Stellar Atmospheres

Lecture 17

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Spectral line formation

SCATTERING IN LINES
TRANSFER EQUATION INCLUDING LINES
THE MILNE-EDDINGTON MODEL
RESIDUAL FLUX OF THE LINE
ABSORPTION AND SCATTERING LINES
SCHUSTER MECHANISM FOR LINE EMISSION

Recap from last lecture

Simple line transfer:

The total absorption coefficient within an arbitrary line is the sum of the line (α_L) and continuum (α_C) contributions i.e. $\alpha_\lambda = \alpha_L + \alpha_C$ as is the total emission coefficient ($\varepsilon_\lambda = \varepsilon_L + \varepsilon_C$). Hence,

$$S_\lambda = (\varepsilon_L + \varepsilon_C)/(\alpha_L + \alpha_C)$$

and

$$d\tau_\lambda = -(\alpha_L + \alpha_C)\, dz \qquad \tau_\lambda = \tau_L + \tau_C$$

So we can write the transfer equation as usual: $\cos\theta \dfrac{dI_\lambda(\theta)}{d\tau_\lambda} = I_\lambda(\theta) - S_\lambda$

The surface specific intensity and surface flux, are obtained as previously.

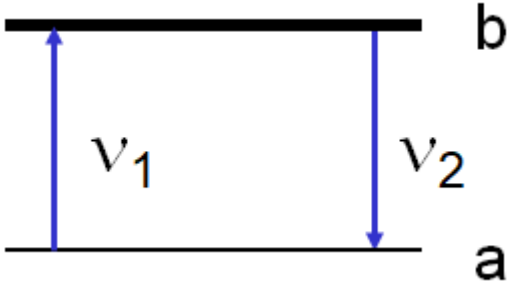
$$I_\lambda(0,\theta) = \int_0^\infty S_\lambda(\tau_\lambda) e^{-\tau_\lambda \sec\theta} \sec\theta \, d\tau_\lambda$$

$$F_\lambda(0) = 2\pi \int_0^1 I_\lambda(0,\theta)\mu d\mu \qquad \mu = \cos\theta$$

Again we need to know $S(\tau)$ to evaluate these integrals.

Scattering in lines

- Special case:
  Coherent scattering: $\nu_1 = \nu_2$
- Common case:
  2-level atom absorbs photon with frequency ν_1, re-emits photon with frequency ν_2; frequencies not exactly equal, because
  - levels a and b have non-vanishing energy width
  - Doppler effect because atom moves
- Non-coherent scattering requires a redistribution function



Transfer Equation including lines

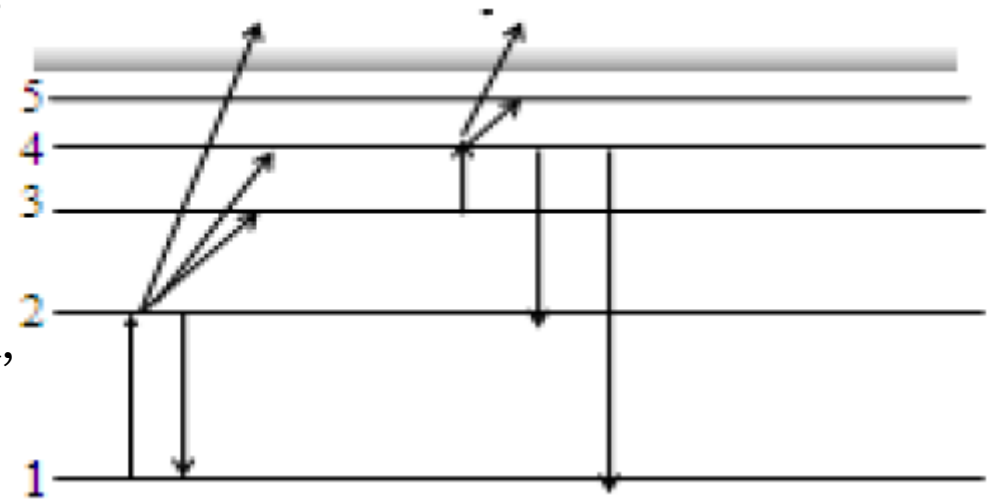
Classical approach:

absorption of photons by line has two parts

1. (**1-ζ**) of absorbed photons are scattered
   (e- returns to original state)
2. **ζ** of absorbed photons are destroyed
   (into thermal energy of gas)
   (for LTE: ζ =1)

Resonance lines (to/from the ground level)

A photon 1→2 returns back to the radiation field, thus dominates **Scattering**



Subordinate lines (to/from higher levels)

A photon 3→4 disappears,

thus dominates **True absorption**

Scattering

- Pure Absorption and Thermal Emission:

$$S(\tau) = \frac{\epsilon}{\alpha} \qquad \text{LTE: } \epsilon_{th} = \alpha_{th} B(\tau)$$

- Pure Scattering:
  For the case of pure scattering, the associated emission becomes completely insensitive to the thermal properties of the gas, and instead depends only on the local radiation field. If the scattering is roughly isotropic, the scattering emissivity ϵ_{sc} in any direction depends on both the opacity and the angle-averaged mean-intensity $\epsilon = \kappa_{sc} \rho J = \alpha_{sc} J$
  This implies then that, for pure-scattering,

$$S(\tau) = J(\tau)$$

- Source Function for Scattering and Absorption:
  The total opacity consists of both scattering and absorption, $\alpha \equiv \alpha_{abs} + \alpha_{sc}$
  The total emissivity likewise contains both thermal and scattering components $\epsilon = \epsilon_{th} + \epsilon_{sc} = \alpha_{th} B + \alpha_{sc} J$. The general source function

$$S(\tau) = \zeta B + (1 - \zeta) J(\tau)$$

absorption fraction $\zeta \equiv \dfrac{\alpha_{abs}}{\alpha_{abs} + \alpha_{sc}}$

The Milne-Eddington model

Consider a case where at the given frequency the total opacity is a combination of both continuum and line processes:

Total absorption coefficient is $\alpha_\nu = \alpha_\nu^C + \alpha_\nu^L + \sigma$ ← scattering in the continuum

$= \alpha_\nu \times \phi_\nu =$ line opacity × line profile

The total optical depth is $d\tau_\nu = -(\alpha_\nu^C + \alpha_\nu^L + \sigma)\, ds$

(larger than in the continuum!)

The correponding emissivities $\epsilon_\nu = \epsilon_\nu^C + \epsilon_\nu^L + \sigma J_\nu$

$$\mu \frac{dI_\nu(\mu)}{d\tau_\nu} = I_\nu(\mu, \tau_\nu) - S_\nu(\tau_\nu)$$

Recall radiative transfer equation

$$\mu \frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + \epsilon_\nu$$

The Milne-Eddington model

Transfer equation:

$$\epsilon = \epsilon_{th} + \epsilon_{sc} = \alpha_{th} B + \alpha_{sc} J$$

-absorbed +thermal +scattered

$$\mu \frac{dI_\nu}{ds} = -(\alpha_\nu^C + \alpha_\nu^L + \sigma) I_\nu + \epsilon_\nu^C + \sigma J_\nu + \zeta \alpha_\nu^L B_\nu + (1-\zeta)\alpha_\nu^L J_\nu$$

+therm. line em. +scat. line emission (coherent)

Without dealing with the general case for the computation of all coefficients we assume:

- LTE in the continuum $\epsilon_\nu^C = \alpha_\nu^C B_\nu(T)$
- scattering negligible in the continuum $\sigma \ll \alpha_\nu^C$

The Milne-Eddington model

$$\mu \frac{dI_\nu}{ds} = -(\alpha_\nu^C + \alpha_\nu^L) I_\nu + \alpha_\nu^C B_\nu + \zeta \alpha_\nu^L B_\nu + (1-\zeta)\alpha_\nu^L J_\nu$$

Using $\beta_\nu \equiv \frac{\alpha_\nu^L}{\alpha_\nu^C}$ $\qquad d\tau_\nu = -(\alpha_\nu^C + \alpha_\nu^L)\, ds = -\alpha_\nu^C (1+\beta_\nu)\, ds$

$$\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - B_\nu \frac{1+\zeta\beta_\nu}{1+\beta_\nu} - \frac{(1-\zeta)\beta_\nu}{1+\beta_\nu} J_\nu = I_\nu - \lambda_\nu B_\nu - (1-\lambda_\nu)\, J_\nu$$

destruction probability → $\lambda_\nu \equiv \frac{1+\zeta\beta_\nu}{1+\beta_\nu}$

$$\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - \lambda_\nu B_\nu - (1-\lambda_\nu)\, J_\nu$$

Milne-Eddington Equation. Solve at each frequency point across profile.

The Milne-Eddington model

$$\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - \lambda_\nu B_\nu - (1 - \lambda_\nu) J_\nu$$

Milne-Eddington assumptions (for analytical solution):

1. β_ν, λ_ν and ζ are constant with depth
2. B_ν is linear in continuum optical depth: $B_\nu = a + b\tau_c$

$$d\tau_c = \frac{d\tau_\nu}{1 + \beta_\nu} \qquad \tau_c = \frac{\tau_\nu}{1 + \beta_\nu}$$

Also, the Eddington approximation $K_\lambda(\tau_\lambda) = \frac{1}{3} J_\lambda(\tau_\lambda)$

Recap: Eddington approximation

11

Lecture 3

K-integral and radiation pressure

7

- K-integral is related to the radiation pressure: $K_\lambda = \frac{1}{4\pi}\oint I_\lambda \cos^2\theta \, d\omega$
- A photon has momentum $p_\lambda = E_\lambda / c$
- Consider photons transferring momentum to a solid wall. Force:

$$F = \frac{dp_{\lambda\perp}}{dt} = \frac{1}{c}\frac{dE_\lambda}{dt}\cos\vartheta$$

- Pressure: $dP_\lambda = \frac{F}{d\sigma} = \frac{1}{c}\frac{dE_\lambda \cos\vartheta}{dt \quad d\sigma} = \frac{1}{c} I_\lambda \cos^2\vartheta \; d\omega \, d\lambda$

$$I_\lambda = \frac{dE_\lambda}{\cos\theta \, d\lambda \, d\sigma \, d\omega \, dt}$$

$$P(\lambda) = \frac{1}{c}\oint_{4\pi} I_\lambda \cos^2\vartheta \; d\omega = \frac{4\pi}{c} K_\lambda$$

Lecture 5

Eddington approximation (1)

15

- Previously we have seen that for the determination of the flux **the anisotropy in the radiation field is very important** because in the flux integral the inward-going intensities are subtracted from the outward-going ones, due to the factor $\cos\theta$.
- But for ***K***, a small anisotropy is unimportant because the intensities are multiplied by the factor $\cos^2\theta$, which does not change sign for inward and outward radiation.
- In order to evaluate ***K*** or c_2 we can approximate the radiation field by an isotropic radiation field of the mean intensity ***J***: ***I*** = ***J*** (by definition). From the definition of K_λ we obtain

$$4\pi K_\lambda = \oint I_\lambda(\tau_\lambda, \theta) \cos^2\theta \, d\omega = J_\lambda(\tau_\lambda) \oint \cos^2\theta \, d\omega = \frac{4\pi}{3} J_\lambda(\tau_\lambda)$$

or after division by 4π,

$$K_\lambda(\tau_\lambda) = \frac{1}{3} J_\lambda(\tau_\lambda)$$

This approximation for the K function is known as the *Eddington approximation*.

Recap: Moments of intensity

- The *mean* intensity J_λ is the directional average (over 4π steradians) of the specific intensity [0-th moment of intensity]:

$$J_\lambda \equiv \frac{1}{4\pi}\oint I_\lambda d\omega = \frac{2\pi}{4\pi}\int_{-1}^{1} I(\mu)\,d\mu = \frac{1}{2}\int_{-1}^{1} I(\mu)\,d\mu$$

- **Eddington** flux $H_{\lambda,,}$ is the directional average (over 4π steradians) of the projection of the specific intensity [1st moment of intensity]:

$$H_\lambda = \frac{1}{4\pi}\oint I_\lambda \cos\theta\, d\omega = \frac{2\pi}{4\pi}\int_{-1}^{1} I(\mu)\,\mu\, d\mu = \frac{1}{2}\int_{-1}^{1} I(\mu)\,\mu\, d\mu$$

- K-integral [2nd moment of intensity] :

$$K_\lambda = \frac{1}{4\pi}\oint I_\lambda \cos^2\theta\, d\omega = \frac{2\pi}{4\pi}\int_{-1}^{1} I(\mu)\,\mu^2\, d\mu = \frac{1}{2}\int_{-1}^{1} I(\mu)\mu^2 d\mu$$

The Milne-Eddington model

$$\frac{1}{2}\int_{-1}^{+1} \ldots [\mu]\, d\mu \;\times$$

$$\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - \lambda_\nu B_\nu - (1-\lambda_\nu) J_\nu$$

$$\times \frac{1}{2}\int_{-1}^{+1} \ldots [\mu]\, d\mu$$

Multiply both sides by $d\mu$ and $\mu\, d\mu$ and integrate:

$$\frac{dH_\nu}{d\tau_\nu} = J_\nu - \lambda_\nu B_\nu - (1-\lambda_\nu) J_\nu = \lambda_\nu (J_\nu - B_\nu)$$

$$\frac{dK_\nu}{d\tau_\nu} = H_\nu = \frac{1}{3}\frac{dJ_\nu}{d\tau_\nu}$$

F_λ - astrophysical flux
H_λ - Eddington flux
$F_\lambda = \pi F_\lambda = 4\pi H_\lambda$

Differentiate again

$$\frac{d^2K_\nu}{d\tau_\nu^2} = \lambda_\nu (J_\nu - B_\nu) = \frac{1}{3}\frac{d^2J_\nu}{d\tau_\nu^2}$$

Eddington approximation

The Milne-Eddington model

$$\frac{1}{3}\frac{d^2 J_\nu}{d\tau_\nu^2} = \lambda_\nu (J_\nu - B_\nu)$$

B_ν is linear in τ, so zero second derivative $\frac{d^2 B_\nu}{d\tau_\nu^2} = 0$

$$\frac{1}{3}\frac{d^2 J_\nu}{d\tau_\nu^2} = \frac{1}{3}\frac{d^2 (J_\nu - B_\nu)}{d\tau_\nu^2} = \lambda_\nu (J_\nu - B_\nu)$$

This can be integrated to give

$$J_\nu - B_\nu = \mathcal{A} e^{-\sqrt{3\lambda_\nu}\tau_\nu} + \mathcal{B} e^{\sqrt{3\lambda_\nu}\tau_\nu}$$

Apply boundary condition at depth:

$$\tau_\nu \to \infty \quad \Rightarrow \quad J_\nu \to B_\nu \quad \Rightarrow \quad \boldsymbol{\mathcal{B} = 0}$$

The Milne-Eddington model

15

$J_\nu - B_\nu = \mathcal{A}$

Now app

From gre

Lecture 5

Eddington approximation (3)

17

- Boundary condition: there is no flux going into the star, i.e. $I(0,\theta)=I^-=0$ for $\pi/2<\theta<\pi$
- We also assume that the outward intensity does not depend upon θ, ie. $I(0,\theta)=I^+ = \text{const}$ for $0<\theta<\pi/2$
- It gives $J(0) = \frac{1}{2\pi} I^+ = \frac{1}{2\pi} F(0)$
- Hence C=$J(0)$= $F(0)/2\pi$ so:

$$S(\tau) = \frac{3}{4\pi}\tau F(0) + C = J(\tau)$$

$$S(\tau) = \frac{1}{\pi}\left(\frac{3}{4}\tau + \frac{1}{2}\right)F(0)$$

$$S(\tau) = \frac{3}{4\pi}\left(\tau + \frac{2}{3}\right)F(0)$$

- To find the depth dependence of T, we also need to assume LTE.

$q(\tau)$ is a slowly varying function (Hopf function), with $q = 1/\sqrt{3}$ at $\tau=0$

The Milne-Eddington model

$$J_\nu - B_\nu = \mathcal{A} e^{-\sqrt{3\lambda_\nu}\tau_\nu} + \mathcal{B} e^{\sqrt{3\lambda_\nu}\tau_\nu}$$

Now apply boundary condition at at surface:

$$\tau_\nu = 0 \quad \Rightarrow \quad J_\nu = B_\nu + \mathcal{A}$$

From grey atmosphere solution (Lecture 5), get $J(\tau=0)$:

Eddington approximation (2)

- Inserting the Eddington approximation into the above equation $\dfrac{dK(\tau)}{d\tau} = \dfrac{F(\tau)}{4\pi}$ we find

$$\frac{dK(\tau)}{d\tau} = \frac{1}{3}\frac{dJ(\tau)}{d\tau} = \frac{F(\tau)}{4\pi} = c_1 \qquad \frac{dJ(\tau)}{d\tau} = \frac{3}{4\pi}F(\tau)$$

The Milne-Eddington model

$$J_\nu - B_\nu = \mathcal{A} e^{-\sqrt{3\lambda_\nu}\tau_\nu} + \mathcal{B} e^{\sqrt{3\lambda_\nu}\tau_\nu}$$

Now apply boundary condition at at surface:

$$\tau_\nu = 0 \quad \Rightarrow \quad J_\nu = B_\nu + \mathcal{A}$$

From grey atmosphere solution (Lecture 5), get $J(\tau=0)$:

$$J(\tau) = \frac{3}{4\pi}\left[\tau + q(\tau)\right]F(0) = 3H\left(0 + \frac{1}{\sqrt{3}}\right) = \sqrt{3}H$$

$$\frac{1}{3}\frac{dJ_\nu}{d\tau_\nu}\bigg|_{\tau_\nu=0} = H_\nu(0) = \frac{1}{\sqrt{3}}J_\nu(0)$$

From $B_\nu = a + b\tau_c$ $\quad J_\nu(\tau_c = 0) = B_\nu + \mathcal{A} = a + \mathcal{A} = \frac{1}{\sqrt{3}}\frac{dJ_\nu}{d\tau_\nu}\Big|_{\tau_\nu=0}$

The Milne-Eddington model

$$J_\nu = B_\nu + \mathcal{A} e^{-\sqrt{3\lambda_\nu}\tau_\nu} = a + b\tau_c + \mathcal{A} e^{-\sqrt{3\lambda_\nu}\tau_\nu}$$

$$\left.\frac{1}{\sqrt{3}}\frac{dJ_\nu}{d\tau_\nu}\right|_{\tau_\nu=0} = \frac{1}{\sqrt{3}}\left[-\mathcal{A}\sqrt{3\lambda_\nu} + \frac{b}{1+\beta_\nu}\right] = a+\mathcal{A}$$

can now solve for $\mathcal{A}$!

$$\mathcal{A} = \frac{\frac{b}{1+\beta_\nu} - \sqrt{3}a}{\sqrt{3} + \sqrt{3\lambda_\nu}}$$

$$\tau_c = \frac{\tau_\nu}{1+\beta_\nu}$$

Define $p_\nu \equiv \frac{b}{1+\beta_\nu}$

$$J_\nu(\tau) = a + p_\nu\tau_\nu + \frac{p_\nu - \sqrt{3}a}{\sqrt{3} + \sqrt{3\lambda_\nu}} e^{-\sqrt{3\lambda_\nu}\tau_\nu}$$

The Milne-Eddington model

Thus, we obtained the fully analytic solution for the mean intensity

$$J_\nu(\tau) = \underbrace{a + p_\nu \tau_\nu}_{B_\nu} + \frac{p_\nu - \sqrt{3}a}{\sqrt{3} + \sqrt{3\lambda_\nu}} e^{-\sqrt{3\lambda_\nu}\tau_\nu}$$

Thermalization depth

$$\tau_\nu \gtrsim \frac{1}{\sqrt{\lambda_\nu}}$$

$J_\nu \rightarrow B_\nu$

$J_\nu < B_\nu$ in outer parts of atmosphere

We can use this to obtain the emergent flux

$$H_\nu(0) = \frac{1}{\sqrt{3}} J_\nu(0) = \frac{a}{\sqrt{3}} + \frac{p_\nu - \sqrt{3}a}{3(1 + \sqrt{\lambda_\nu})} = \frac{p_\nu + a\sqrt{3\lambda_\nu}}{3(1 + \sqrt{\lambda_\nu})}$$

Residual flux of the line

$$H_\nu(0) = \frac{p_\nu + a\sqrt{3\lambda_\nu}}{3(1+\sqrt{\lambda_\nu})}$$

$$B_\nu = a + b\tau_c \qquad \beta_\nu \equiv \frac{\alpha_\nu^L}{\alpha_\nu^C} \qquad \tau_c = \frac{\tau_\nu}{1+\beta_\nu}$$

$$p_\nu \equiv \frac{b}{1+\beta_\nu} \qquad \lambda_\nu \equiv \frac{1+\zeta\beta_\nu}{1+\beta_\nu}$$

Residual flux (relative intensity)

$$r_\nu = \frac{F_\nu}{F_c} = \frac{H_\nu(0)}{H_c(0)}$$

for continuum H_c: $\beta_\nu = 0 \quad \Rightarrow \quad p_\nu = b \qquad \lambda_\nu = 1$

$$H_c(0) = \frac{1}{3}\frac{(b + a\sqrt{3})}{2}$$

$$r_\nu = 2\frac{p_\nu + a\sqrt{3\lambda_\nu}}{(1+\sqrt{\lambda_\nu})(b+a\sqrt{3})}$$

Non-negligible scattering in continuum

$$H_\nu(0) = \frac{p_\nu + a\sqrt{3\lambda_\nu}}{3(1+\sqrt{\lambda_\nu})}$$

$B_\nu = a + b\tau_c \quad \beta_\nu \equiv \frac{\alpha_\nu^L}{\alpha_\nu^C} \quad \tau_c = \frac{\tau_\nu}{1+\beta_\nu}$

$$p_\nu \equiv \frac{b}{1+\beta_\nu} \qquad \lambda_\nu \equiv \frac{\zeta^C + \zeta^L\beta_\nu}{1+\beta_\nu}$$

for continuum H_c: $\beta_\nu = 0 \Rightarrow p_\nu = b \quad \lambda_\nu = \zeta^C$

without proof

$$H_c(0) = \frac{(b + a\sqrt{3\zeta^C})}{3(1+\sqrt{\zeta^C})}$$

$$r_\nu = \left(\frac{p_\nu + a\sqrt{3\lambda_\nu}}{b + a\sqrt{3\zeta^C}}\right)\left(\frac{1+\sqrt{\zeta^C}}{1+\sqrt{\lambda_\nu}}\right)$$

Various special cases

$$r_\nu = 2\frac{p_\nu + a\sqrt{3\lambda_\nu}}{(1+\sqrt{\lambda_\nu})(b+a\sqrt{3})}$$

$$B_\nu = a + b\tau_c \qquad \beta_\nu \equiv \frac{\alpha_\nu^L}{\alpha_\nu^C} \qquad \tau_c = \frac{\tau_\nu}{1+\beta_\nu}$$

$$p_\nu \equiv \frac{b}{1+\beta_\nu} \qquad \lambda_\nu \equiv \frac{1+\zeta\beta_\nu}{1+\beta_\nu}$$

This general result contains interesting behaviours in various special cases:

a) case $\zeta = 1$ (LTE: pure absorption lines)

b) case $\zeta = 0$ (extreme non-LTE: pure scattering lines)

c) Schuster Mechanism: Line Emission from Continuum Scattering Layer

$$r_\nu = \left(\frac{p_\nu + a\sqrt{3\lambda_\nu}}{b + a\sqrt{3\zeta^C}}\right)\left(\frac{1+\sqrt{\zeta^C}}{1+\sqrt{\lambda_\nu}}\right)$$

Pure absorption lines (LTE)

$$r_\nu = 2\frac{p_\nu + a\sqrt{3\lambda_\nu}}{(1+\sqrt{\lambda_\nu})(b+a\sqrt{3})}$$

$$B_\nu = a + b\tau_c \qquad \beta_\nu \equiv \frac{\alpha_\nu^L}{\alpha_\nu^C} \qquad \tau_c = \frac{\tau_\nu}{1+\beta_\nu}$$

$$p_\nu \equiv \frac{b}{1+\beta_\nu} \qquad \lambda_\nu \equiv \frac{1+\zeta\beta_\nu}{1+\beta_\nu}$$

a) pure absorption in line: $\zeta = 1$

$$\lambda_\nu \equiv \frac{1+\zeta\beta_\nu}{1+\beta_\nu} = 1 \quad \Rightarrow \quad r_\nu = \frac{p_\nu + a\sqrt{3}}{b+a\sqrt{3}} = \frac{\frac{b}{1+\beta_\nu} + a\sqrt{3}}{b+a\sqrt{3}}$$

For strong lines: $\beta_\nu \gg 1$

$$r_\nu = \frac{a\sqrt{3}}{b+a\sqrt{3}} = \frac{a}{b/\sqrt{3}+a} = \frac{B_\nu(\tau_\nu = 0)}{B_\nu(\tau_\nu = 1/\sqrt{3})} \neq 0$$

For grey atmosphere, strongest lines:

$$S_\lambda(\tau_\lambda) = \frac{3}{4\pi}(\tau_\lambda + \frac{2}{3})F_\lambda(0)$$

a/b=2/3 → $r_\nu \approx 0.54$

Non-zero because we see B_ν at upper level with non-zero temperature

Thus, in LTE, the residual flux is non-zero even for strong absorption lines. However, resonance lines such as Na D have R~10^{-3} – 10^{-4}

Pure scattering lines (extreme NLTE)

$$r_\nu = 2\frac{p_\nu + a\sqrt{3\lambda_\nu}}{(1+\sqrt{\lambda_\nu})(b+a\sqrt{3})}$$

$$B_\nu = a + b\tau_c \qquad \beta_\nu \equiv \frac{\alpha_\nu^L}{\alpha_\nu^C} \qquad \tau_c = \frac{\tau_\nu}{1+\beta_\nu}$$

$$p_\nu \equiv \frac{b}{1+\beta_\nu} \qquad \lambda_\nu \equiv \frac{1+\zeta\beta_\nu}{1+\beta_\nu}$$

b) pure scattering in line: $\zeta = 0$

$$\lambda_\nu \equiv \frac{1+\zeta\beta_\nu}{1+\beta_\nu} = \frac{1}{1+\beta_\nu}$$

$$r_\nu = 2\frac{\frac{b}{1+\beta_\nu} + a\sqrt{\frac{3}{1+\beta_\nu}}}{(1+\sqrt{\frac{1}{1+\beta_\nu}})(b+a\sqrt{3})}$$

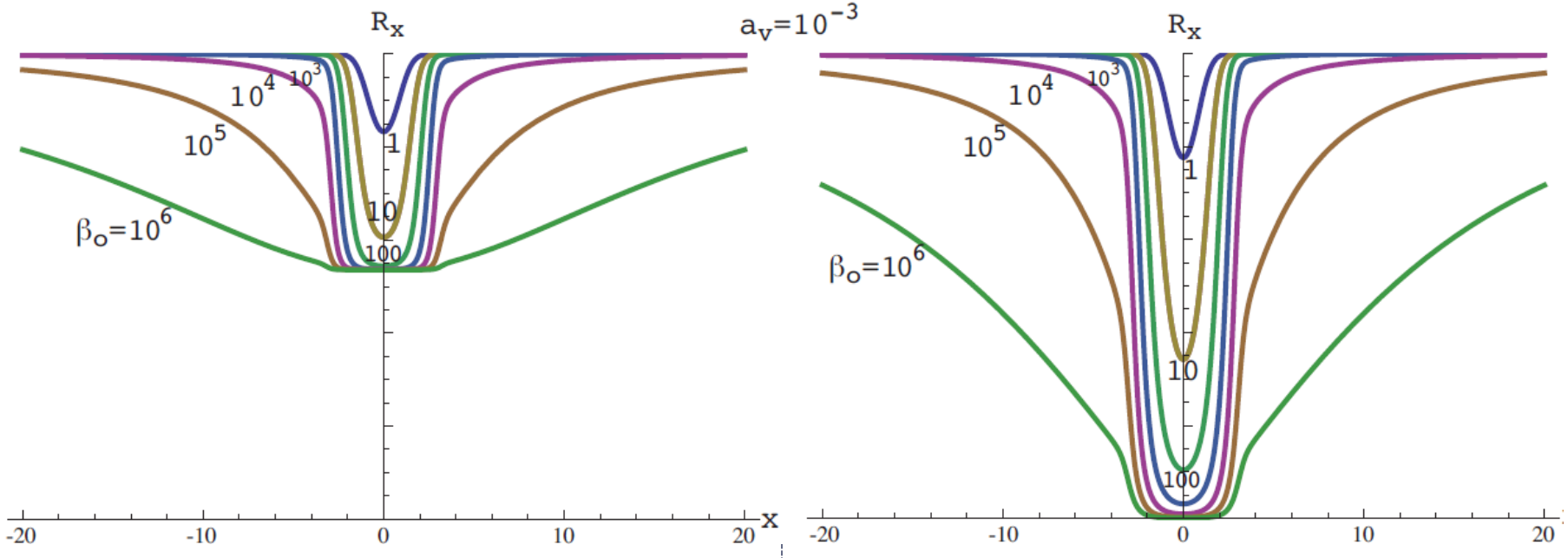
For strong lines: $\beta_\nu \gg 1$, $r_\nu \to 0$

Scattering removes all photons → no photon emerges from surface. Core of strong scattering lines are **dark**!

The residual flux Rx vs frequency x

$\zeta = 1$ (LTE)

$\zeta = 0$ (non-LTE)



Line emission from continuum scattering layer

$$r_\nu = \left(\frac{p_\nu + a\sqrt{3\lambda_\nu}}{b + a\sqrt{3\zeta^C}}\right)\left(\frac{1+\sqrt{\zeta^C}}{1+\sqrt{\lambda_\nu}}\right)$$

$$B_\nu = a + b\tau_c \qquad \beta_\nu \equiv \frac{\alpha_\nu^L}{\alpha_\nu^C} \qquad \tau_c = \frac{\tau_\nu}{1+\beta_\nu}$$

$$p_\nu \equiv \frac{b}{1+\beta_\nu} \qquad \lambda_\nu \equiv \frac{\zeta^C + \zeta^L\beta_\nu}{1+\beta_\nu}$$

c) pure scattering in continuum: $\zeta^C = 0$

$$\lambda_\nu \equiv \frac{\zeta^C + \zeta^L\beta_\nu}{1+\beta_\nu} = \frac{\zeta^L\beta_\nu}{1+\beta_\nu} \quad \Rightarrow \quad r_\nu = \frac{\frac{1}{1+\beta_\nu} + \frac{a}{b}\sqrt{3\lambda_\nu}}{1+\sqrt{\lambda_\nu}}$$

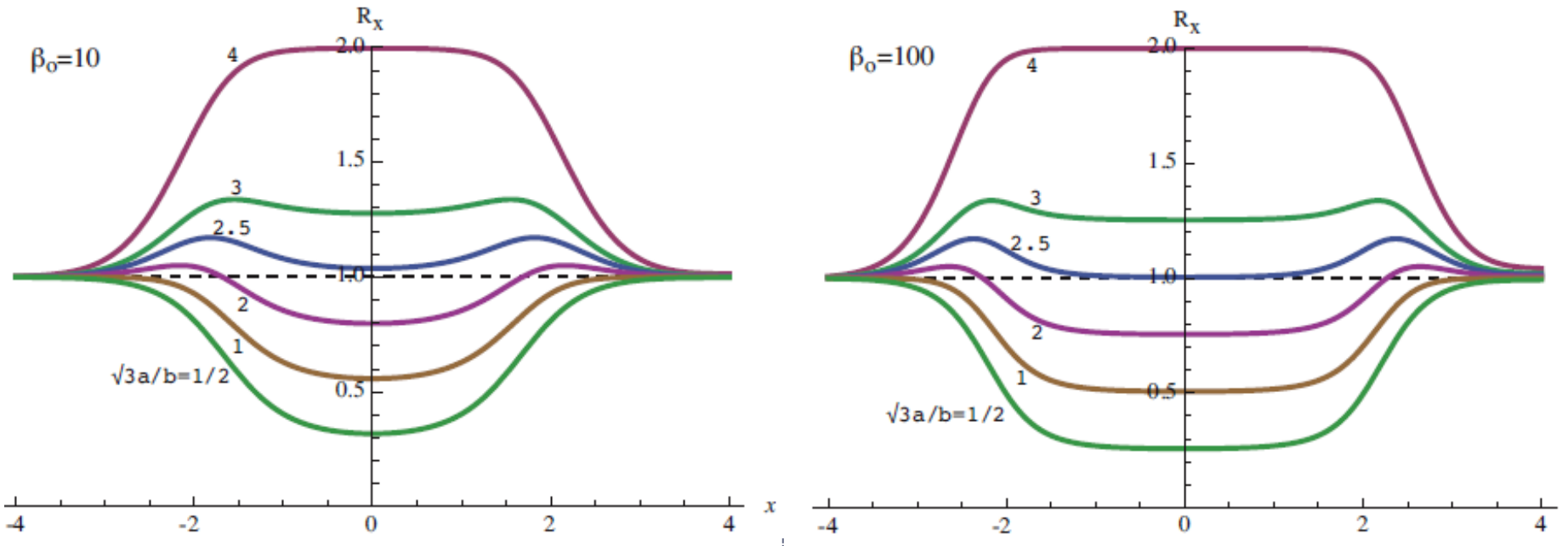
If the line opacity is also pure scattering, $\zeta^L = 0$, then $\lambda_\nu = 0$ $\quad r_\nu = \dfrac{1}{1+\beta_\nu} < 1$

always in absorption

But for $\zeta^L = 1$ and for strong lines $\beta_\nu \gg 1$ $\quad r_\nu \to \dfrac{\sqrt{3}a}{2b}$

For a weak temperature gradient with small b/a, can exceed unity, implying a net line emission instead of absorption.

Line profiles for Schuster model



Scattering makes the continuum source function low near the surface, $S_c(0) - J_c(0) \ll B(0)$, which implies a weak continuum flux. The line can potentially be brighter, but only if the decline from the negative temperature gradient term is not too steep.

Summary

- We obtained Transfer Equation including lines and taking into account Scattering in lines.
- We solved it using the Milne-Eddington model.
- We then obtained Residual flux of the line.
- Finally, we discussed interesting special cases such as pure absorption and pure scattering lines.
- We also tried to explain emission lines applying Schuster mechanism for line emission.

Next lecture: Theoretical curve of growth

- Standard diagnostic tool for the determination of metal abundances in cool stars.
- We will apply a simple model which allows equivalent widths to be calculated analytically.