

Discrete FT of a sinusoid

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The power spectrum of a sinusoid $A \sin(2\pi\nu_{\text{sine}}t_k + \phi)$:

$$\boxed{|a_j|^2} = \frac{1}{4}A^2N^2 \left(\frac{\sin \pi x}{\pi x}\right)^2 \left[\left(\frac{\pi x/N}{\sin \pi x/N}\right)^2 + \left(\frac{\pi x/N}{\sin [\pi(2j+x)/N]}\right)^2 + \right. \\ \left. + 2 \left(\frac{\pi x/N}{\sin \pi x/N}\right) \left(\frac{\pi x/N}{\sin [\pi(2j+x)/N]}\right) \cos [(N-1)(2\pi(j+x)/N) + 2\phi] \right]$$
$$x = (\nu_{\text{sine}} - \nu_j)T$$

$$\boxed{\approx \frac{1}{4}A^2N^2 \left(\frac{\sin \pi x}{\pi x}\right)^2}$$

$$x/N \ll 1 \text{ and } 0 \ll j/N \ll \frac{1}{2}$$

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