

Spectral Analysis with Unevenly-Spaced Data

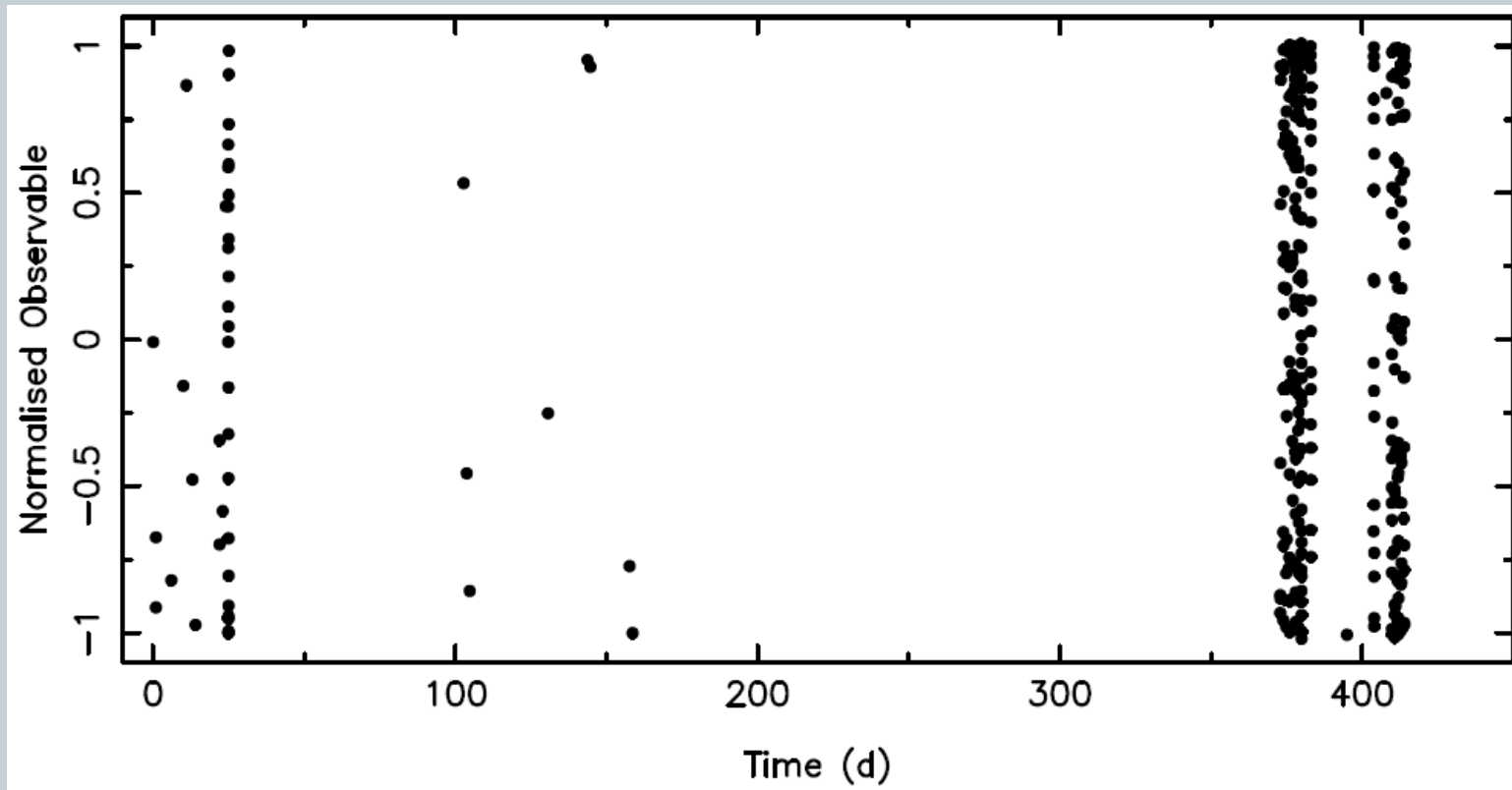
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DISCRETE FOURIER TRANSFORM
LOMB-SCARGLE PERIODOGRAM

Spectral Analysis with Unevenly-Spaced Data

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- Gapped data representing a typical time series for a ground-based single-site observational campaign:



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FOURIER ANALYSIS WITH UNEQUALLY-SPACED DATA*

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(Received 22 March; in revised form 11 November, 1974)

Abstract. The general problems of Fourier and spectral analysis are discussed. A discrete Fourier transform $F_N(\nu)$ of a function $f(t)$ is presented which (i) is defined for arbitrary data spacing; (ii) is equal to the convolution of the true Fourier transform of $f(t)$ with a spectral window. It is shown that the 'pathology' of the data spacing, including aliasing and related effects, is all contained in the spectral window, and the properties of the spectral windows are examined for various kinds of data spacing. The results are applicable to power spectrum analysis of stochastic functions as well as to ordinary Fourier analysis of periodic or quasiperiodic functions.

Discrete Fourier transform

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- Time series, x_k , $k=0, \dots, N-1$
- **Evenly** spaced data:
 - The discrete Fourier transform decomposes the signal into N sine waves, a_j , $j= -N/2+1, \dots, N/2$

$$a_j = \sum_{k=0}^{N-1} x_k e^{i2\pi jk/N} \quad j = -\frac{N}{2} + 1, \dots, \frac{N}{2}$$

- **Unevenly** spaced data:
 - The discrete Fourier transform decomposes the signal into M sine waves, a_j , $j= 1, \dots, M$

$$a_j = \sum_{k=0}^{N-1} x_k e^{i2\pi v_j t_k} \quad j = 1, \dots, M$$

M and v_j are now arbitrary.

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- PSD is computed as the squared Fourier amplitudes:

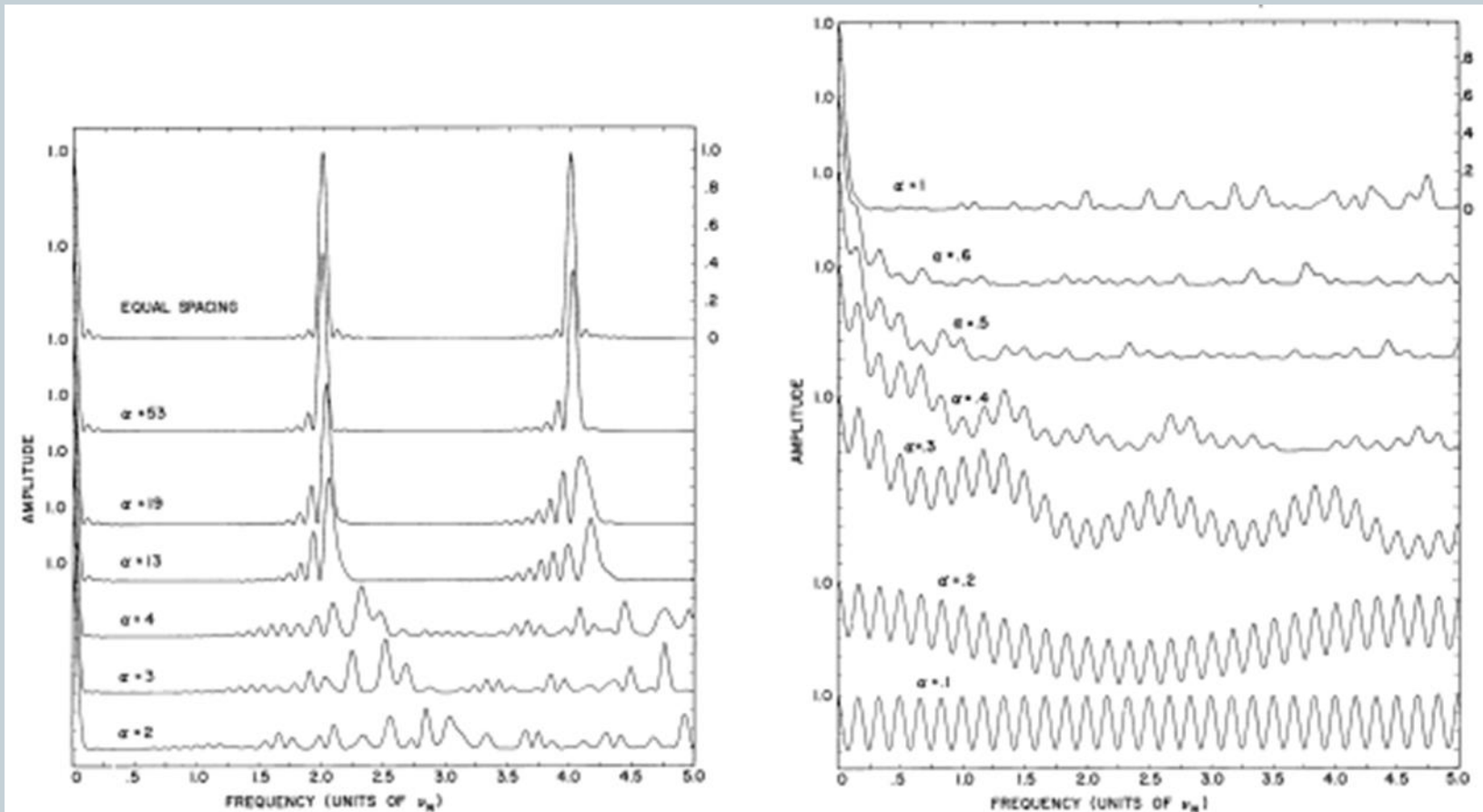
$$P_j = (\textit{Normalization})|a_j|^2$$

- **Deeming:**
the “pathology” of the data spacing, including aliasing and related effects, is all contained in the spectral window.

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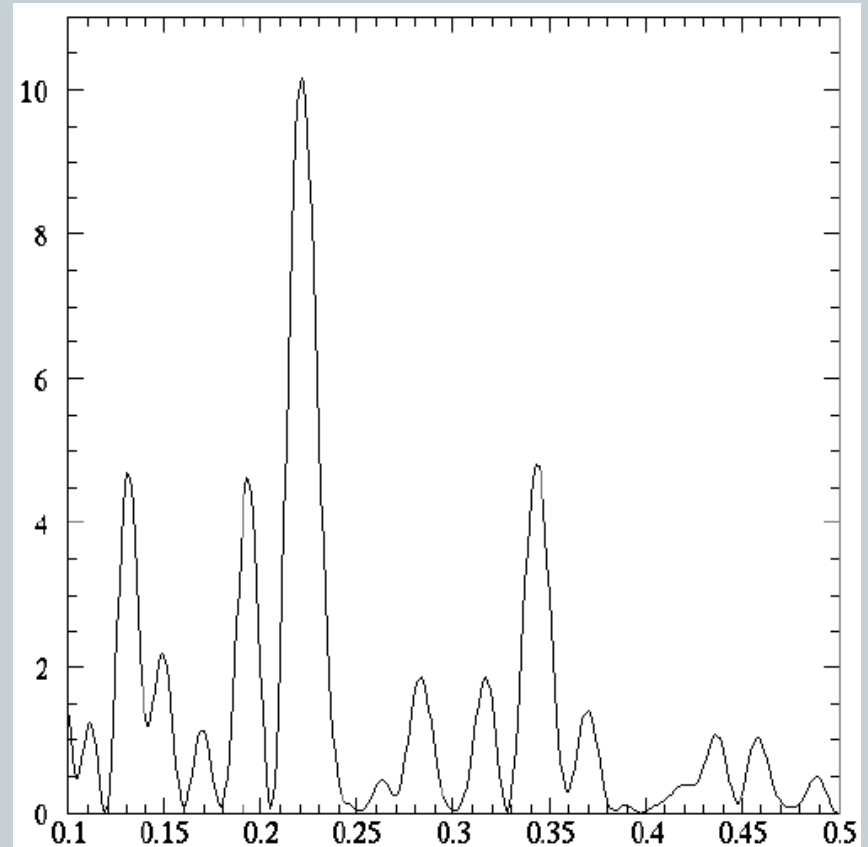
- The variation of spectral window shape:



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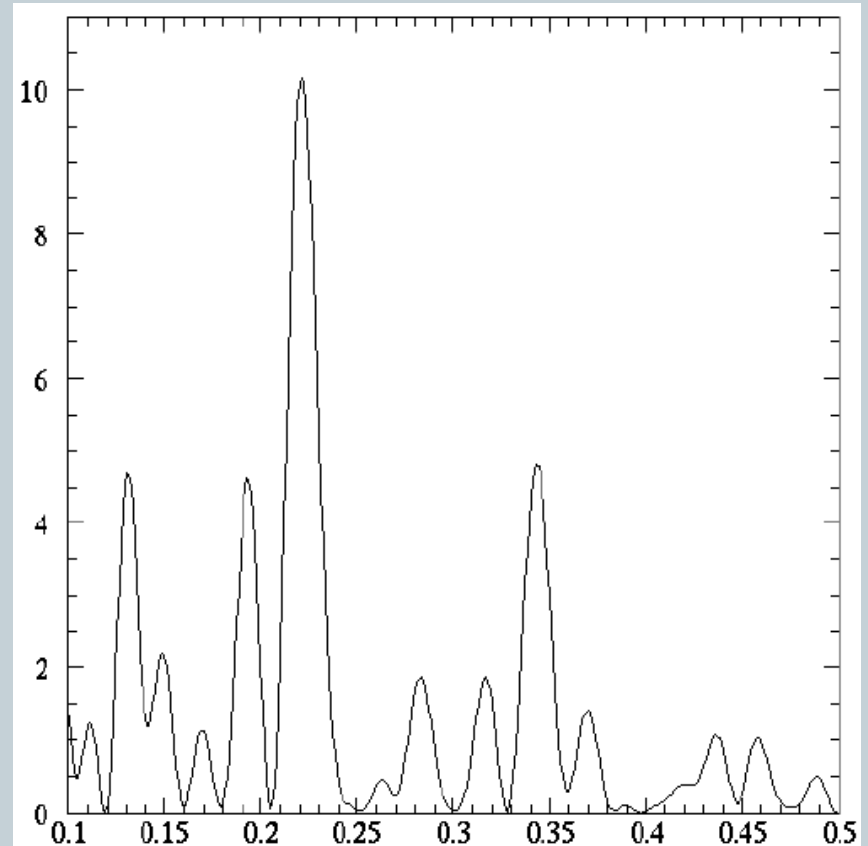
- How to determine the significance of peaks found in power spectra of unevenly-spaced data?
- How many *independent* frequencies do we use? In the plot to the right, 1000 values of v_j are plotted ($M=1000$), but most frequencies are ***not*** independent!



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- Most frequencies are ***not*** independent!
- Best solution: use Monte Carlo data sets to estimate the probability that the largest peak is bigger than P_{det} .
- Second-best rule of thumb: Each peak has a width of $d\omega = 2\pi/T$, where T is the length of the entire data set. Here $T=365$ days, so $d\omega = 0.017$. We then estimate that there should be approximately $(0.5-0.1)/0.017 = 23$ independent frequencies over this range.



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- **BE VERY SKEPTICAL OF CLAIMS FOR PERIODICITIES THAT COINCIDE WITH NATURAL FREQUENCIES OF DETECTORS OR OBSERVERS (eg. 1-day, 7-day, 1-year).**

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- The dependence of the PSD on the data length, T , is different for periodic, non-periodic, and stochastic functions:

$|a_j| \propto T^0$ non-periodic

$|a_j| \propto T^1$ periodic

$|a_j| \propto T^{1/2}$ stochastic