Stellar Atmospheres
Lecture 16

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Spectral line formation

SIMPLE LINE TRANSFER
SCHUSTER-SCHWARZSCHILD MODEL
THEORY OF LINE FORMATION
CURVE OF GROWTH

Summary of the previous lecture

- Final profile is a convolution of all the key broadening processes
- Convolution of Lorentzian profiles: $\Gamma_{total}=\Sigma\Gamma_i$
- Convolution of Lorentzian and Doppler broadening yields a Voigt profile
- Pressure/collisional broadening via linear Stark broadening (only for hydrogenic ions), quadratic Stark broadening (interaction with electrons – hot stars) or Van der Waals broadening (interaction between neutral atoms – cool stars)
- Inglis-Teller relation allows estimate of N_e from overlapping Balmer lines in hot stars
- Non-pressure broadening mechanisms include microscopic (thermal Doppler), macroscopic (rotational Doppler), turbulent, Zeeman, instrumental.
- Line profiles typically have characteristic Voigt profiles – Gaussian (thermal) cores and Lorenzian (pressure) wings

Broadening of spectral lines

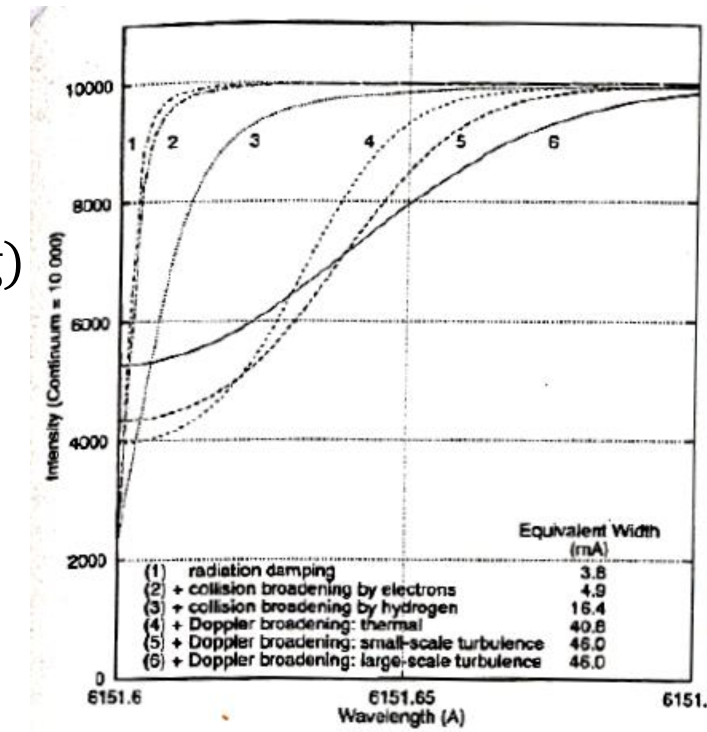
There are numerous broadening mechanisms which influence the apparent shape of spectral lines:

microscopic

1. Natural broadening
2. Thermal broadening
3. Microturbulence
   (treated like extra thermal broadening)
4. Collisions (important for strong lines)
5. Isotopic shift, *hfs*, Zeeman effect

macro

6. Macroturbulence
7. Rotation
8. Instrumental broadening



Holweger 1996

Line depth

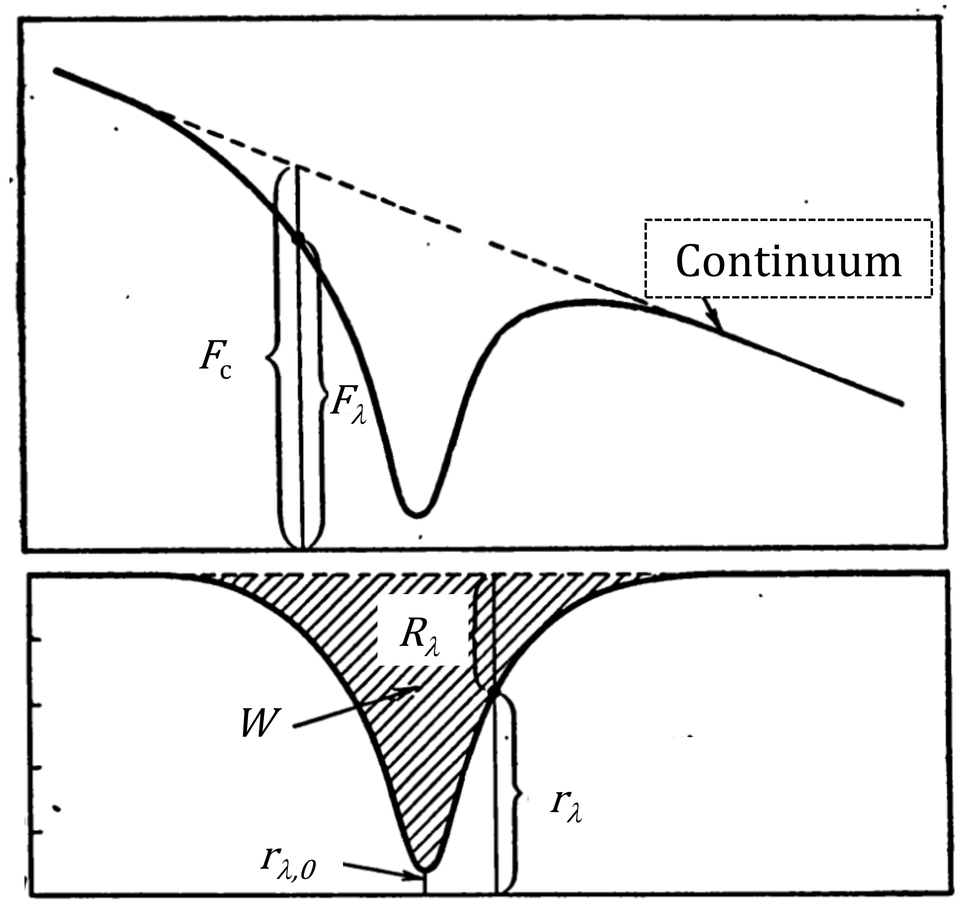
Relative intensity r_λ
(*not very common term, usually applied to emission lines*):

$$r_\lambda = \frac{F_\lambda}{F_c}$$

The line depth R_λ:

$$R_\lambda = \frac{F_c - F_\lambda}{F_c} = 1 - \frac{F_\lambda}{F_c}$$

The largest $R_{\lambda,0}$ — the central line depth

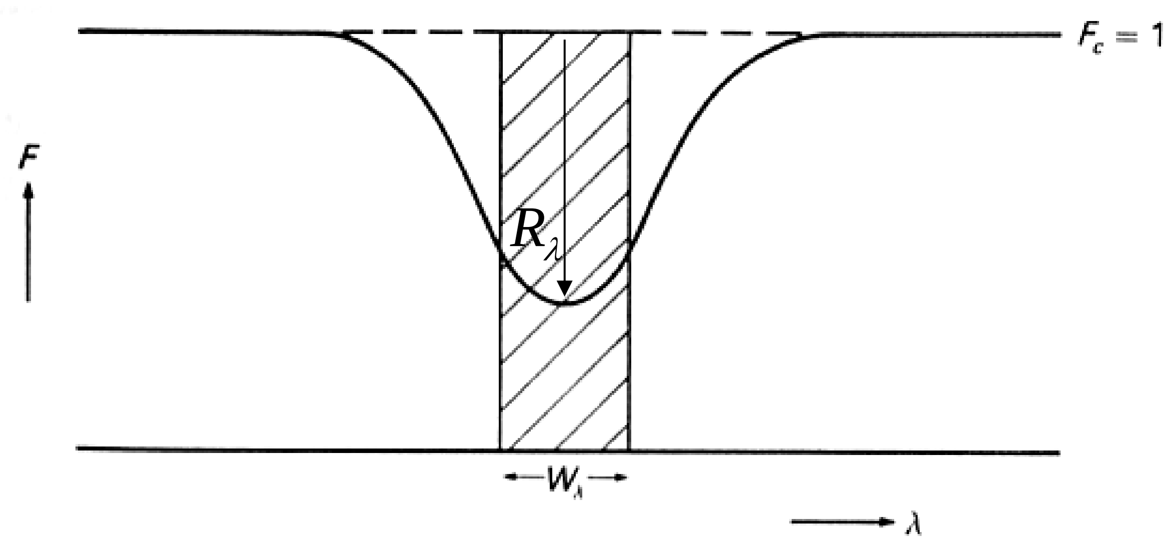


Equivalent Width

- The total area in a spectral line divided by the continuum flux F_c is called the line **equivalent width**, i.e. an integral over a line **depth** R_λ

$$W_\lambda = \int \frac{F_c - F_\lambda}{F_c} d\lambda = \int R_\lambda d\lambda$$

- The division by the continuum flux means that this is a measurement of the flux in units of the continuum – the equivalent width is identical to a rectangular line of width W_λ.



Schuster-Schwarzschild model

We now turn to the solution of the transfer equation for *both* line and continuum radiation. We will adopt the Schuster-Schwarzschild model, which assumes that the line is formed above the continuum and that continuous opacity plays only indirect role.

The total absorption coefficient within an arbitrary line is the sum of the line (α_L) and continuum (α_C) contributions i.e. $\alpha_\lambda = \alpha_L + \alpha_C$ as is the total emission coefficient ($\varepsilon_\lambda = \varepsilon_L + \varepsilon_C$). Hence,

$$S_\lambda = (\varepsilon_L + \varepsilon_C)/(\alpha_L + \alpha_C)$$

and

$$\mathrm{d}\tau_\lambda = -(\alpha_L + \alpha_C)\,\mathrm{d}z \qquad\qquad \tau_\lambda = \tau_L + \tau_C$$

So we can write the transfer equation as usual:

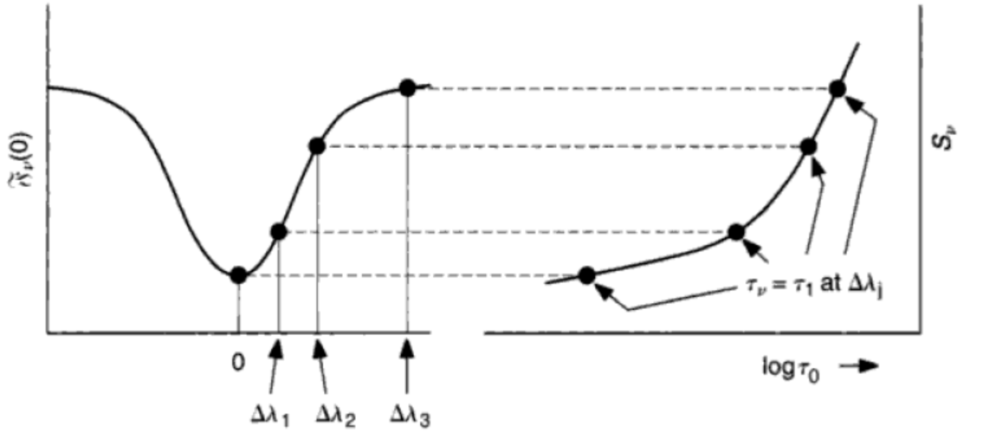
$$\cos\theta \frac{dI_\lambda(\theta)}{d\tau_\lambda} = I_\lambda(\theta) - S_\lambda$$

Line source function

- We have seen earlier that the emergent flux from the stellar surface is π times the Source function at an optical depth of 2/3

$$F_\lambda(0) = \pi S_\lambda(\tau = 2/3)$$

- Across a line profile, α_λ varies, being larger towards the centre. The condition τ_λ=2/3 is true higher up in the atmosphere for λ near line centre, and holds for progressively deeper layers for λ further into the wing.
- Assuming S_λ is a slowly varying function of λ (i.e. constant over the line width), $\pi S_\lambda(\tau_1 = 2/3) = F_\lambda(0)$ provides a mapping between S_λ as a function of τ_λ and F_λ as a function of λ.



Theory of line formation

- Because of larger absorption in the line, it is formed higher up in the atmosphere where T is lower=> absorption line.

$$\tau_\lambda = \tau_L + \tau_C$$

- Consider **weak** lines: $\alpha_L << \alpha_C \rightarrow \alpha_\lambda = \alpha_C (1 + \alpha_L / \alpha_C)$

  We can evaluate S_λ by a Taylor expansion:

$$S_\lambda(\tau_\lambda = 2/3) \approx S_\lambda(\tau_C = 2/3) + \left.\frac{dS_\lambda}{d\tau_c}\right|_{\tau=2/3} \Delta\tau_C$$

- $\tau_\lambda = \alpha_\lambda t \rightarrow \tau_C = (\tau_L + \tau_C) \frac{\alpha_C}{\alpha_L + \alpha_C} \approx \frac{2}{3} \frac{\alpha_C}{\alpha_L + \alpha_C} \approx \frac{2}{3}\left(1 - \frac{\alpha_L}{\alpha_C}\right)$
- $\tau_C = \tau_\lambda + \Delta\tau_C = \frac{2}{3} + \Delta\tau_C \rightarrow \Delta\tau_C = -\frac{2}{3}\frac{\alpha_L}{\alpha_C}$

Theory of line formation

- The line equivalent width is then (LTE)

$$W_\lambda = \int \frac{F_c - F_\lambda}{F_c} d\lambda = \int d\lambda \frac{B_\lambda(\tau_c = 2/3) - B_\lambda(\tau_\lambda = 2/3)}{B_\lambda(\tau_c = 2/3)} =$$

$$W_\lambda = \int d\lambda \frac{dB_\lambda(\tau_c = 2/3)}{d\tau_c}\bigg|_{\tau_c = 2/3} \left(\frac{2}{3}\frac{\alpha_L}{\alpha_C}\right) \frac{1}{B_\lambda(\tau_c = 2/3)} =$$

$$W_\lambda = \frac{2}{3}\int d\lambda \frac{d\ln B_\lambda(\tau_c = 2/3)}{d\tau_c}\bigg|_{\tau_c = 2/3} \left(\frac{\alpha_L}{\alpha_C}\right)$$

Weakly depends on λ

$$W_\lambda = \frac{2}{3}\frac{1}{\alpha_C} \frac{d\ln B_\lambda(\tau_c = 2/3)}{d\tau_c}\bigg|_{\tau_c = 2/3} \times \int_0^\infty \alpha_L d\lambda$$

If there is no temperature gradient with the temperature decreasing outwards, then there are no absorption lines in the spectrum.

The profile mimics the shape of α_L. Line strength can be increased by decreasing the continuous absorption α_C or by increasing the line absorption α_L.

Theory of line formation

$$W_\lambda = \frac{2}{3}\frac{1}{\alpha_C}\frac{d\ln B_\lambda(\tau_c = 2/3)}{d\tau_c}\bigg|_{\tau_c = 2/3} \times \int_0^\infty \alpha_L d\lambda$$

$$\alpha_L = \sigma_L n, \quad N = \int n dr = \frac{n}{\alpha_c}\int \alpha_c dr = \tau_c \frac{n}{\alpha_c} \approx \frac{2}{3}\frac{n}{\alpha_c} \Rightarrow W_\lambda \propto N$$

For optically thin lines with $\alpha_L << \alpha_C$, $W_\lambda \propto N$

Strong lines

For $\alpha_L << \alpha_C$, the line is optically thin and its strength increases proportionally with α_L / α_C. If $\alpha_L/\alpha_C>1$, the line becomes optically thick, reaching a maximum depth R_λ. For very thick lines with $\alpha_L/\alpha_C=\infty$, the intensity in the line centre is given by the source function $S_\lambda(\tau_\lambda=0)$, or $B_\lambda(\tau_\lambda= 0)$ in LTE. This is **not** zero since $T(\tau_\lambda= 0)$ is non-zero.

If non-LTE applies, when $S_\lambda \neq B_\lambda$, $S_\lambda(\tau_\lambda=0)$ may tend towards zero, for instance, in resonance lines.

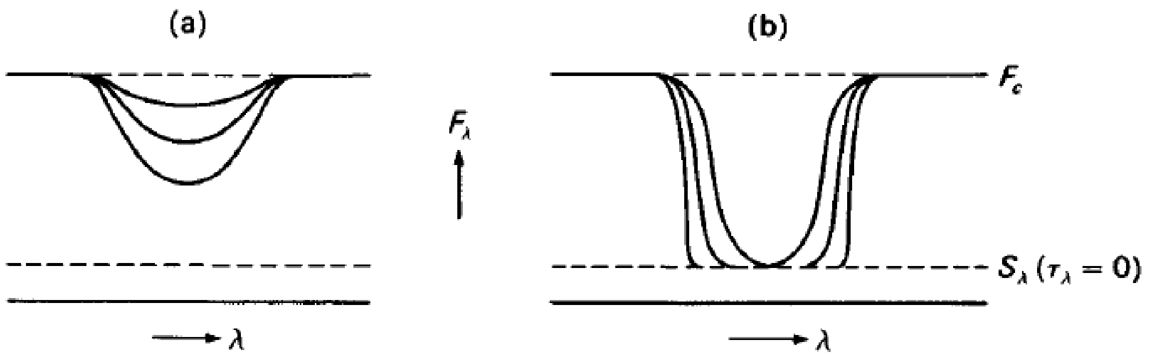


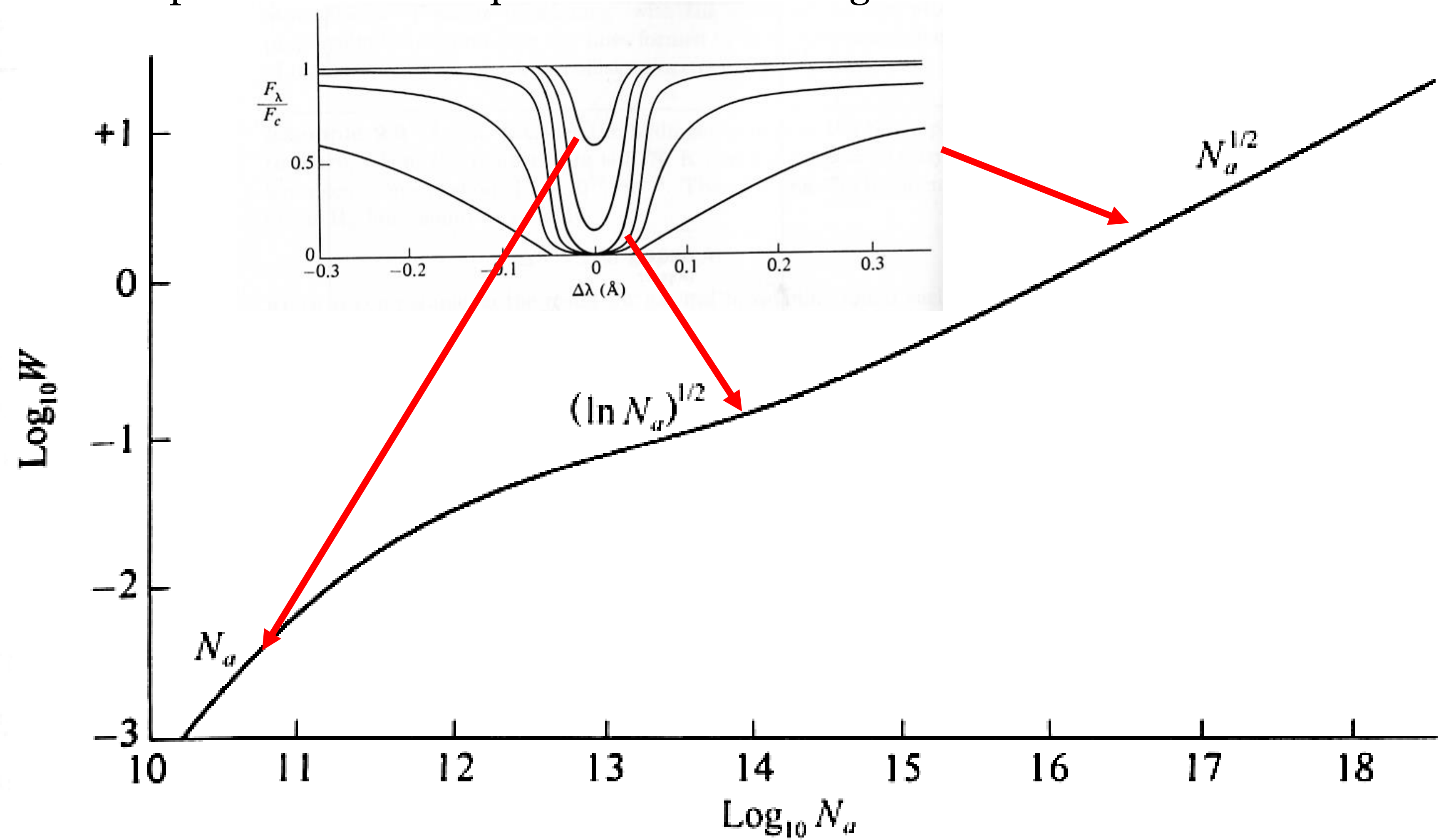
Fig. 10.12. Changes of the line profile with increasing κ_L/κ_c for (a) optically thin and (b) optically thick lines.

Curve of Growth

- The **Curve of growth** describes how the equivalent width (line strength) W_λ depends on the number of absorbing atoms or ions.
- For weak, optically thin lines, as the abundance doubles, the line equivalent width also doubles in strength: $W_\lambda \sim N$ – this is the LINEAR part of the curve of growth
- As the abundance continues to increase, the Doppler core of the line becomes optically thick and saturates. The wings of the line, which are still optically thin, deepen, which occurs with little change in the line equivalent width and so produces a PLATEAU in the curve of growth, $W_\lambda \sim (\ln N)^{1/2}$,
- Ultimately, the damping wings become optically thick, increasing the equivalent width, $W_\lambda \sim (N)^{1/2}$. This is the DAMPING or SQUARE ROOT part of the curve of growth.

Curve of Growth

Curve of growth for the K line of CaII. As N increases, the functional dependence of the equivalent width changes.



Methodology

- Using the curve of growth and a measured equivalent width we can derive the number of absorbing atoms.
- The Boltzmann and Saha equations convert this value into the total number of atoms of that element in the photosphere.
- To reduce errors, it is advisable to locate several lines on a curve of growth

Thermal and Pressure effects

- The exact form of the curve of growth depends on the ratio of pressure to thermal broadening, $\alpha = \gamma / 2\Delta\lambda_D$ For increasing Doppler line width, saturation occurs for larger W_λ, whilst the damping part will start earlier if α (i.e. γ) is larger.