Stellar Atmospheres
Lecture 19

VITALY NEUSTROEV

ASTRONOMY RESEARCH UNIT
UNIVERSITY OF OULU

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Recap from last lecture

Summary

- If LTE does not hold, Saha-Boltzmann no longer describes excitation and ionization conditions – need to solve rate equations for statistical equilibrium – much more complicated!

- Non-LTE is necessary for hot stars, coronae of cool stars, M-type stars (as well as in nebulae and ISM).

NonLTE: Statistical equilibrium

$$\underbrace{n_i \sum_{j \neq i} (R_{ij} + C_{ij})}_{\text{lines}} + \underbrace{n_i (R_{ik} + C_{ik})}_{\text{ionization}} = \underbrace{\sum_{j \neq i} n_j (R_{ji} + C_{ji})}_{\text{lines}} + \underbrace{n_p (R_{ki} + C_{ki})}_{\text{recombination}}$$

$$n_l (B_{lu} \int_0^\infty \varphi_\nu J_\nu d\nu + C_{lu}) = n_u (A_{ul} + B_{ul} \int_0^\infty \varphi_\nu J_\nu d\nu + C_{ul})$$

The line source function

scattering term

thermal term

$$S_\nu^{\text{line}} = (1-\epsilon)\int_0^\infty \varphi_{\nu'} J_{\nu'} d\nu' + \epsilon\, B_\nu(T) = \frac{\int_0^\infty \varphi_{\nu'} J_{\nu'} d\nu' + \epsilon'\, B_\nu(T)}{1+\epsilon'}$$

$$\epsilon := \frac{\left(1-e^{-h\nu/kT}\right) C_{ul}/A_{ul}}{1+\frac{C_{ul}}{A_{ul}}\left(1-e^{-h\nu/kT}\right)} = \frac{\epsilon'}{1+\epsilon'}$$

destruction probability

$$\frac{N_u}{N_l} = \frac{g_u}{g_l} e^{-(E_u-E_l)/kT}$$

Collisional rates

For an electron with kinetic energy E exciting an atom

- Excitation:

$$C_{ij} = n_e \int_{E_{ij}}^{\infty} \sigma_{ij}(u) u f(u) du = n_e \int_{E_{ij}}^{\infty} \sigma_{ij}(E) \sqrt{\frac{2E}{m}} f(E) dE \propto \frac{n_e}{T^{3/2}} \int_{E_{ij}}^{\infty} \sigma_{ij}(E) e^{-E/kT} E \, dE \propto \frac{n_e}{T^{1/2}} e^{-E_{ij}/kT}$$

$$\sigma_{ij}(E) \propto 1/E$$

- De-excitation:

$$C_{ji} = n_e \int_0^{\infty} \sigma_{ji}(u) u f(u) du = n_e \int_0^{\infty} \sigma_{ji}(E) \sqrt{\frac{2E}{m}} f(E) dE$$

where $f(E)$ is the (Maxwellian) energy distribution of the colliding particles.

- In TE, the principle of detailed balance gives

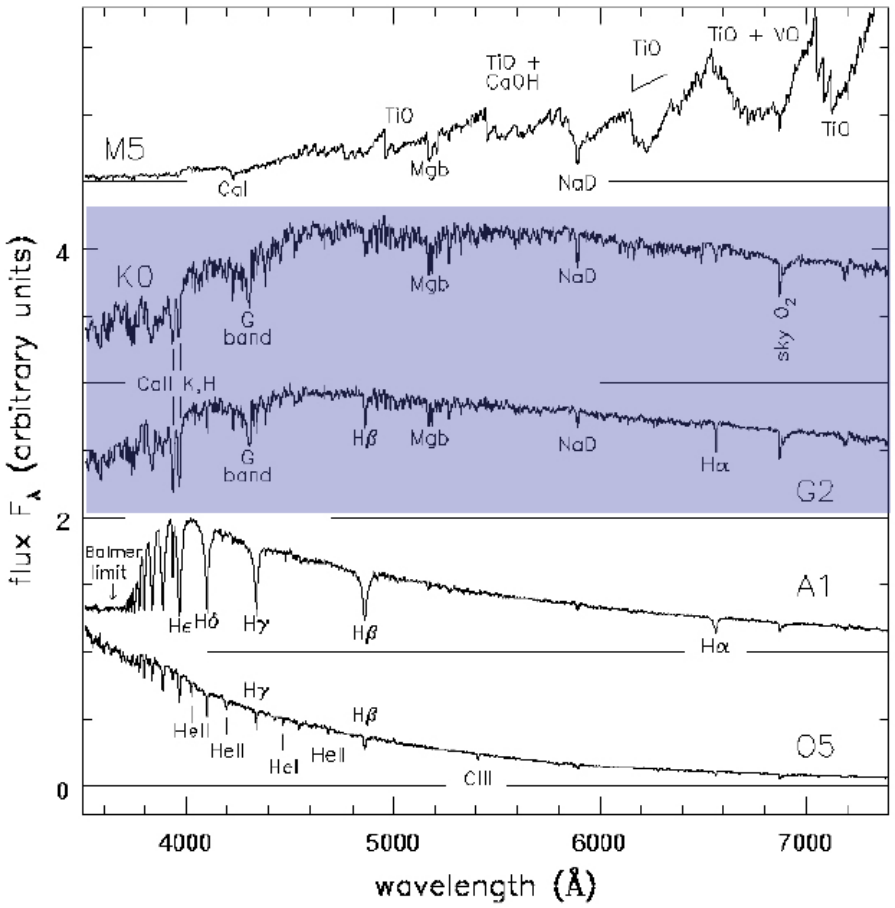
$$n_i C_{ij} = n_j C_{ji} \Rightarrow \frac{C_{ij}}{C_{ji}} = \frac{n_j}{n_i} = \frac{g_j}{g_i} e^{-E_{ij}/kT}$$

- Even if there is no TE, but we have a Maxwellian velocity distribution $\frac{C_{ij}}{C_{ji}} = \frac{g_j}{g_i} e^{-E_{ij}/kT}$

$$\frac{C_{ul}}{C_{lu}} = \frac{g_u}{g_l} e^{-(E_u - E_l)/kT}$$

Spectral type sequence

Spectral Types: temperature sequence



T ~ 4000K

Molecules!

Mainly neutral metal lines

T ~ 6000K

Ionised Metal lines

T < 11 000K

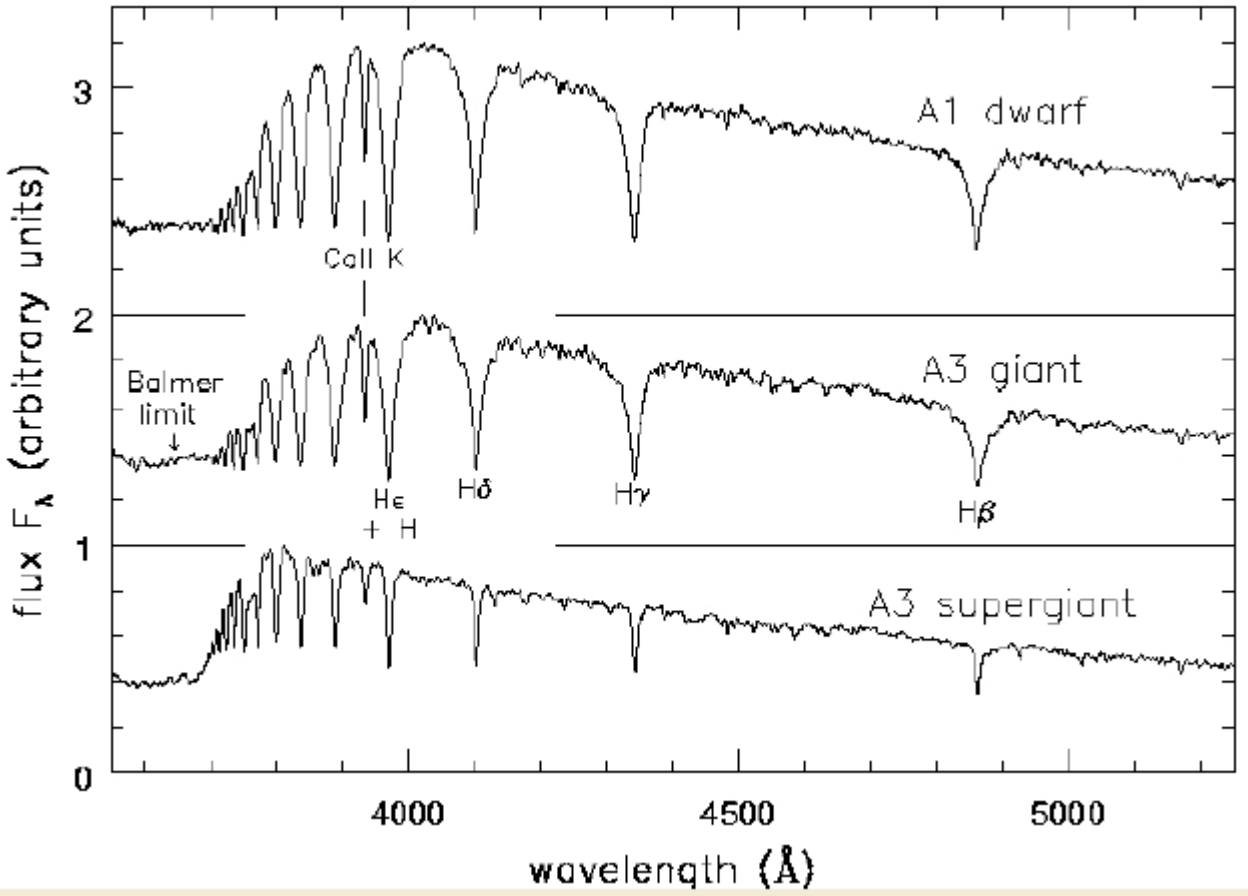
Dominated by neutral H

T ~ 30 000K

Highly ionised species

Line Broadenings

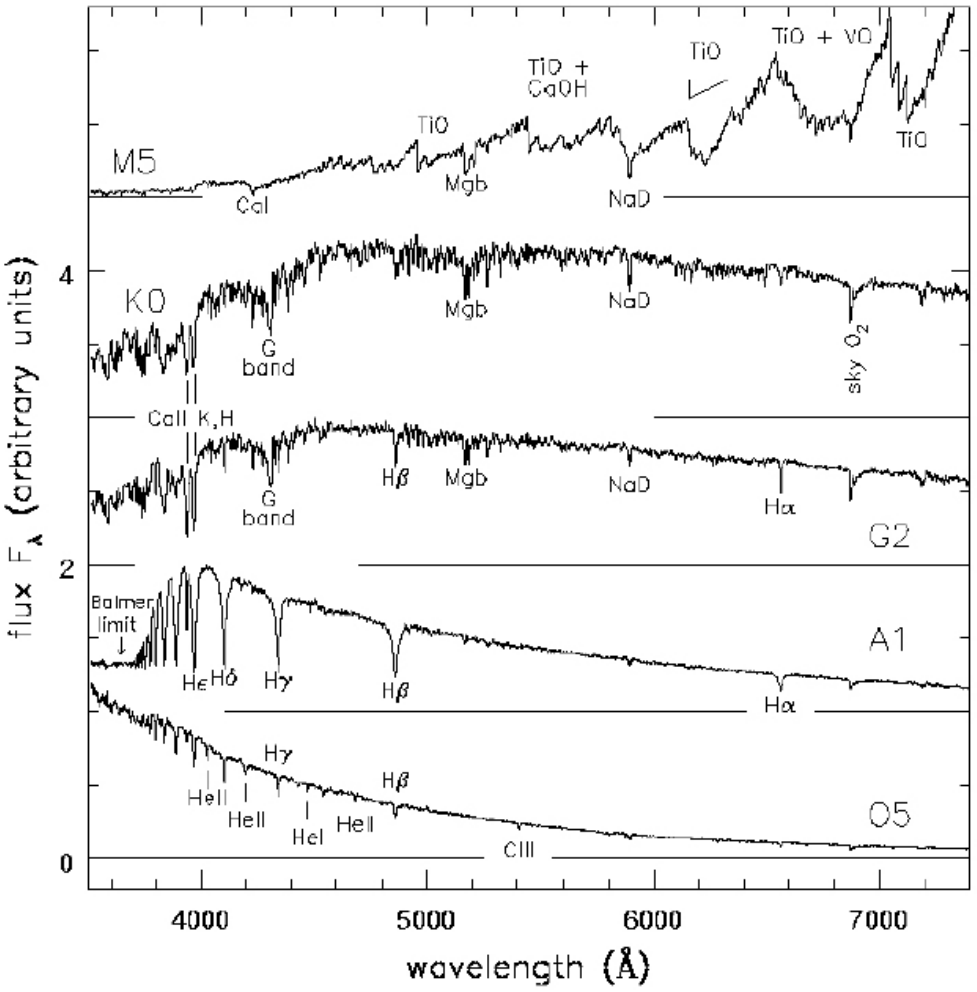
For example:
Stark Effect



He and Metals

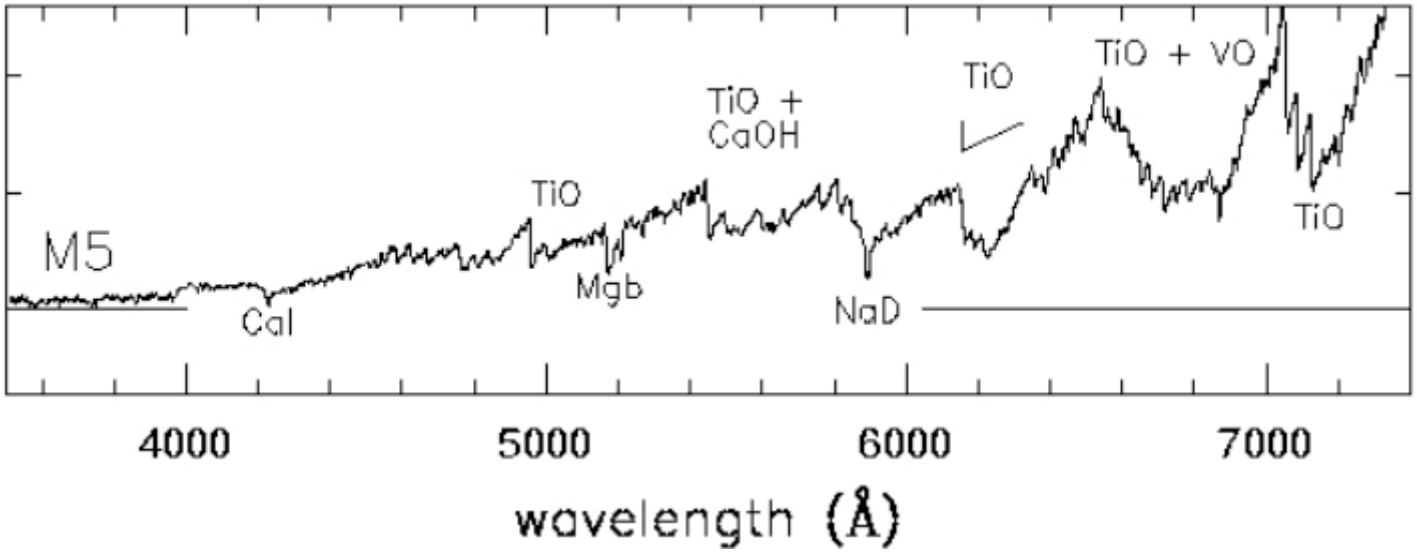
Metal are strongests when temperature is low enough that lower ionization stages are populated.
The metal lines become progressively stronger as the temperature cools and dominate in the F, G, K stars.

Helium is the second most abundant element, but only in the hottest stars (O and B) do He atoms show up in their excited levels where they can absorb visible light. For the very hottest O stars we also see HeII lines.

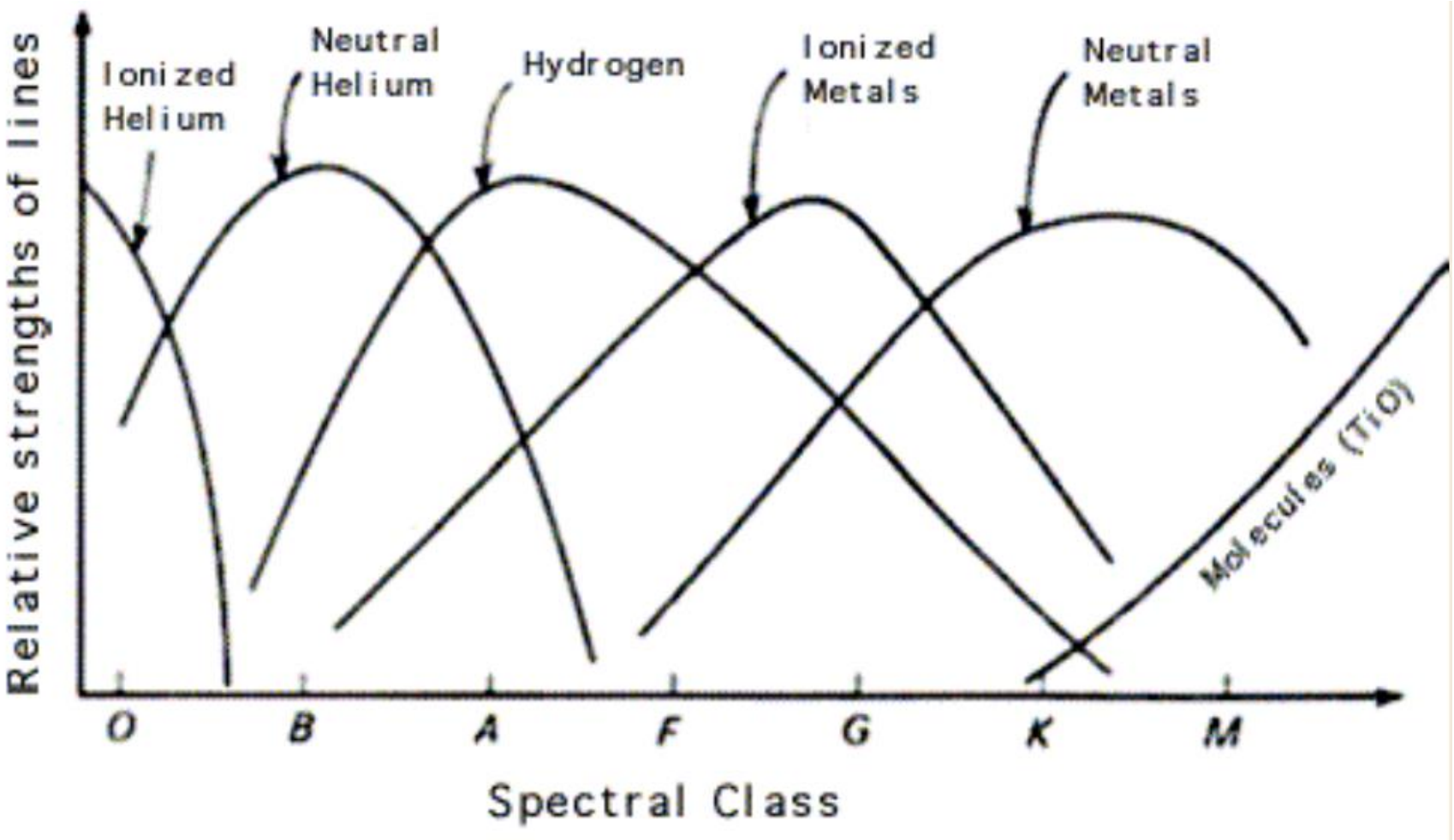


Molecular Bands

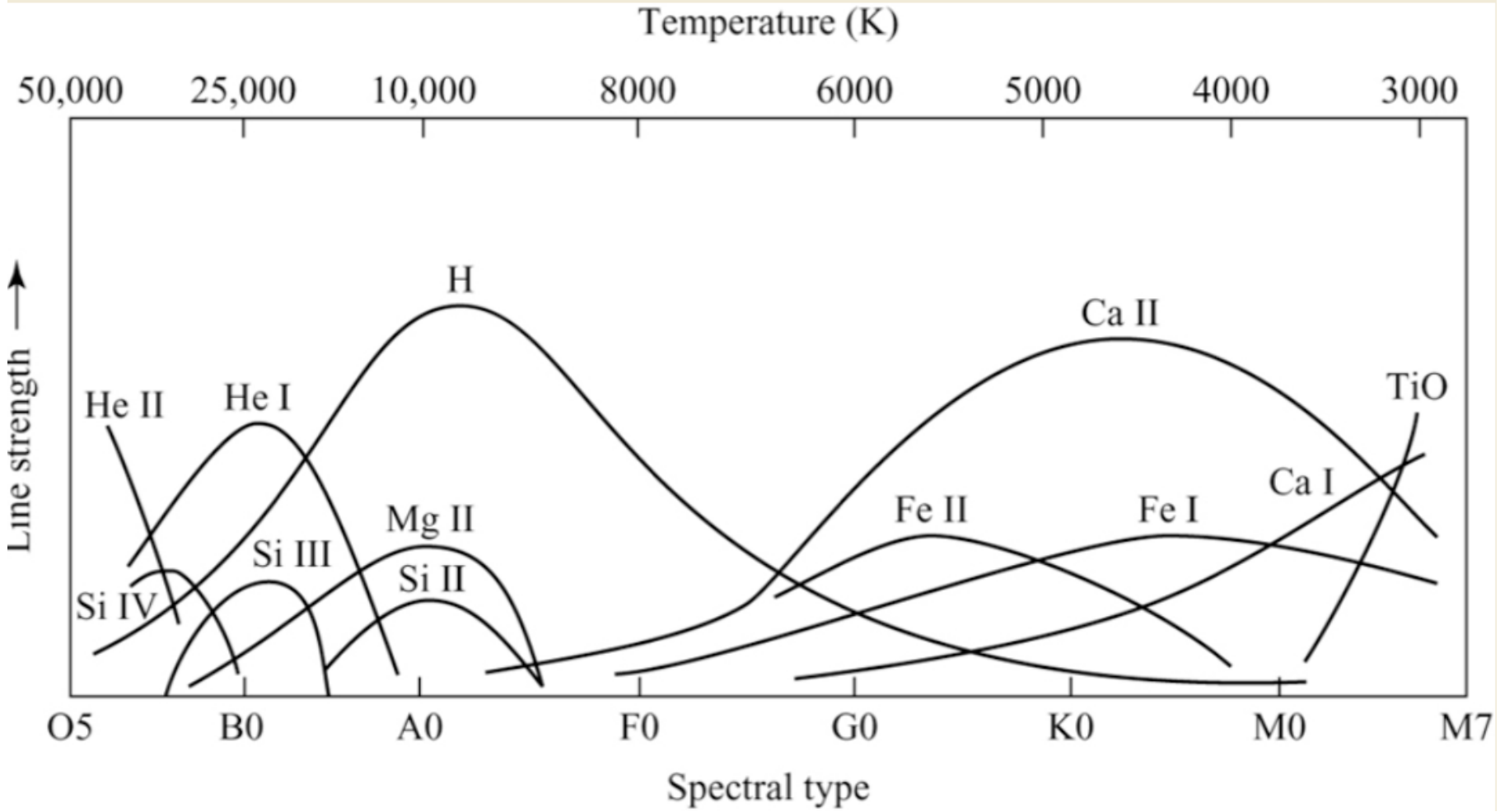
For very cool stars (M, L, T type) the atmospheres are sufficiently cool that simple molecules can form. These can absorb not only in electronic transitions, but also in vibrational and rotational modes. These create "bands" of absorption which can reduce the flux in vast portions of the spectrum. In M stars, TiO is a common important molecule. In L and T stars, other molecules such as CO, H_2O and CH_4 become important.



Relative Strength of Spectral Lines



Relative Strength of Spectral Lines



Measuring temperatures and surface gravities

DIRECT MEASUREMENT OF RADII
DETERMINING T_{EFF} AND SURFACE GRAVITY
MODEL-INDEPENDENT METHODS
MODEL-DEPENDENT METHODS
ATMOSPHERIC MODELS
PHOTOMETRIC METHODS
SPECTROSCOPIC METHODS

Fundamental parameters

Stellar parameters:

- Luminosity (L)
- Mass (M)
- Radius (R)

In most cases, cannot be measured directly

Atmosphere parameters:

- Effective Temperature ($T_{\rm eff}$)
- Surface gravity ($\log g$)

Can help in measuring L & M

- Chemical composition (metallicity, element abundances)

~90% of stars in the Galaxy are "normal" (close to the Sun)

Surface Flux, Luminosity and T_{eff}

- Integral over frequency at outer boundary (Surface Flux):

$$F_s = \int_0^\infty F_\lambda d\lambda$$

- Multiplied by stellar surface area yields the Luminosity, total energy radiated away by the star

$$\boldsymbol{L = 4\pi R^2 F_s}$$

- The total energy arriving above the Earth's atmosphere is its observed flux, $F_\oplus$, corrected for the distance to the star, neglecting interstellar absorption:

$$L = 4\pi d^2 F_\oplus \quad \rightarrow \quad F_s = F_\oplus (d/R)^2$$

- The Stefan-Boltzmann law, $F = \sigma T^4$, or alternatively $\boldsymbol{L/4\pi R^2 = \sigma T^4}$ defines the "effective temperature" of a star, i.e. the temperature which a black body would need to radiate the same amount of energy as the star.

Model-independent methods

Direct measurements:

$f_\oplus$ – the flux measured at the Earth ($F_\oplus$ - bolometric flux at the Earth)
F_S – the flux emitted from the stellar surface
d – the distance from us to the star
R – the radius of the star
θ – the angular radius of the star, R/d

Example:
$d = 1.3$ pc, $R = 700000$ km
$\theta = 0.004$ arcsec !!

$$4\pi d^2 F_\oplus = 4\pi R^2 F_S$$

We can relate this equation to the effective temperature

$$F_\oplus = \int_0^\infty f_\oplus(\nu)\, d\nu = \theta^2 \sigma T_{eff}^4$$

If θ is measured and the distance d is known, e.g. from parallax (Gaia, Hipparcos, etc.), then we can obtain R and L.

Inteferometric radii

- We have already introduced interferometry with regard to limb darkening (Lecture 4).
- Several ground-based optical and IR interferometers are currently in operation.
- Reliable diameters generally restricted to nearby late-type giants with large angular radii on the sky.
- ~200 star radii are measured with an accuracy better than 10%

- **VLTI** (Paranal, Chile): currently the most advanced optical/IR interferometer in operation. Combines large apertures of individual 8m VLT telescopes with dedicated auxiliary 1.8m telescopes.

The New Set at Paranal - The VLT, the VST Dome and the AT1

ESO PR Photo 02d/04 (30 January 2004) © European Southern Observatory

The AT1 Positioned Next to the VLTI Laboratory

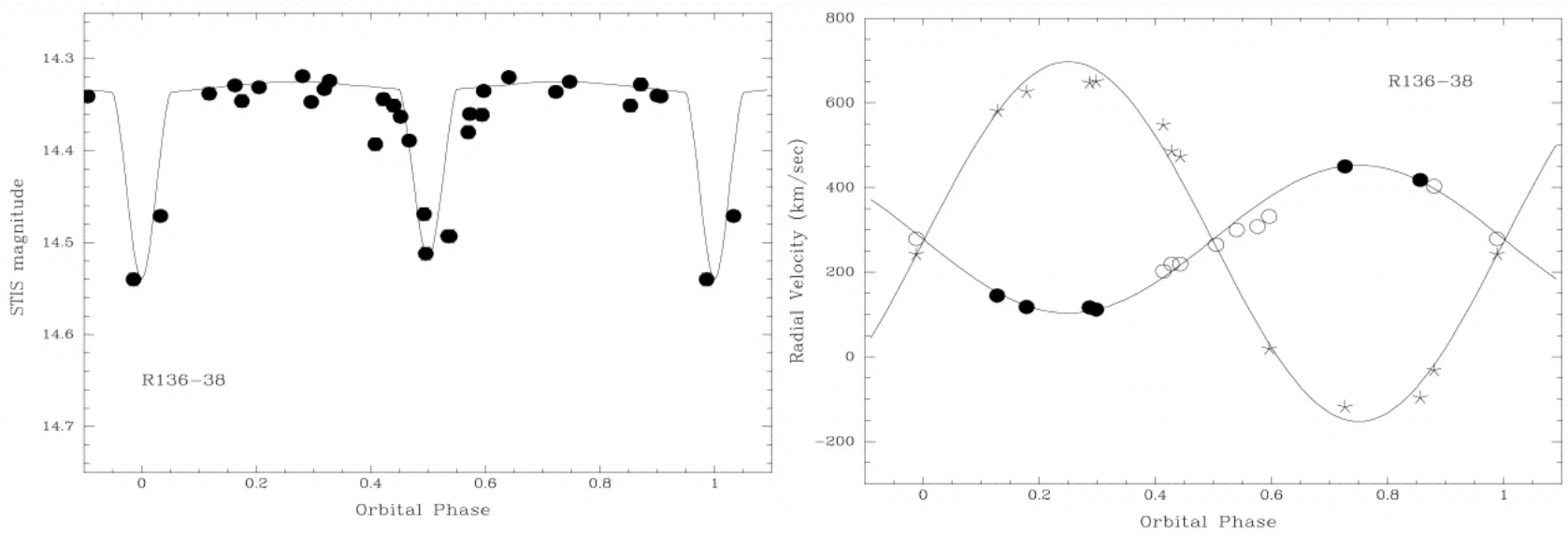
ESO PR Photo 02b/04 (30 January 2004) © European Southern Observatory

Radii from other direct methods

- **Occultations**
  - Moon used as “knife-edge”
  - Diffraction pattern recorded as flux vs. time
  - Precision ~ 0.5 mas
  - A few hundred radii have been determined
- **Eclipsing binaries**
  - Photometry gives ratio of radii to semi-major axes. Useful simulation at http://www.midnightkite.com/binstar/StarLightPro.exe
  - Velocities from spectra give semi-major axes (i=90°)

Binary Masses

Accurate radii and masses can be obtained from analysis of photometric light curves (R, i) & spectroscopic orbit information ($M \sin^3 i$). If eclipsing $i \cong 90°$ → R, M.



R136-38 (Massey et al. 2002, ApJ 565, 982) light curve analysis of O3V+O6V in LMC (P=3.4 day): 9.3$R_\odot$ (primary) 6.4$R_\odot$ (secondary)

Model-independent methods

Direct measurements:

$$4\pi d^2 F_\oplus = 4\pi R^2 F_S \qquad g = \frac{GM}{R^2}$$

$$F_\oplus = \int_0^\infty f_\oplus(\nu)\, d\nu = \theta^2 \sigma T_{eff}^4$$

Difficult to reliably measure $F_\oplus$ because of interstellar absorption in UV (especially beyond the Lyman continuum)

Model-dependent methods

Using, e.g., model atmospheres and/or theoretical evolution tracks.

- T_{eff} from Bolometric Corrections
  - Lecture 11
- Log g from parallaxes

$$\log g/g_{\odot} = \log \mathcal{M}/\mathcal{M}_{\odot} + 4\log T_{\rm eff}/{\rm T}_{,\odot} + 0.4({\rm M_{bol}} - {\rm M_{bol,\odot}})$$

- Method of IR fluxes (Blackwell & Shallis 1977)

$$4\pi d^2 F_{\oplus} = 4\pi R^2 F_{*} \quad \rightarrow \quad \frac{F_{\oplus}}{F_S} = \frac{R^2}{d^2} = \theta^2 = \frac{f_{\oplus}}{f_S}$$

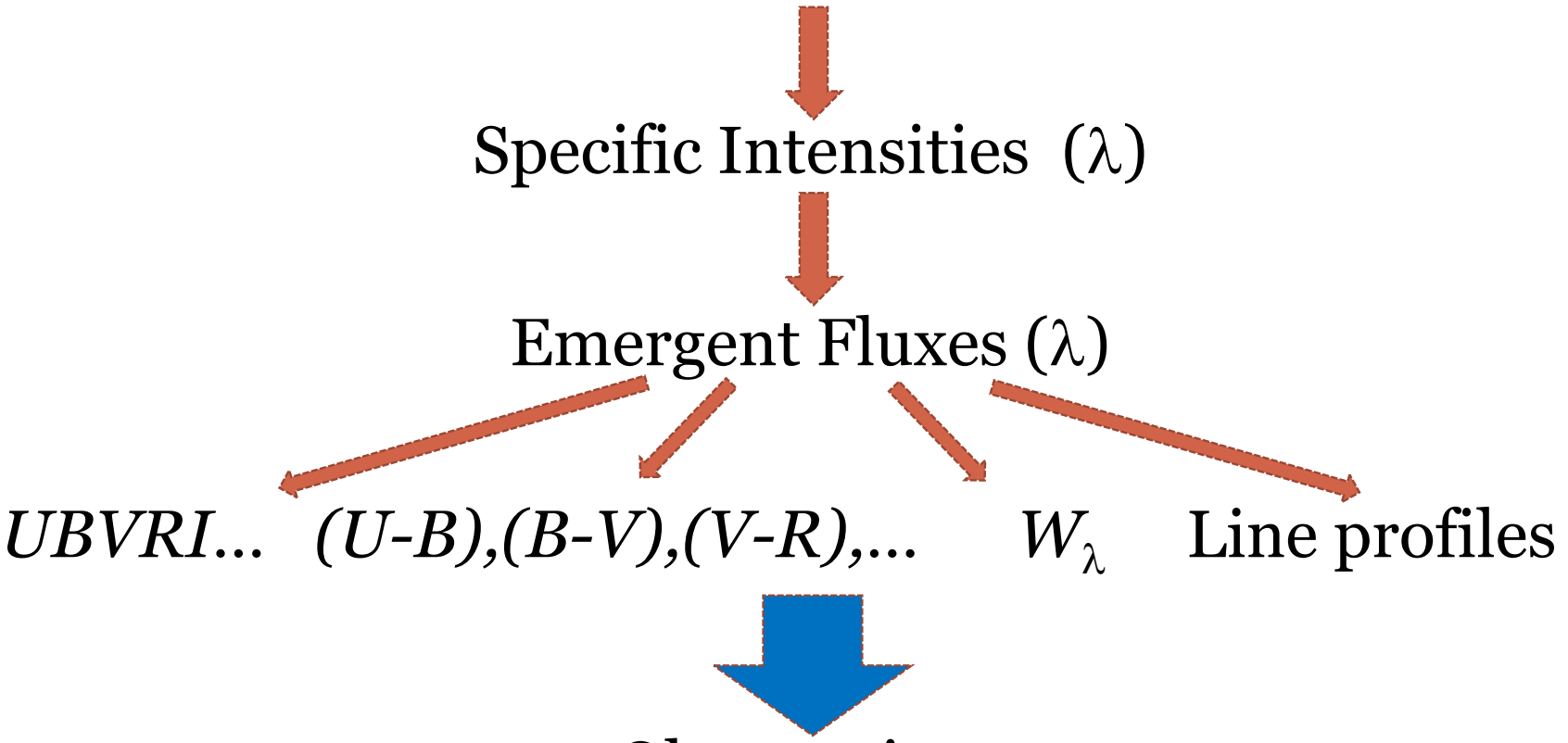
$$T_{eff}^4 = \frac{F_{\oplus}}{\theta^2 \sigma}$$

Also correct for monochromatic fluxes

Alonso et al. : T_{eff}(IRFM) for 1000+ stars

Atmospheric Models

Model atmospheres ($T_{\rm eff}$, log g, chem. composition)



Atmospheric Models

- For most stars, Kurucz LTE atmosphere models, accounting for `line blanketing' from metals, generally suffice **http://kurucz.harvard.edu/grids.html**
- For early-type stars, several non-LTE line blanketed models exist: TLUSTY http://tlusty.oca.eu for plane-parallel O stars, or for O stars with extended atmospheres WMbasic http://www.usm.uni-muenchen.de/people/adi/Programs/Programs.html.
- For very late-type stars, opacity from molecules are important e.g. PHOENIX http://phoenix.phyast.uga.edu/

Kurucz models

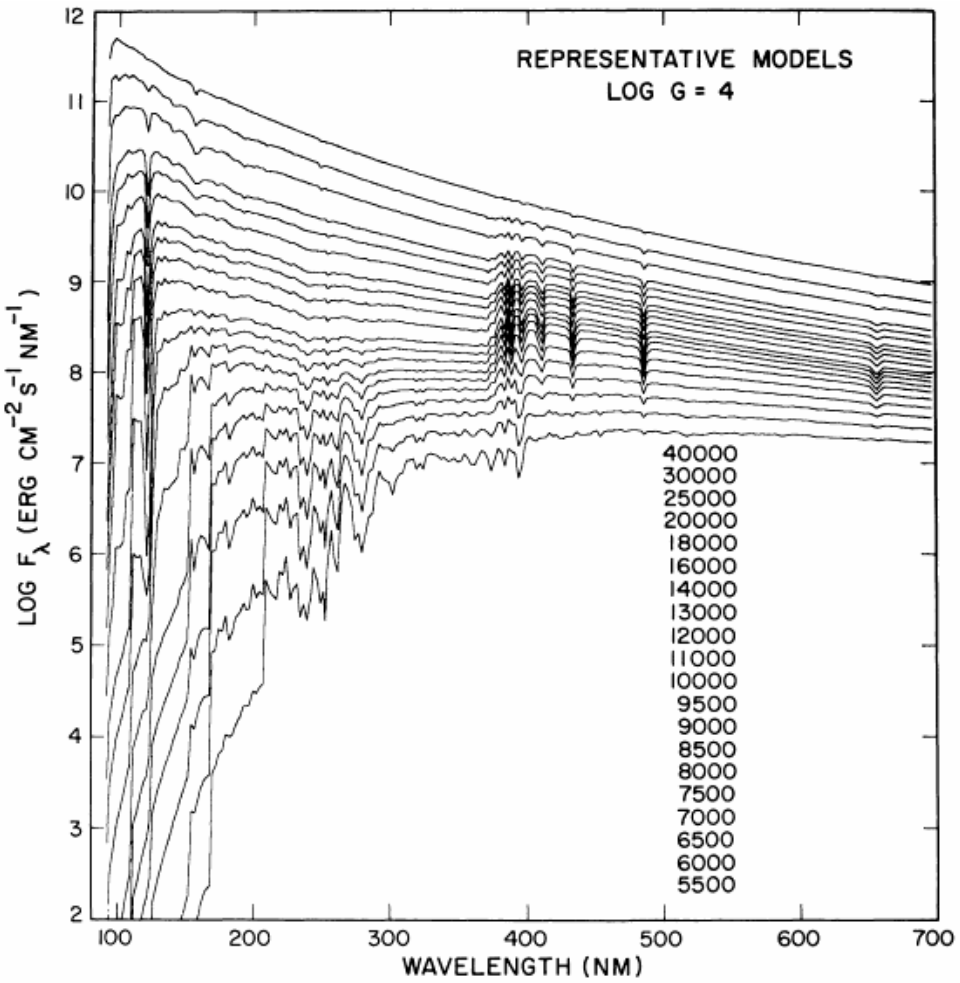


FIG. 16.—The flux F_λ redward of the Lyman limit as a function of T_{eff}

Atmospheric Models

- To determine T_{eff} and log g, one has to use spectral characteristics which are insensitive to chem. composition.
- At least one parameter should have a stronger dependence on T_{eff} than on log g, and another one in the opposite way.
- The more parameters the better.

- If T_{eff} is fixed then $g = \frac{GM}{R^2} = \frac{4\pi GM\sigma T_{eff}^4}{L}$ → $L \sim \frac{M}{g} T_{eff}^4$

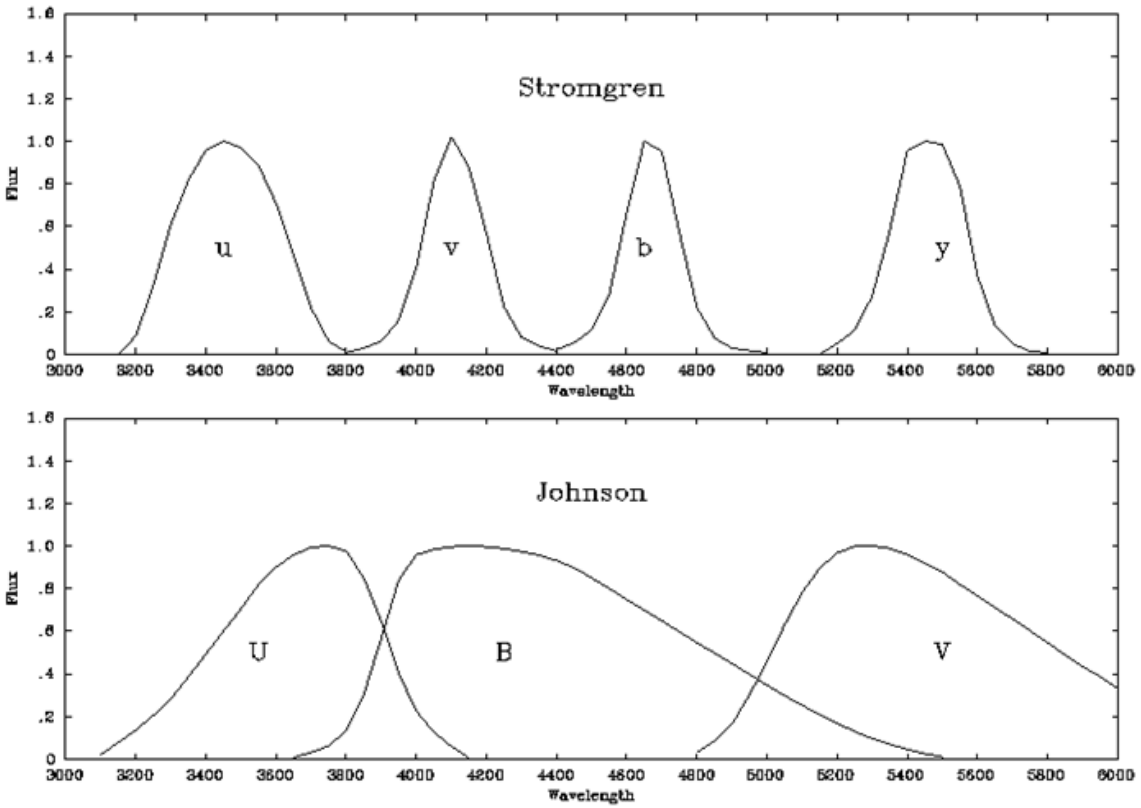
  using $L \sim M^n$, we get $g \sim M^{(1-n)}$

g – a luminosity criterium

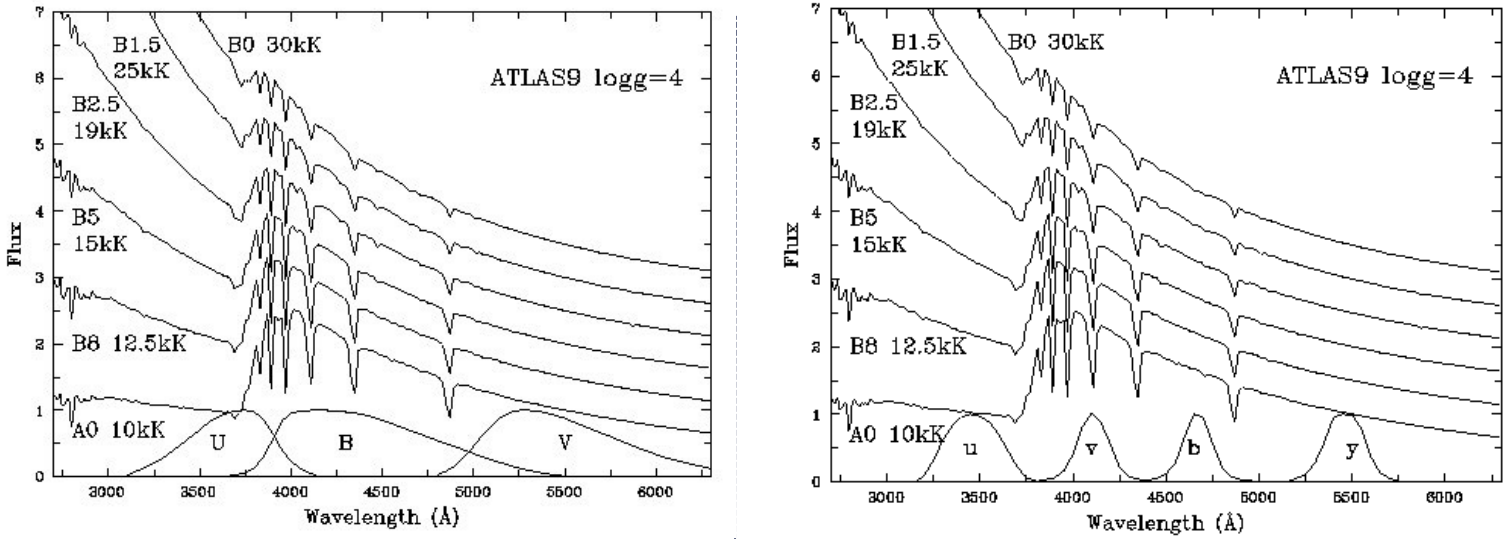
Photometric Methods

Alternative photometric systems to Johnson *UBV* are available – notably Strömgren (1963) *ubvy*.

These are narrower filters and are rather more useful in extracting T_{eff} and log *g* than *UBV*.



UBV versus uvby photometry



A comparison of synthetic Kurucz models for the Balmer jump in B dwarfs with the usual Johnson *UBV* filters (left) and Stromgren *ubvy* filters (right). The *U* filter is sensitive to radiation on both sides of the discontinuity, whilst the narrow Stromgren *u* filter samples light below 3647 & *v* filter samples light above, so the *u*-*v* colour provides T_{eff}.

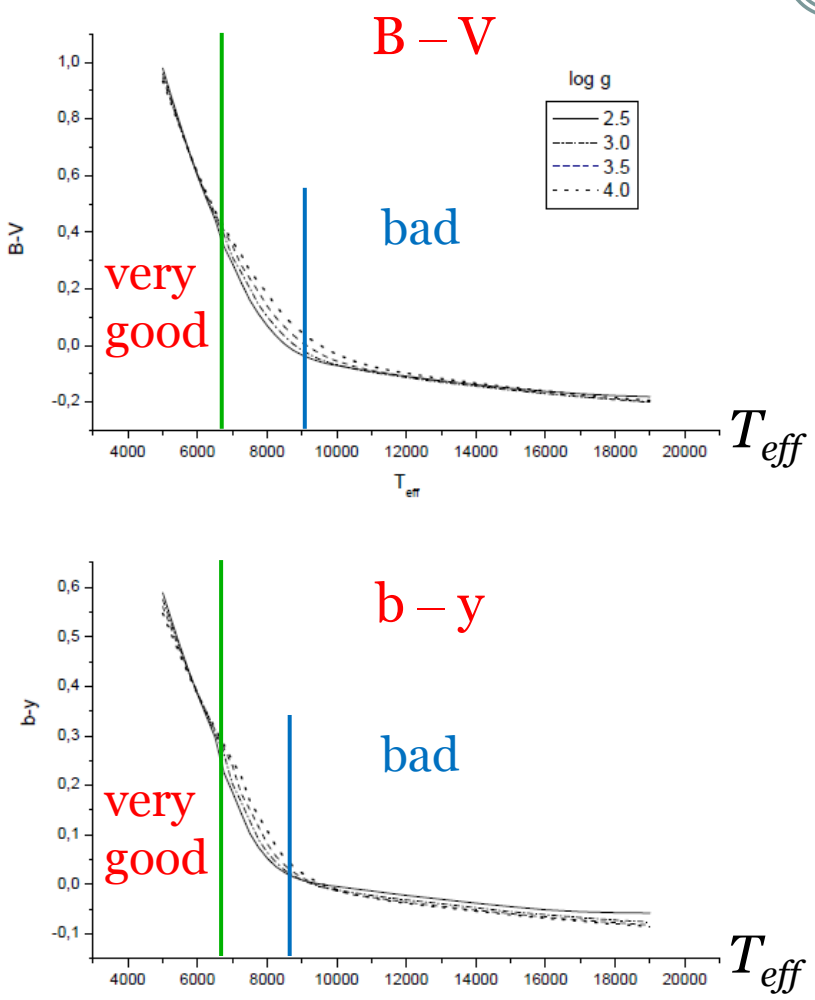
Photometric Methods

T_{eff} from photometry

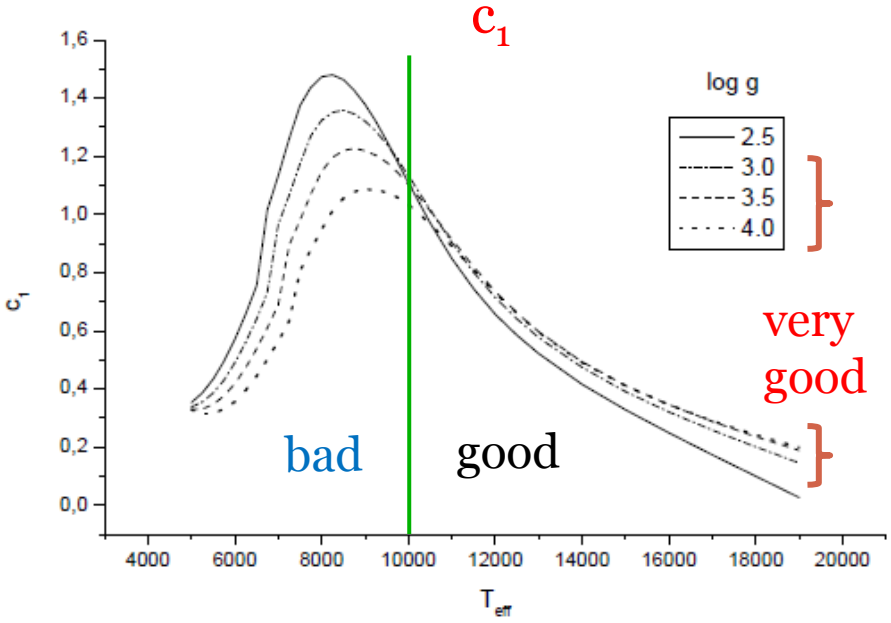
- The slope of the Paschen continuum, F_{4000}/F_{7000},
- $c_1 = (u - v) - (v - b)$ for A0 stars and earlier
- $b - y$, $B - V$, $V - K$ for F stars and later

$\Big\}$ $f(T_{eff})$

Photometric Methods



Kurucz atmosphere models

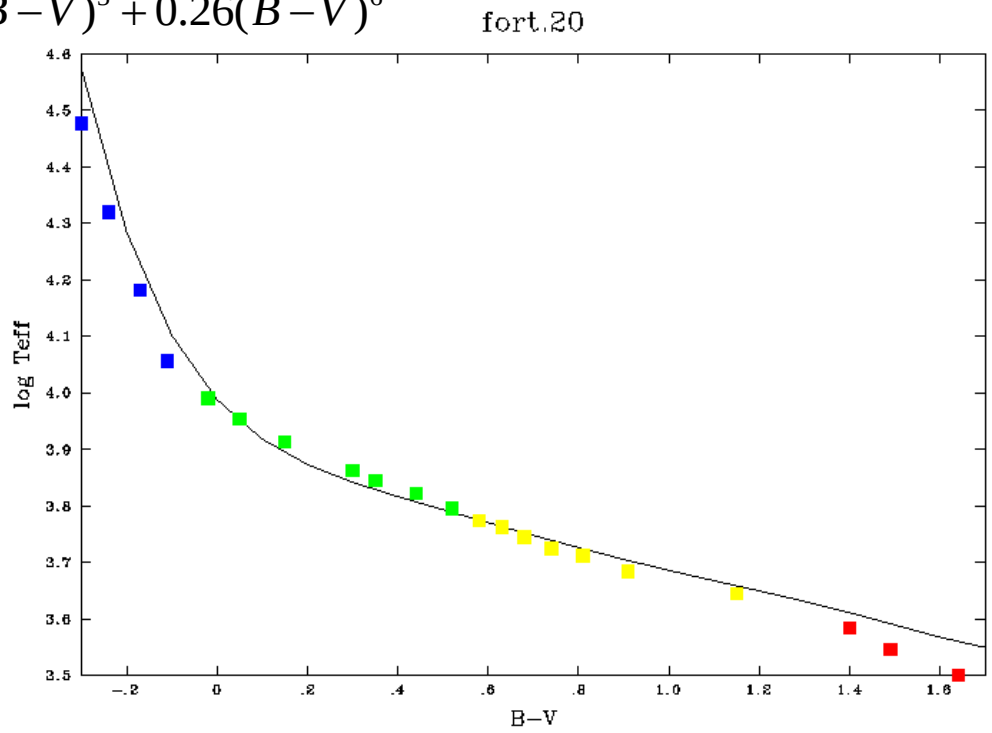


Temperatures from photometry

Observed *B-V* colour index generally allows T for normal stars (0<B-V<1.5).

$$\log T_{eff} = 3.988 - 0.881(B-V) + 2.142(B-V)^2 - 3.614(B-V)^3$$
$$+ 3.2637(B-V)^4 - 1.4727(B-V)^5 + 0.26(B-V)^6$$

Beyond this range most flux is originating in the UV or IR so B-V becomes insensitive to temperature.



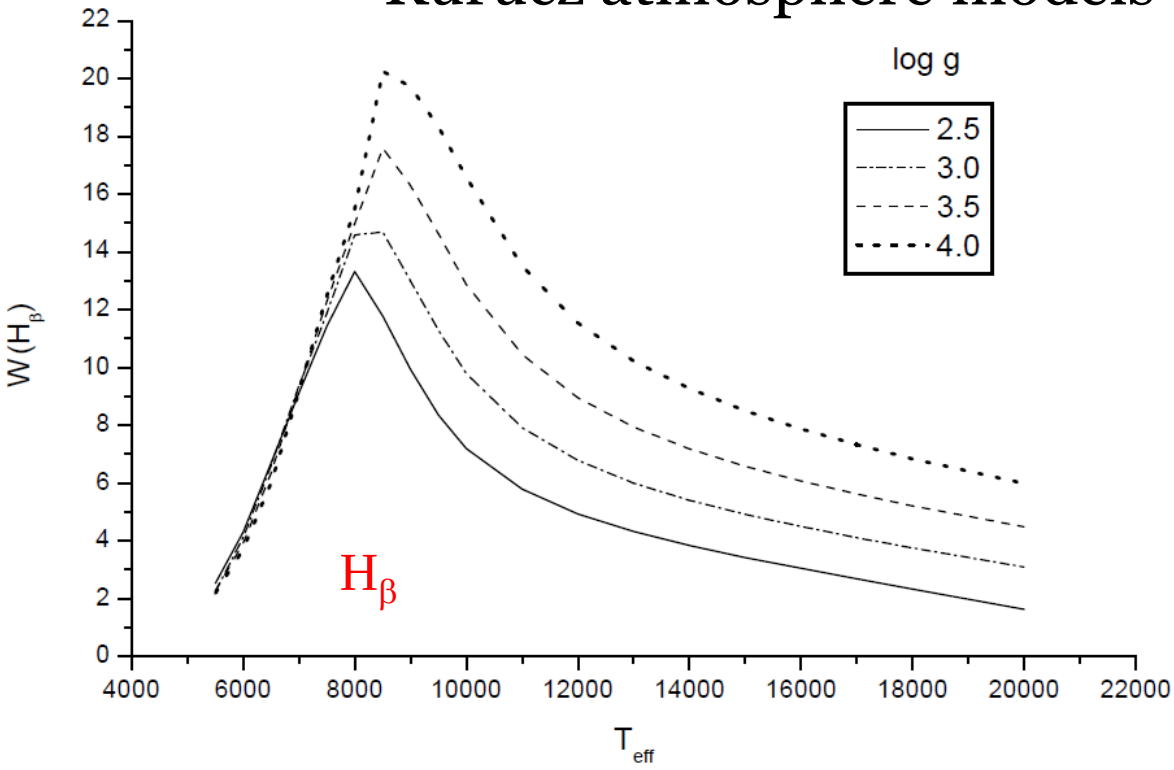
Spectroscopic methods

Equivalent Widths of Balmer lines

Good indicators of T_{eff} when T_{eff}<9000 K

If T_{eff} is higher, then indicators of log g.

Kurucz atmosphere models



Spectroscopic methods

Wings of strong metal lines

Broadening:

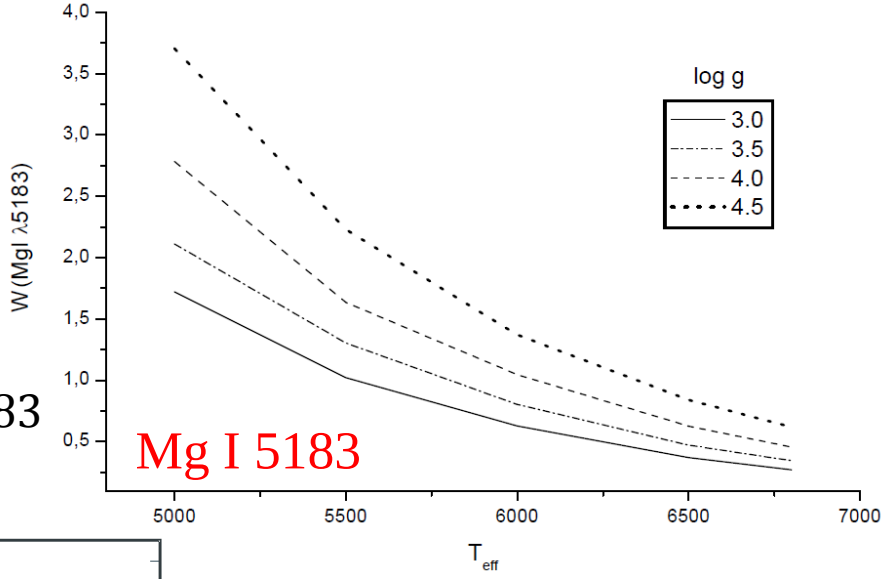
Cool stars - **Van der Waals**

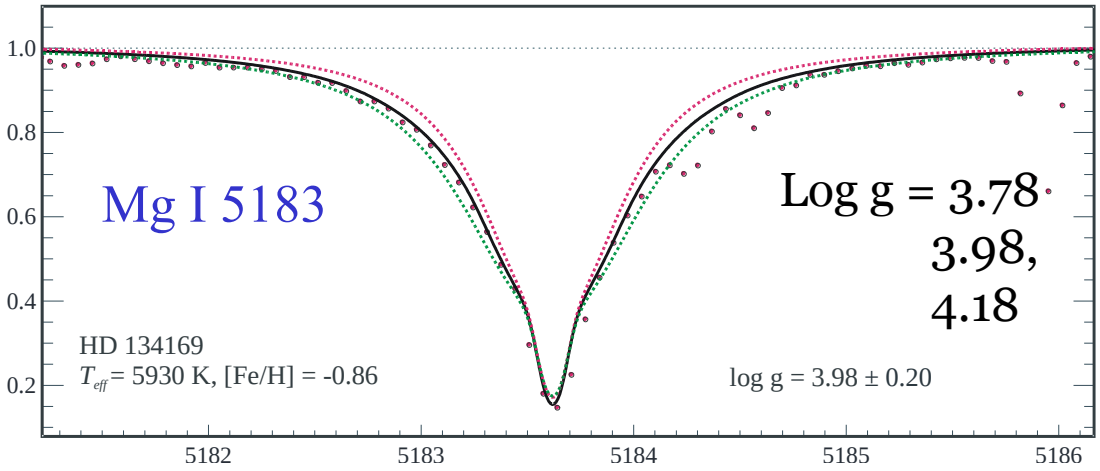
Hot stars - **Quadratic Stark**

Good indicators of g:

G, K, M stars: Na D, Ca I 4226, Mg I 5172, 5183

F, A, B stars: resonance lines Ca II, Mg II





For example, Mg I 5183 is a good indicators of log g when T_{eff}<6000 K.

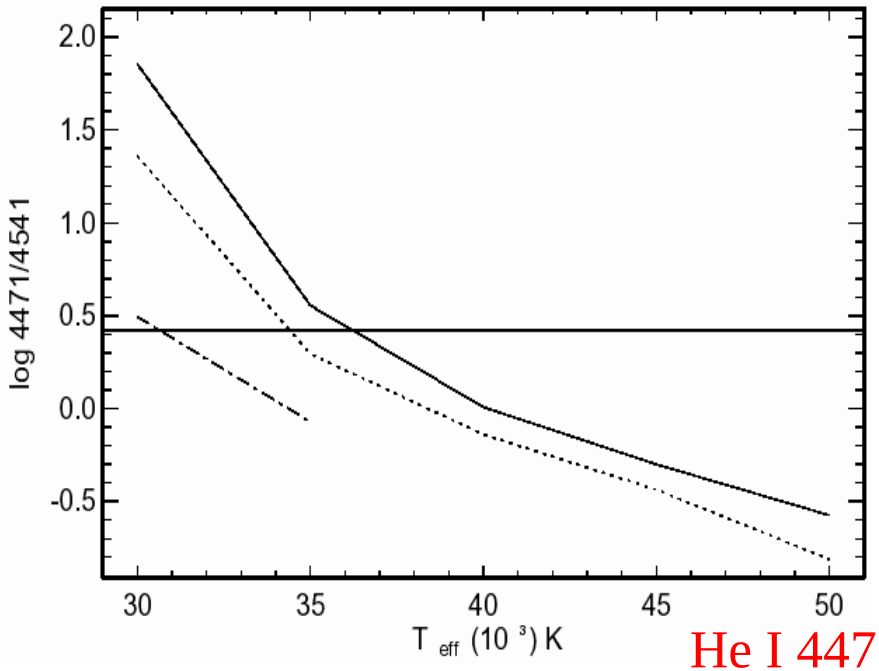
Spectroscopic methods

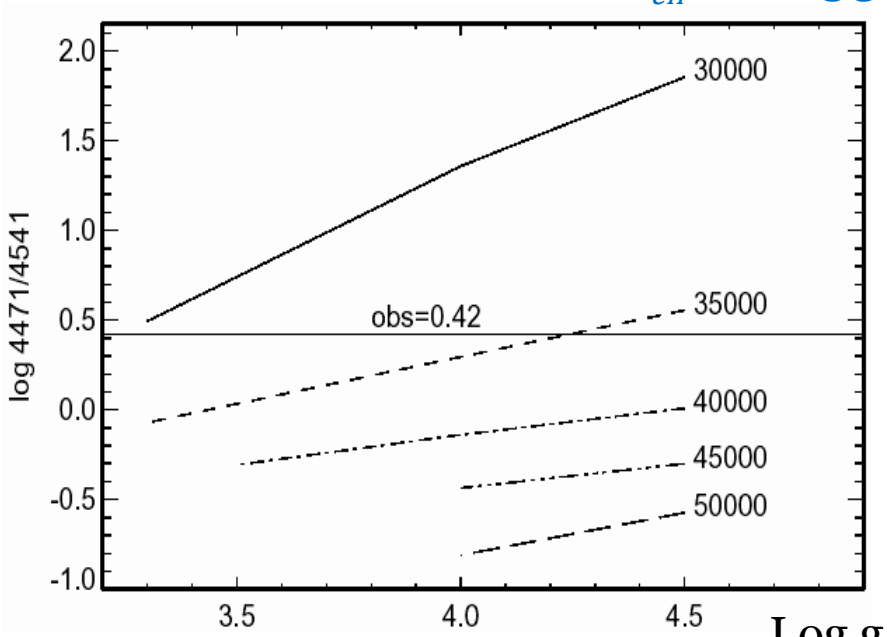
Equivalent Width ratios of species in two consecutive ionization states

G, K, M stars: Fe I and Fe II

O, B stars: He I and He II, Si III and Si IV

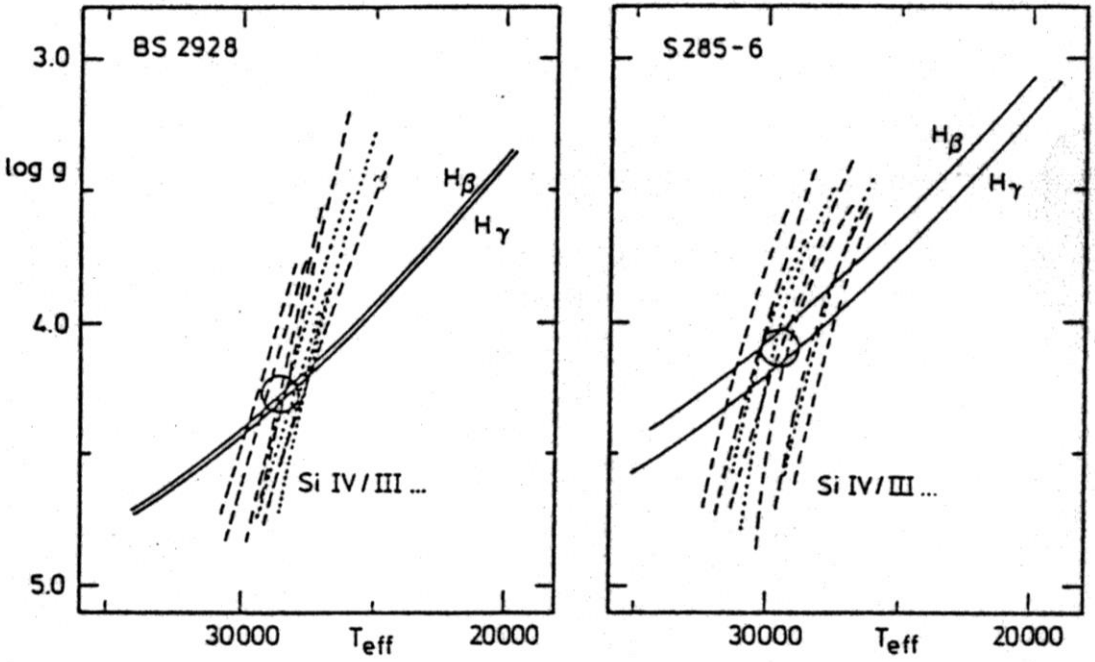
He I 4471/ He II 4511 can be an indicator of both T_{eff} and log g

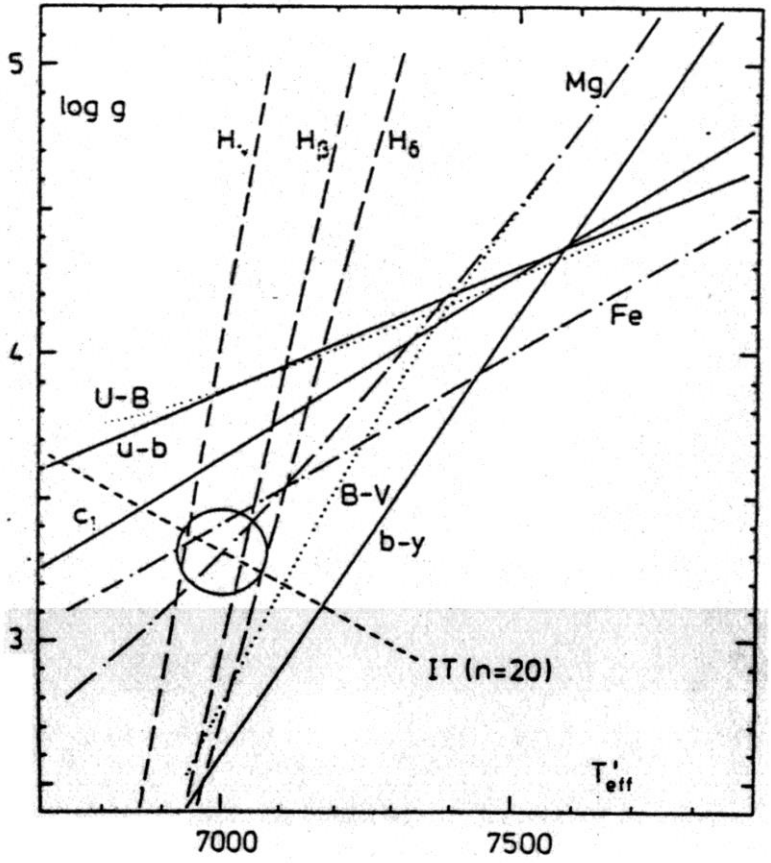




He I 4471/ He II 4511

Examples





The different criteria for determining T_{eff} and $\log g$ are collected in the corresponding parameter plane with the final stellar parameters obtained from the mean intersection point