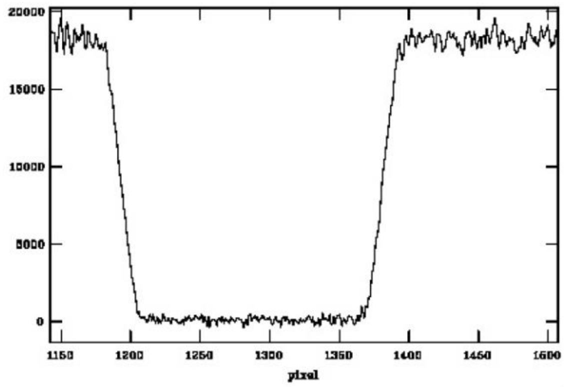
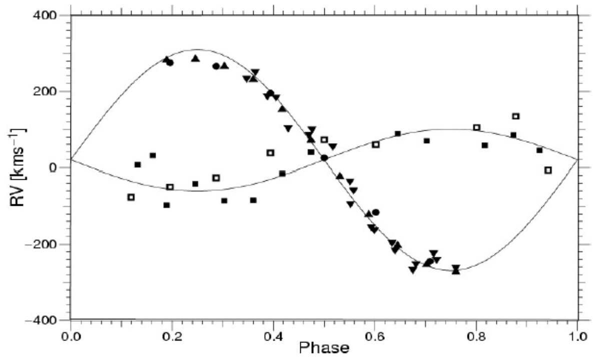
Interacting binary stars

Problem 1.

The pre-cataclysmic binary consisting of a hot white dwarf and a cool late type star shows very deep eclipses of $\Delta V \approx 5.8$ mag. From the light and velocity curves, it is determined that the orbital period is 0.130080 days, and the maximum radial velocities of the white dwarf and the secondary star are 80 km/s and 290 km/s, respectively. Furthermore, the time period between first contact and minimum light (t_b - t_a) is 85 sec, the length of the primary minimum (t_c-t_b) is 485 sec.



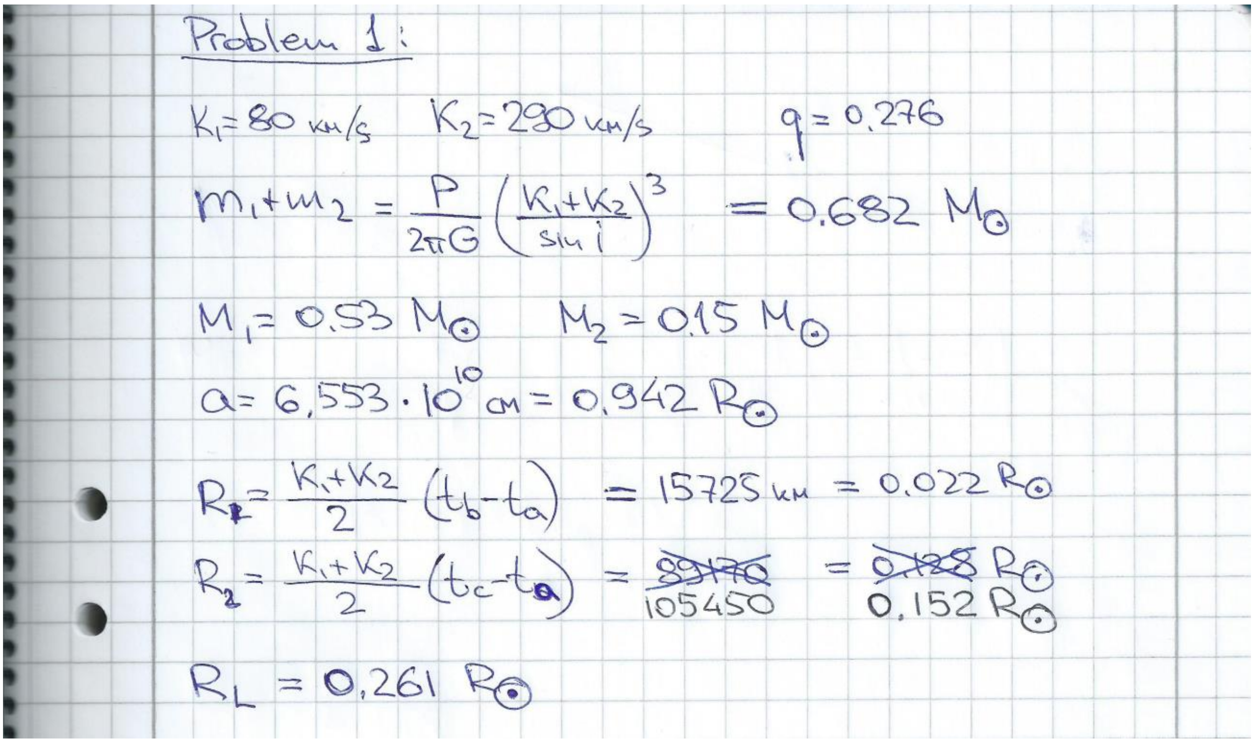




From this information, and assuming circular orbits, find the

1. Binary separation.
2. Ratio of stellar masses.
3. Sum of the masses (assume the inclination angle of the orbit $i \approx 90°$).
4. Individual masses.
5. Individual radii.
6. Does the secondary star fill its Roche lobe? Compare the volume radius of the Roche lobe of the secondary with the radius of the star.
7. If the system with these system parameters is at a semi-detached configuration and the mass transfer is occurring, does the stream strike the surface of the white dwarf directly?

Solution:


Problem 1:

$K_1 = 80$ km/s $K_2 = 290$ km/s $q = 0.276$

$m_1 + m_2 = \frac{P}{2\pi G}\left(\frac{K_1+K_2}{\sin i}\right)^3 = 0.682\ M_\odot$

$M_1 = 0.53\ M_\odot$ $M_2 = 0.15\ M_\odot$

$a = 6.553 \cdot 10^{10}$ cm $= 0.942\ R_\odot$

$R_1 = \frac{K_1+K_2}{2}(t_b - t_a) = 15725$ km $= 0.022\ R_\odot$

$R_2 = \frac{K_1+K_2}{2}(t_c - t_a) =$ ~~89170~~ 105450 $=$ ~~0.128~~ 0.152 $R_\odot$

$R_L = 0.261\ R_\odot$

Problem 2.

The separation between the centres of mass of the binary components can be calculated using this form of Kepler's third law:

$$a = 3.53 \times 10^{10} M_1^{\frac{1}{3}} (1+q)^{\frac{1}{3}} P_{Orb}^{\frac{2}{3}}(h) \quad \text{cm}$$

where M_1 is the mass of the primary in solar masses, and $q \equiv M_2/M_1$.

Compute a and compare it with the solar radius for the following types of interacting binaries:

a) Cataclysmic variable: $M_1 = 0.61\ M_\odot$, q=0.15, P_{Orb}=1.768 h
b) Cataclysmic variable: $M_1 = 0.73\ M_\odot$, q=0.80, P_{Orb}=5.882 h
c) Black hole binary: $M_1 = 6.61\ M_\odot$, q=0.06, P_{Orb}=7.772 h
d) Black hole binary: $M_1 = 7.5\ M_\odot$, q=0.12, P_{Orb}=16.452 h

Solution:

a) $a = 4.586 \cdot 10^{10}$ cm $= 0.659\ R_\odot$
b) $a = 1.260 \cdot 10^{11}$ cm $= 1.812\ R_\odot$
c) $a = 2.650 \cdot 10^{11}$ cm $= 3.811\ R_\odot$
d) $a = 4.642 \cdot 10^{11}$ cm $= 6.674\ R_\odot$