Stellar Atmospheres
Lecture 15

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Spectral line formation

PRESSURE BROADENING

CONVOLUTION OF DIFFERENT BROADENING PROCESSES

INGIS-TELLER RELATION

ROTATIONAL AND INSTRUMENTAL BROADENING

Natural and Thermal Broadenings

From the previous lecture:

- Natural Line Broadening: $\phi_\nu = \dfrac{\Gamma/4\pi}{(\nu-\nu_0)^2+(\Gamma/4\pi)^2}$ $\qquad \Gamma = \sum_{i<j} A_{ji}$

  Lorentzian profile with FWHM $\Delta\lambda_{1/2} = \dfrac{\lambda_0^2}{c}\Delta\nu_{1/2} = \dfrac{\lambda_0^2}{c}\dfrac{\Gamma}{2\pi} \approx f_{ij}\times 7\times 10^{-4}$ Å

- Doppler broadening $\phi(\nu) = \dfrac{1}{\sqrt{\pi}\Delta\nu_D} e^{-(\nu-\nu_0)^2/\Delta\nu_D^2}$ $\qquad \Delta\nu_D = \dfrac{\nu_0}{c}u_{th} = \dfrac{\nu_0}{c}\sqrt{\dfrac{2kT}{m}}$

  Gaussian line profile with FWHM $\Delta\nu_{1/2} = 1.67\Delta\nu_D$

  $\Delta\lambda_{1/2} = \dfrac{\lambda_0^2}{c}\Delta\nu_{1/2} = 7.1\times 10^{-7}\lambda_0(\text{Å})\sqrt{(T/\mu)}$ Å

Comparing broadenings

- Thermal (Doppler):
  - $\Delta\lambda_{th}$=0.02 Å (at λ_o=5000 Å, T=6000K, Fe)
  - $\Delta\lambda_{th}$=0.5 Å (at λ_o=5000 Å, T=50000K, H)
- Radiation damping:
  - $\Delta\lambda_{FWHM}$=a few $\times 10^{-4}$ Å

- But: decline of Gauss profile in wings is much steeper than for Lorentz profile:

$$\text{Gauss}\,(10\Delta\lambda_{th}) \quad : \quad e^{-10^2} \approx 10^{-43}$$

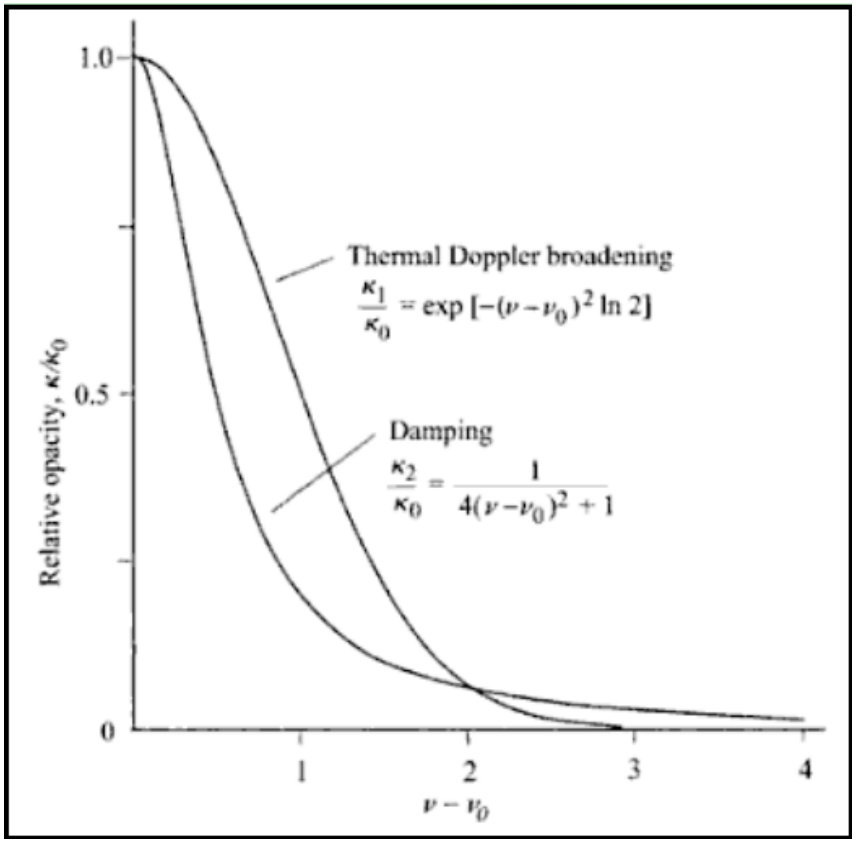
$$\approx$$

$$\text{Lorentz}\,(1000\Delta\lambda_{rad}) : \; 1/1000^2 \approx 10^{-6}$$

- In the line wings the Lorentz profile is dominant

Broadening mechanisms profiles

- Different broadening mechanisms have the form of
  - A Lorentzian function (natural profile and broadening, some pressure brodenings)
  - A Gaussian function (thermal broadening, instrumental broadening, etc.)
  - Other functions are possible (e.g., Linear Stark broadening)
- Generally, we have to consider both (all) types of profiles. For example, the pressure damping profile is negligible in the line core, but the Doppler profile decreases very steeply in the wings, whilst the damping profile decreases only as $1/\Delta\lambda^2$
- The Gaussian dominates the line core (or is confined to it), while the Lorentzian profile dominates in the line wings out to several times the FWHM.



Joint effect of different mechanisms

Mathematically: **convolution** $(f_A * f_B)(x) = \int_{-\infty}^{\infty} f_A(y)\, f_B(x-y)\, dy$

Properties:

- commutative: $f_A * f_B = f_B * f_A$

- Fourier transformation: $F(f_A * f_B) = normfactor \cdot F(f_A) \cdot F(f_B)$
  where F denotes the Fourier transform of f.

i.e., in Fourier space the convolution is a multiplication

Application to profile functions

Convolution of two Gaussian profiles

$$G_A(x) = 1/A\sqrt{\pi}\ \ e^{-x^2/A^2} \quad G_B(x) = 1/B\sqrt{\pi}\ \ e^{-x^2/B^2}$$

$$G_C(x) = G_A(x) * G_B(x) = 1/C\sqrt{\pi}\ \ e^{-x^2/C^2} \quad \text{with} \quad C^2 = A^2 + B^2$$

Result: Gauss profile with quadratic summation of half-widths.

Convolution of two Lorentzian profiles (e.g., radiation + collisional damping)

$$L_A(x) = \frac{A/\pi}{x^2 + A^2} \quad L_B(x) = \frac{B/\pi}{x^2 + B^2}$$

$$L_C(x) = L_A(x) * L_B(x) = \frac{C/\pi}{x^2 + C^2} \quad \text{with} \quad C = A + B$$

Result: Lorentz profile with sum of half-widths

Voigt profile

Convolving Gauss and Lorentz profile (e.g. thermal + natural broadening)

$$G(\nu) = \frac{1}{\Delta\nu_D\sqrt{\pi}}\, e^{-(\nu-\nu_0)^2/\Delta\nu_D{}^2} \qquad L(\nu) = \frac{\gamma/4\pi^2}{(\nu-\nu_0)^2 + (\gamma/4\pi)^2}$$

$V = G * L$ depends on $\nu, \Delta\nu, \gamma, \Delta\nu_D$: $V(\nu) = \int_{-\infty}^{\infty} G(\nu')L(\nu-\nu')d\nu'$

Transformation: $\mathrm{v} := (\nu-\nu_0)/\Delta\nu_D$ $a := \gamma/(4\pi\Delta\nu_D)$ $y := (\nu'-\nu_0)/\Delta\nu_D$

$$G(y) = \frac{1}{\Delta\nu_D\sqrt{\pi}}\, e^{-y^2} \qquad L(y) = \frac{a/\Delta\nu_D\pi}{y^2+a^2} \qquad V = \frac{1}{\Delta\nu_D\sqrt{\pi}}\frac{a}{\pi}\int_{-\infty}^{\infty}\frac{e^{-y^2}}{(\mathrm{v}-y)^2+a^2}dy$$

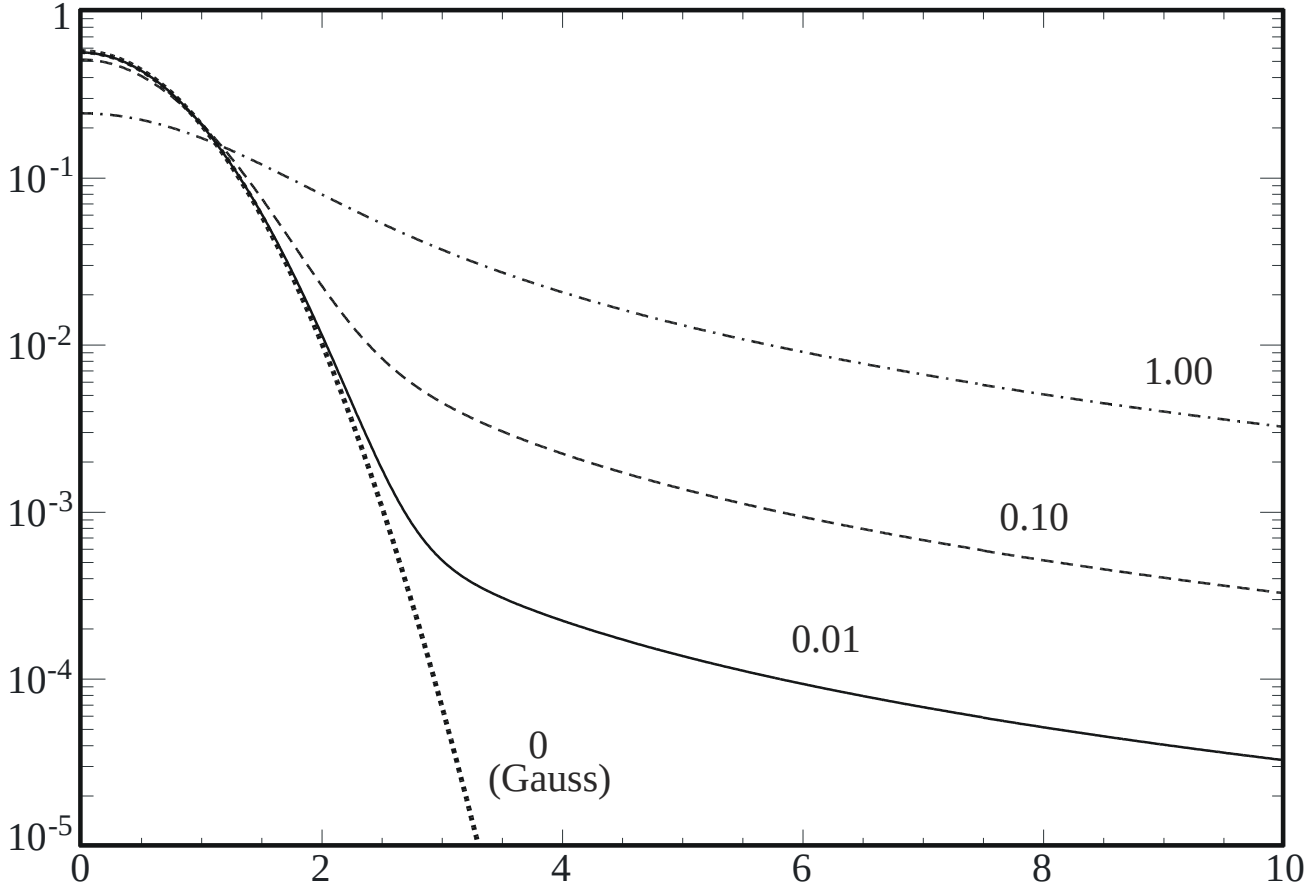
Def: $V = \frac{1}{\Delta\nu_D\sqrt{\pi}} H(a,\mathrm{v})$ with $H(a,\mathrm{v}) = \frac{a}{\pi}\int_{-\infty}^{\infty}\frac{e^{-y^2}}{(\mathrm{v}-y)^2+a^2}dy$

Voigt function, no analytical representation possible.

(approximate formulae or numerical evaluation)

Normalization: $\int_{-\infty}^{\infty} H(a,\mathrm{v})d\mathrm{v} = \sqrt{\pi}$

The Voigt func for various *a*

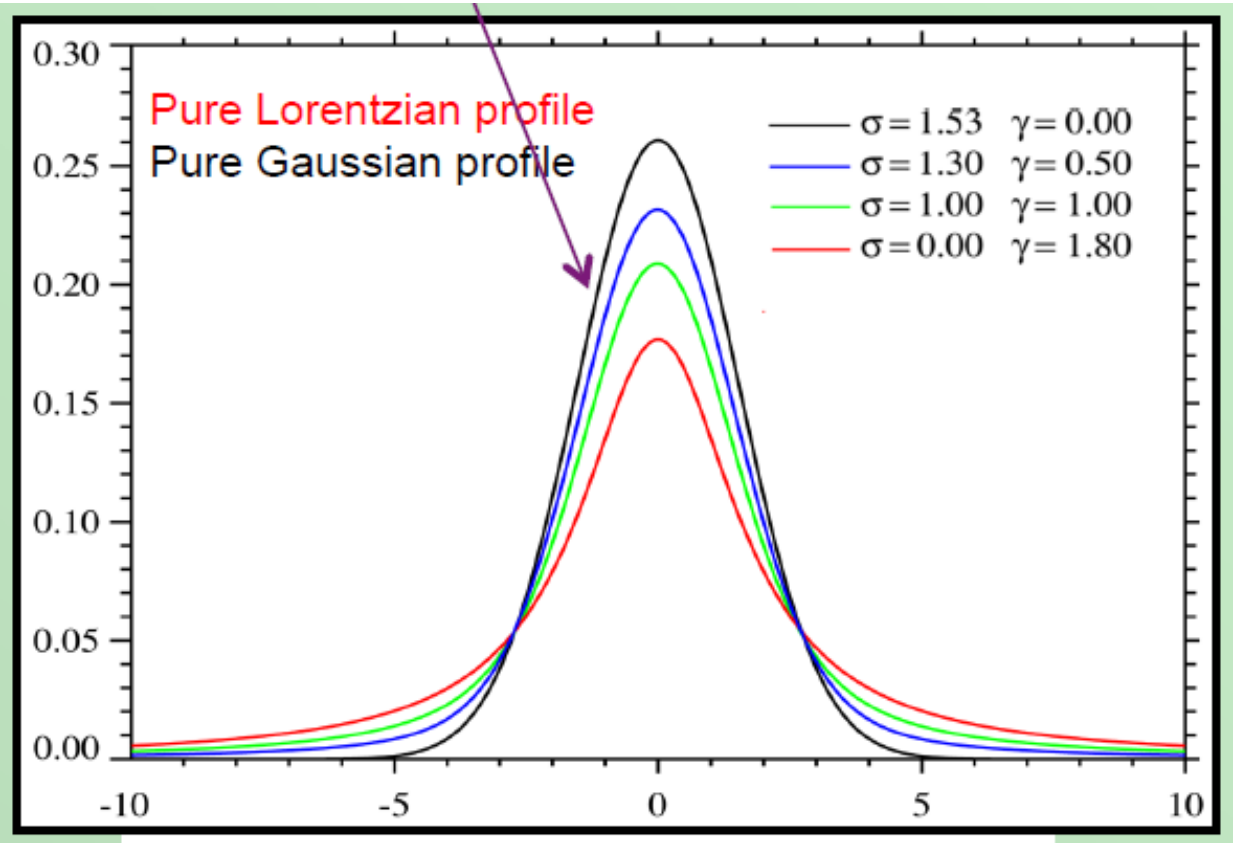


The final form of the combined Voigt profile depends on $\alpha=\gamma/2\Delta\nu_D$ the ratio of the damping widths $\gamma/2$ to the Doppler width $\Delta\nu_D$

As **a rule of thumb**, the damping wings start to contribute a distance $-(\log \alpha)\Delta\lambda_D$ from the line centre

The Voigt func for various *a*

Voigt profiles



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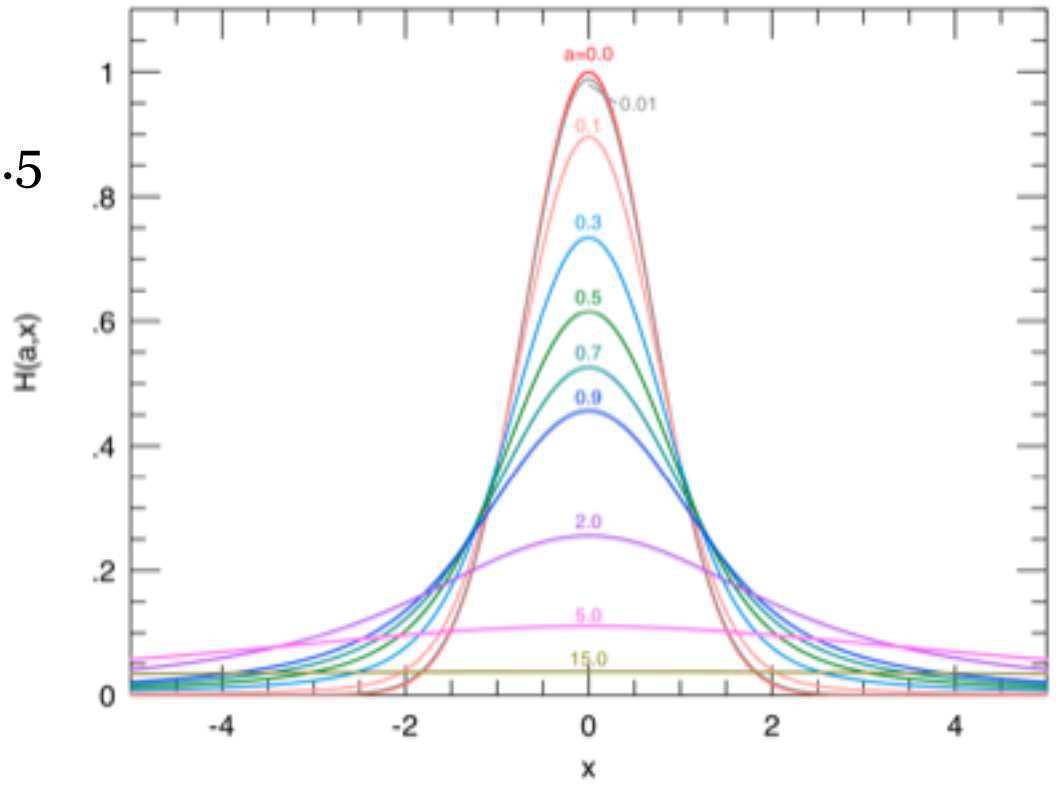
Calculation of a Voigt profile

No analytical representation is possible, but…

- IDL:
  IDL> u=findgen(201)/40.-2.5
  IDL> v=voigt(0.5,u)
  IDL> plot,u,v

- Python



Broadening of spectral lines

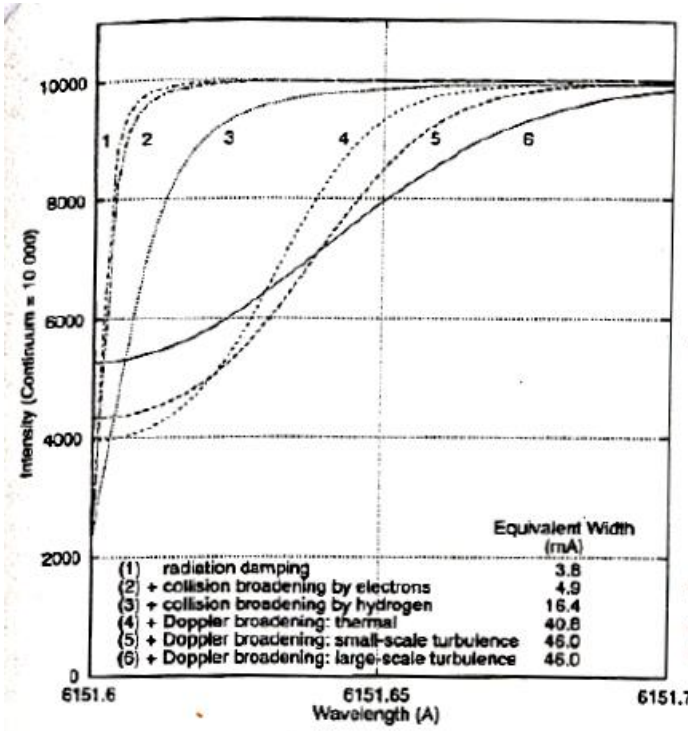
There are numerous broadening mechanisms which influence the apparent shape of spectral lines:

microscopic

1. Natural broadening
2. Thermal broadening
3. Microturbulence
   (treated like extra thermal broadening)
4. Collisions (important for strong lines)
5. Isotopic shift, *hfs*, Zeeman effect

macro

6. Macroturbulence
7. Rotation
8. Instrumental broadening



Holweger 1996

Back to Pressure broadenings…

- The orbitals of an atom can be perturbed in a collision with a neutral atom or encounter with the electric field of an ion.

n =	name	interaction of
2	linear Stark effect	hydrogen-like ions + p, e
3	resonance broadening	neutral atoms with each other, H+H
4	quadratic Stark effect	ions + e, p
6	van der Waals broadening	metals + H

resonance broadening (n=3)
quadratic Stark effect (n=4)
van der Waals broadening (n=6)

} impact broadening approximations

Lorentz profile

$$\Delta\nu = \frac{C_n}{r^n}$$

(Ansatz): constants C_n are determined by laboratory measurements, or calculations → Let‘s discuss in a bit more detail

Collisional Broadening

- Frequency of collisions $= 1/T_0$
- Suppose collisions occur if particles pass within distance = impact parameter ρ_0

$$\frac{1}{T_0} = N\pi\rho_0^2 v$$

N = #perturbers/cm^3, v = relative velocity cm/s

- Then damping parameter is

$$\Gamma = 2N\pi\rho_0^2 v$$

We used $\sigma=\pi a_0^2$ for direct collisions

Weisskopf approximation (1)

- perturber is a classical particle
- path is a straight line
- no transitions caused in atom
- interaction creates a phase shift or frequency shift given by

$$\Delta\omega = \frac{C_p}{r^p}$$

- p exponents of astronomical interest: 3,4,6

Weisskopf approximation (2)

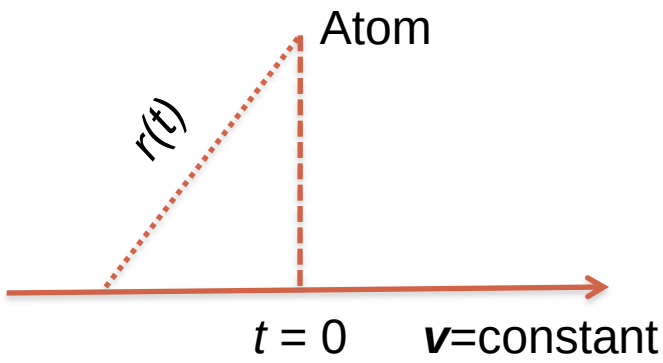
Total phase shift

$$\eta(\rho) = C_p \int_{-\infty}^{+\infty} \frac{dt}{r^p} = C_p \int_{-\infty}^{+\infty} \frac{dt}{[\rho^2 + v^2 t^2]^{p/2}} = \frac{C_p}{v\rho^{p-1}} \psi_p$$

$$\psi_p = \sqrt{\pi}\, \frac{\Gamma[(p-1)/2]}{\Gamma[p/2]}$$

p	ψ_p
2	π
3	2
4	π/2
6	3π/8

$$r(t) = [\rho_0^2 + v^2 t^2]^{1/2}$$



perturber path

Weisskopf approximation (3)

- Assume that only collisions that produce a phase shift $> \eta_0$ are effective in broadening:
  then impact parameter is

$$\rho_0 = \left(\frac{C_p \psi_p}{\eta_0 \nu} \right)^{\frac{1}{p-1}}$$

- Weisskopf assumed $\eta_0 = 1$, yields damping

$$\Gamma_W = 2\pi N \nu \left(\frac{C_p \psi_p}{\nu} \right)^{\frac{2}{p-1}}$$

  depends on ρ, T

- Ignores weak collisions $\eta < \eta_0$

Better Impact Model: Lindholm-Foley

- Includes effects of multiple weak collisions, which introduce a phase shift $\Delta\omega_o$; $\Gamma_{LF} > \Gamma_W$

$$I(\omega) = \frac{\Gamma/(2\pi)}{(\omega - \omega_0 - \Delta\omega_0)^2 + (\Gamma/2)^2}$$

p	3	4	6
Γ	$2\pi^2 C_3 N$	$11.37\ C_4^{2/3}\ v^{1/3}\ N$	$8.08\ C_6^{2/5}\ v^{3/5}\ N$
$\Delta\omega_0$	0	$9.85\ C_4^{2/3}\ v^{1/3}\ N$	$2.94\ C_6^{2/5}\ v^{3/5}\ N$

- Impact theory fails for small ρ

Impact broadenings (n=3,4,6)

- **Resonance Broadening** (n=3) occurs between identical species, restricted to upper/lower level having an electric dipole transition to ground state (resonance line):

$$\Delta\lambda_{1/2} = 8.6\text{x}10^{-30}(g_i / g_k)^{1/2}\lambda^2\lambda_{res}f_{res}N_i$$

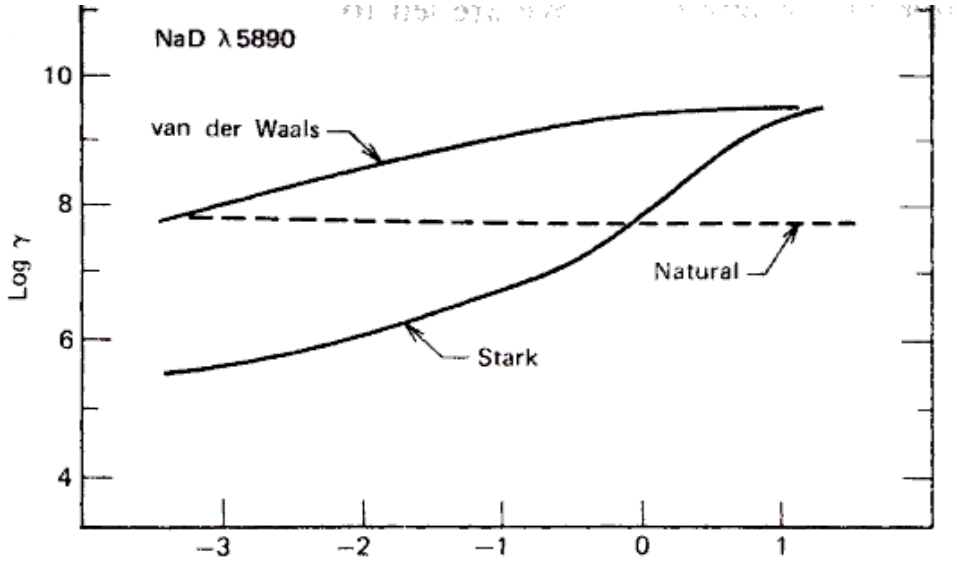
- **Quadratic Stark broadening** (n=4): Interaction of electron or proton with a system without dipole moment. The frequency shift depends on the square of the local electric field generated by passing electrons. With C_4 a constant obtained from laboratory data,

$$\log\gamma_4 = 19 + \frac{2}{3}\log C_4 + \log P_e - \frac{5}{6}\log T$$

- **Van der Waals broadening** (n=6): A momentary dipole on one neutral atom induces a change in lifetime, by inducing a dipole on the other. Because of its overwhelming abundance, **neutral H** acts as a perturber. For C_6 a constant (excitation and ionization dependent),

$$\log\gamma_6 = 19.6 + \frac{2}{5}\log C_6 + \log P_g - \frac{7}{10}\log T$$

Example



A comparison of quadratic Stark and van der Waals broadening for NaI 5890 at various optical depths in the Sun. The latter dominates here, and greatly exceeds the natural width by a factor of about 30.

In general,

- **Quadratic Stark broadening** (n=4) affects most lines in **hot stars** since **electron** pressure approaches gas pressure.
- **Van der Waals broadening** (n=6) affects most lines in **cool stars** since this involves interactions between **neutral atoms**

Linear Stark broadening (n=2)

- Atoms do **not** generally have permanent electric dipole moments. If there were such a moment, the Stark effect would be *linear*. Such a moment can occur only for two or more levels of the same energy (they are degenerate) but different orbital quantum numbers. This happens only for single electron atoms (H, He^{+}, Li^{2+}, …).

- The frequency shift depends on the the local electric field generated by passing electrons.

- Unfortunately, impact theory is no longer satisfactory and we have to consider the distribution of electric fields. In the star there is no a uniform field – there is an average field distribution felt by an average atom (**statistical** Stark effect). This distribution is called the Holtsmark distribution.

Holtsmark Statistical Theory

- Ensemble of perturbers instead of single
- more particles, more chances for strong field
- e- attracted to ions, reduce perturbation by Debye shielding
- in stellar atmospheres density is low, number of perturbers is large, and Holtsmark distribution is valid

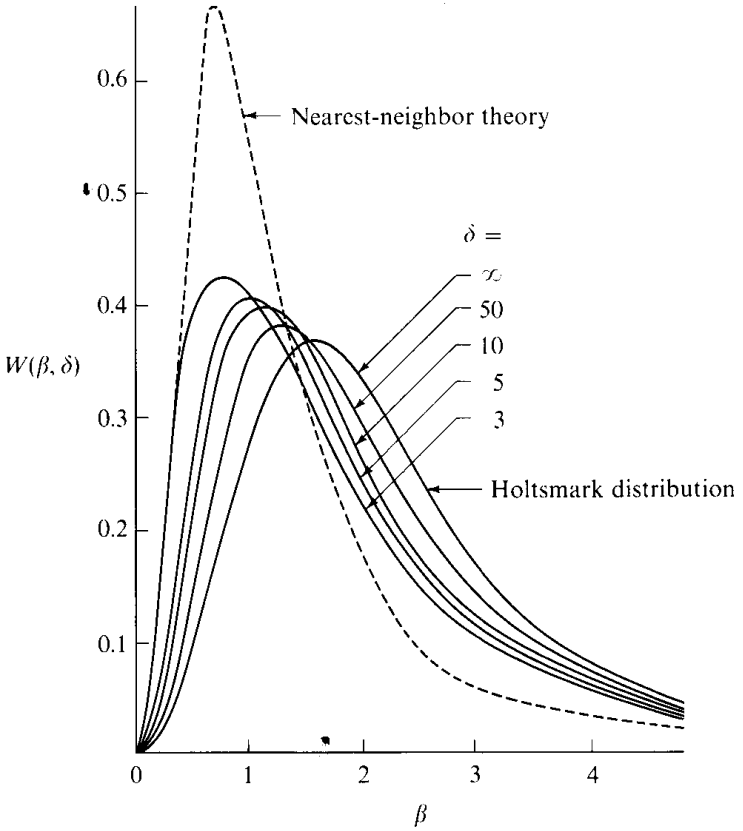


FIGURE 9-1
Probability distribution of field strength at a test point, including shielding effects; δ is the number of charged particles within the Debye sphere. From (**205**), by permission.

Hydrogen: Linear Stark Effect

- Each level degenerate with $2n^2$ sublevels.
- Perturbing field will separate sublevels.
- Observed profile is a superposition of components weighted by relative intensities and shifted by field probability function.
- $\Delta\lambda_{1/2} \approx 2.5\text{x}10^{-9}\alpha_{1/2}N_e^{2/3}$ where $\alpha_{1/2}$ is a half-width parameter widely used for plasma diagnostics (NIST).
- For Hα (n=2 to 3) in the Sun (P_e=20 dyne/cm^2, T=5800K), $\Delta\lambda_{\text{FWHM}}$=0.5 Å, i.e. a width 1000 times the natural width.
- Hot stars have very **high** electron pressures, so the Linear Stark effect greatly affects H I lines in hot stars (including white dwarfs), and is also relevant for hydrogenic ions (e.g. HeII lines) in O stars.

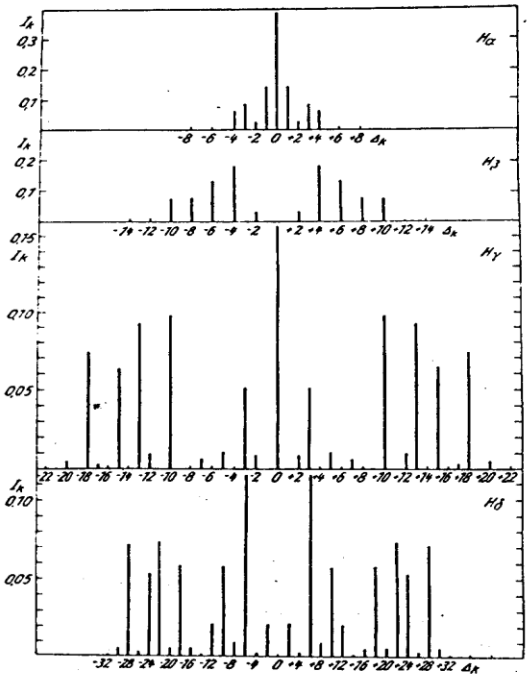
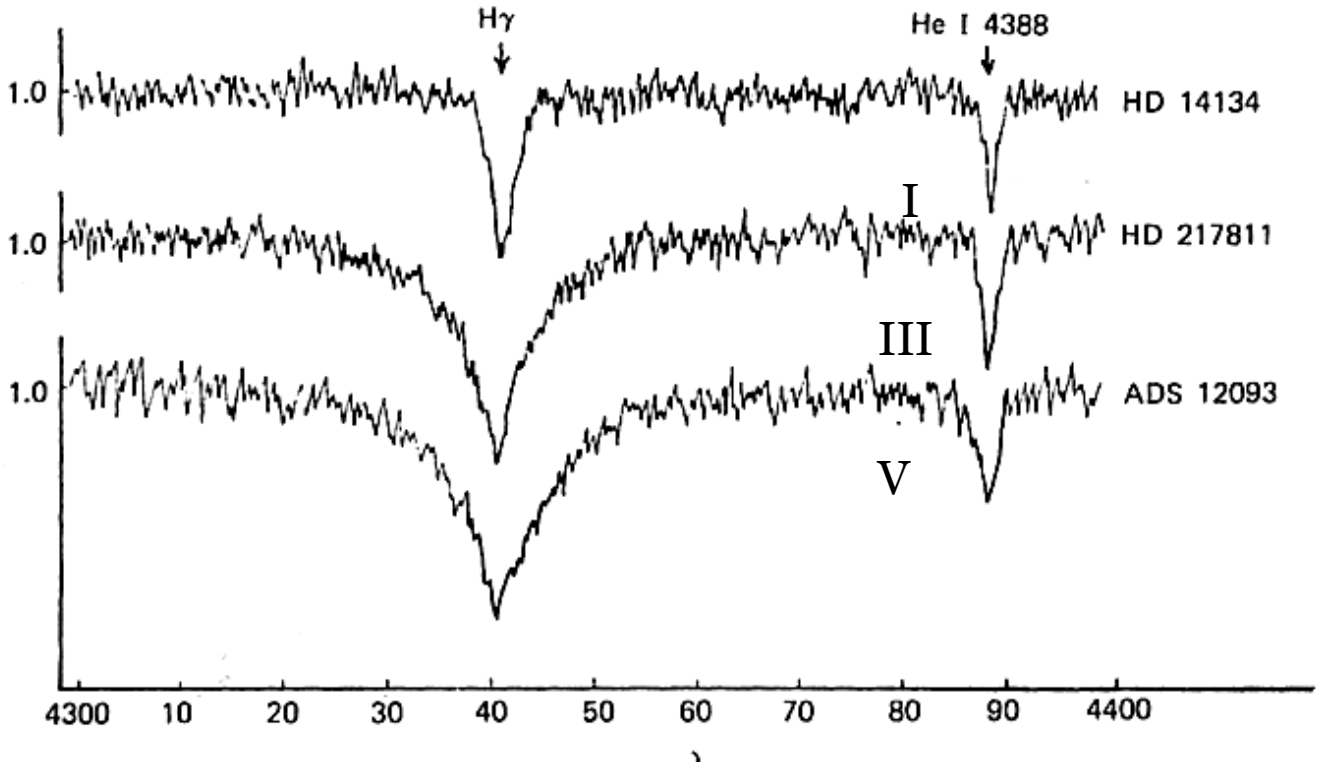


Fig. 11.1. The Stark effect splitting of the different Balmer lines (according to Unsöld, 1955, p. 320.)

Linear Stark broadening: examples (1)

Example of linear Stark broadening in early B stars – increased Hγ line width for increased pressure (this effect becomes significant for T_{eff}>7500K).



Linear Stark broadening: examples (2)

Vidal, Cooper & Smith (1973):

H I + p quasistatic approach;

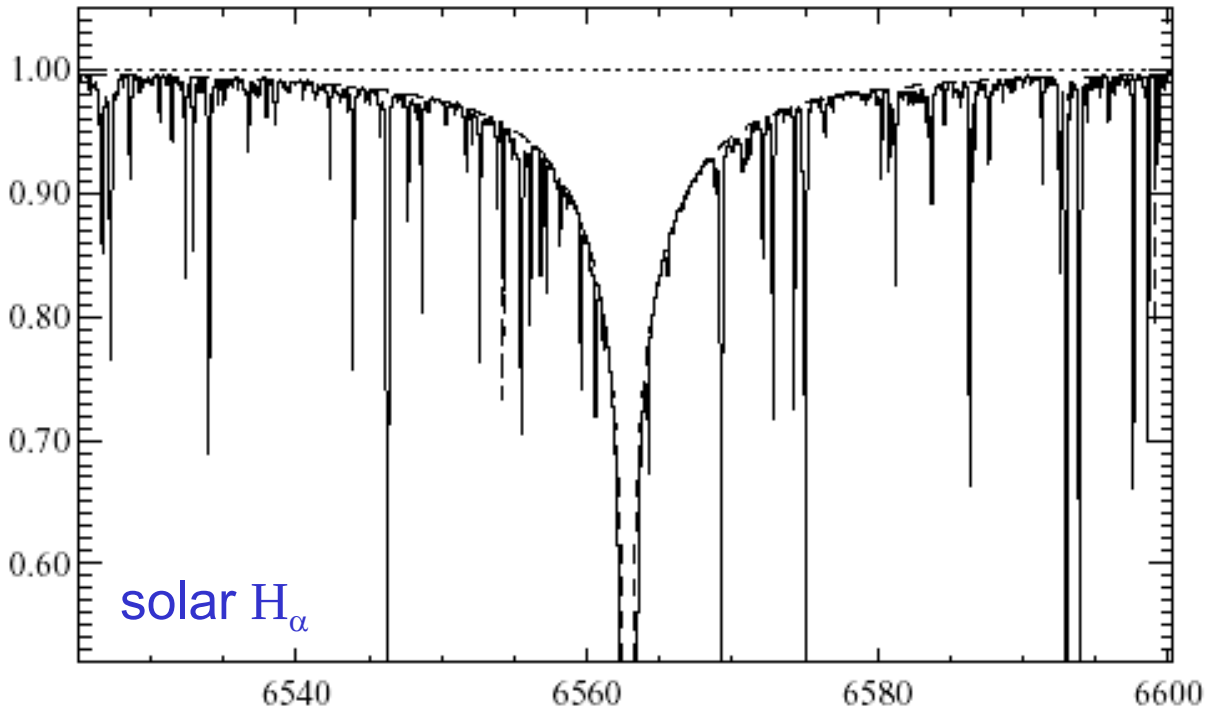
H I + e collisional approximation – in a core

quasistatic approach - in wings

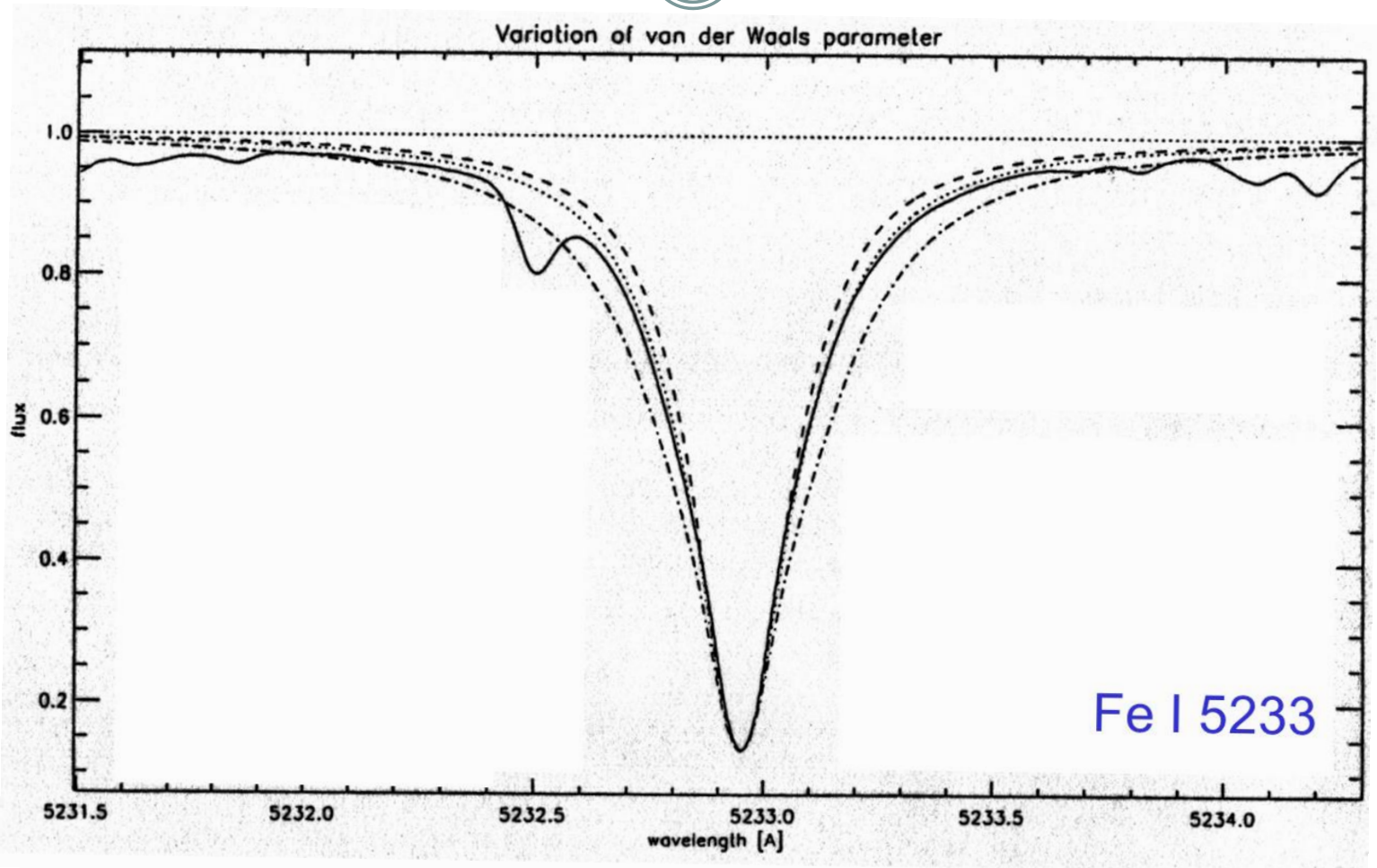
Observations

(solid line) and theory

(dash line)



van der Waals broadening: example



Example: log C_6 varies from -31.40 (top), -31.10 (middle), to -30.50 (bottom)

New Developments in the Theory of Pressure-Broadening

- Linear Stark broadening

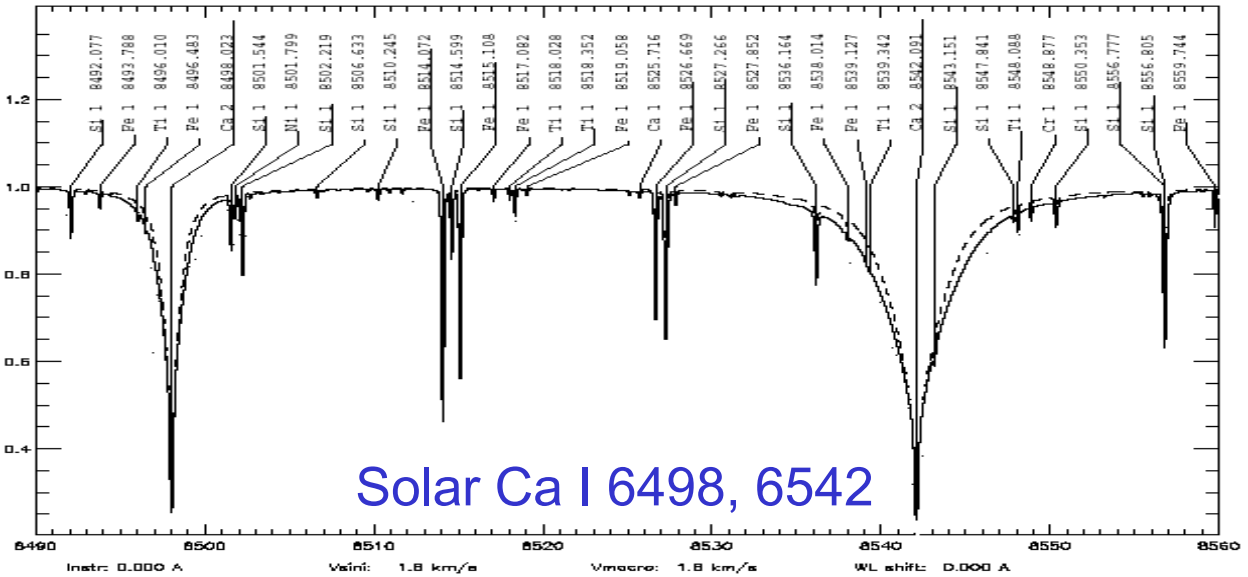
Stehle & Hutcheon (1999, A&AS, 140, 93) – tables of Stark profiles

- van der Waals broadening

Anstee & O'Mara (1995, MNRAS, 276, 859) and following papers

$$\gamma_6 / 4\pi = N_H (4/\pi)^{\alpha/2} \Gamma\left((4-\alpha)/2\right) \mathrm{v}\, \sigma_0 (\mathrm{v}/\mathrm{v}_0)^{-\alpha}$$

σ_0, α - tabulated parameters



Observations and Theory of *Anstee&O'Mara* are consistent!

Dash line – approximation of *Unsold* (1955)

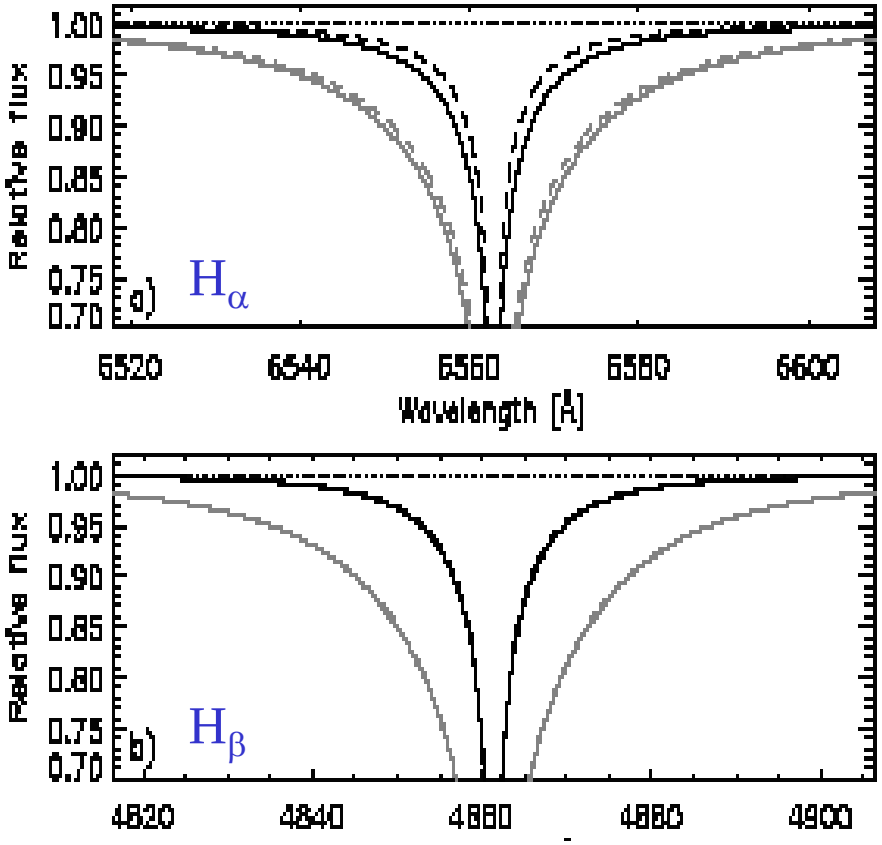
♦ Resonance Broadening

Barklem et al. (2000, A&A, 363, 1091)

Influence of resonance broadening on the line profiles of H_α and H_β

♦ Quadratic Stark broadening

Papers by *Dimitrijevic et al.*



$T_{eff} = 5780$ K, $\log g = 4.44$

$T_{eff} = 7000$ K (grey)

Dash line:

Without resonance broadening

Inglis-Teller relation

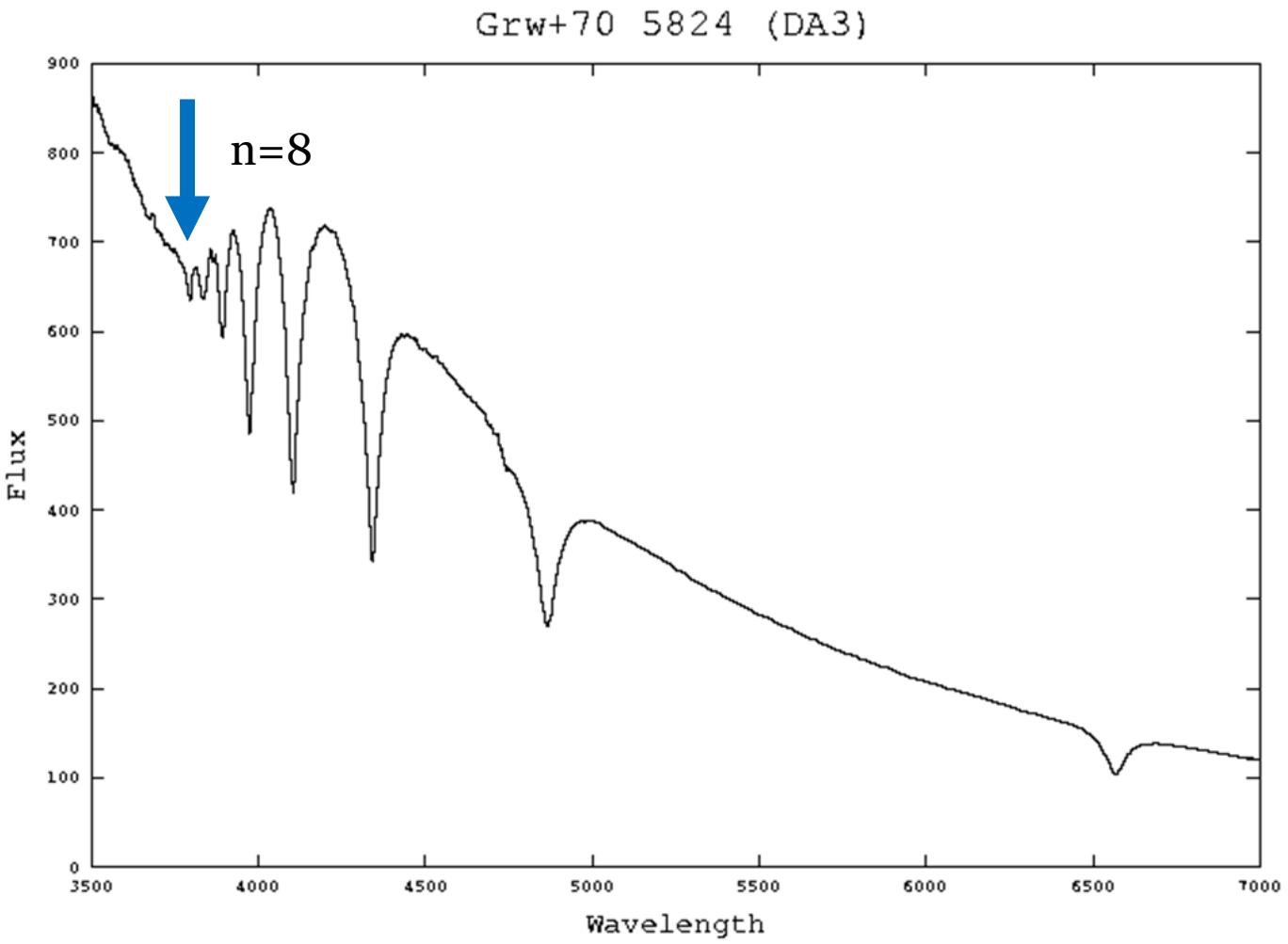
- Balmer lines, due to linear Stark broadening, overlap with each other close to the series limit, merging into a quasi-continuum at frequencies well below the nominal threshold.
- If linear Stark broadening is the dominant mechanism, one can estimate the N_e from the highest frequency Balmer line n_{max} that is still visible – the Inglis & Teller (1939) relation:

$$\log N_e = 23.26 - 7.5 \log n_{\max}^{Balmer}$$

Star	SpT	n_{max}	Log N_e
α Cyg	A2I	29	12.2
Sirius	A2V	18	13.8
τ Sco	B0V	14	14.6
White dwarf	DA	8	16.4

From Mihalas (1970)

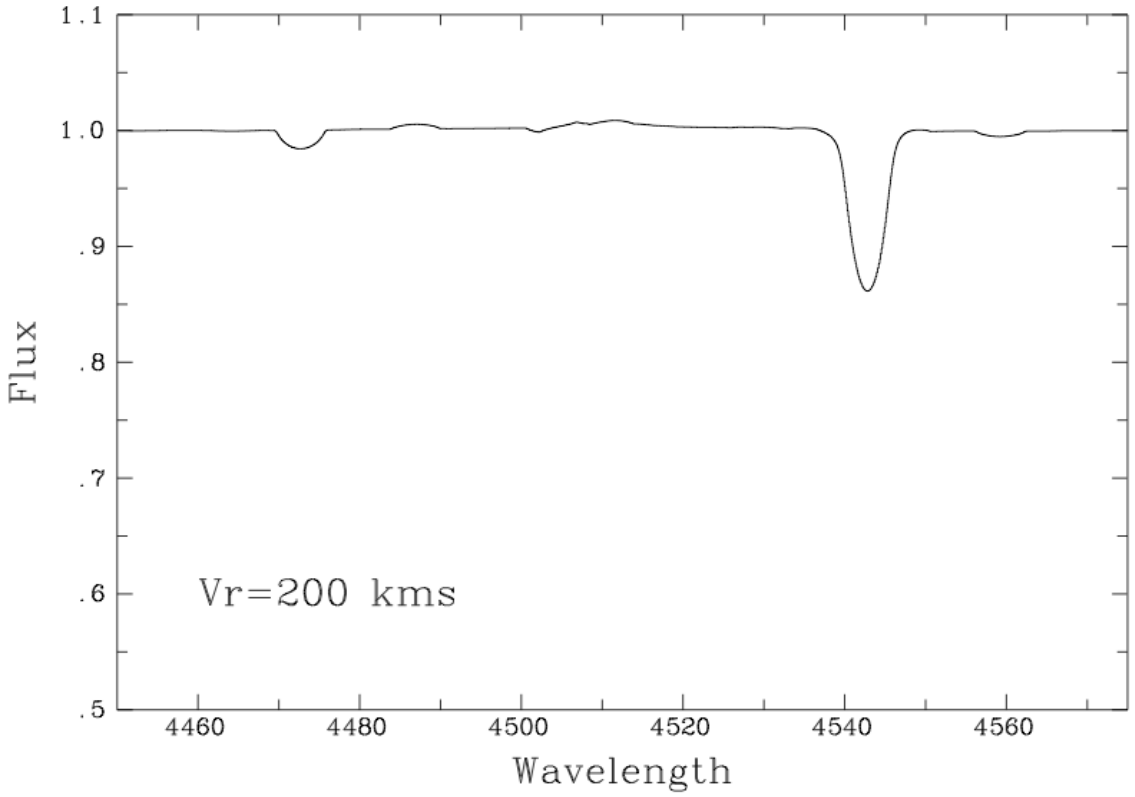
Inglis-Teller in White Dwarfs



Rotational broadening

Thermal Doppler broadening describes the microscopic motion of individual particles in the atmosphere. The other scale extreme is **macroscopic** broadening of the lines caused by the **rotation** of the whole star. The maximum (critical) rotation velocity $V_c = \sqrt{(GM/R_e)}$ where R_e is the equatorial radius.

Successive synthetic models allowing for Doppler and Stark broadening are shown here for $V_{rot}\sin i = 0$, 100, 200 km/s.



$V_{rot} \sin i$?

Many early-type OB stars are observed to be rotating rapidly (Be stars close to critical rotation), so this is the major broadening mechanism in these stars. Why sin(i)? **Inclination is rarely known**, except for eclipsing binaries.

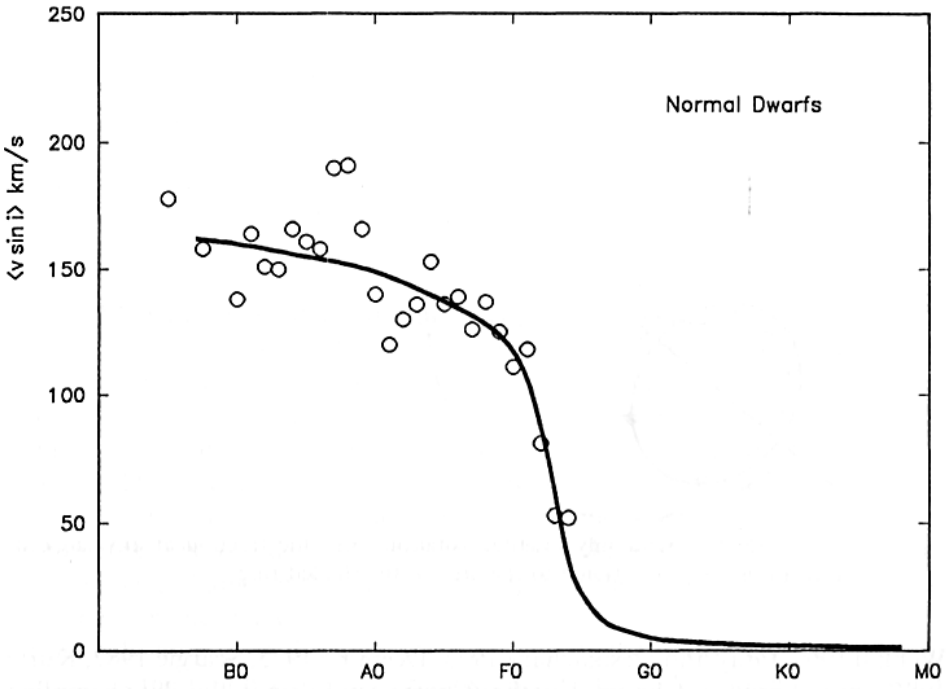


Fig. 17.16. The average rotation rates are shown for spectral intervals as a function of spectral type. (Data are from Uesugi and Fukuda (1982), Soderblom (1983), and Gray (1982b, 1984b).)

Other broadening mechanisms

- **Turbulent Broadening:** In addition to microscopic (thermal) and macroscopic (rotation) motions, there are other motions in stellar atmospheres which are introduced, operating on microscopic (**microturbulence**) and macroscopic (**macroturbulence**) scales, via convolutions with Gaussian velocity distribution
- **Isotope splitting:** Different isotopes have different nuclear mass and so slightly different term energies – the effect is greatest for hydrogen (e.g. deuterium vs hydrogen).
- **Zeeman splitting:** Magnetic fields split magnetically sensitive lines – at optical wavelengths the splitting is seen as line broadening, towards the IR the splitting becomes more noticeable since it increases as λ^2 versus λ for Doppler broadening.

Turbulent broadening

- Added to thermal broadening in quadrature. The Gaussian line profile (normalized to unity) remains.
  Recall the convolution of two Gaussian profiles!

$$\phi(\nu) = \frac{1}{\sqrt{\pi}\sqrt{\Delta\nu_D^2 + \xi_t^2}} \exp[-(\nu - \nu_0)^2 / (\Delta\nu_D^2 + \xi_t^2)]$$

  where ξ_t is a microturbulence velocity.

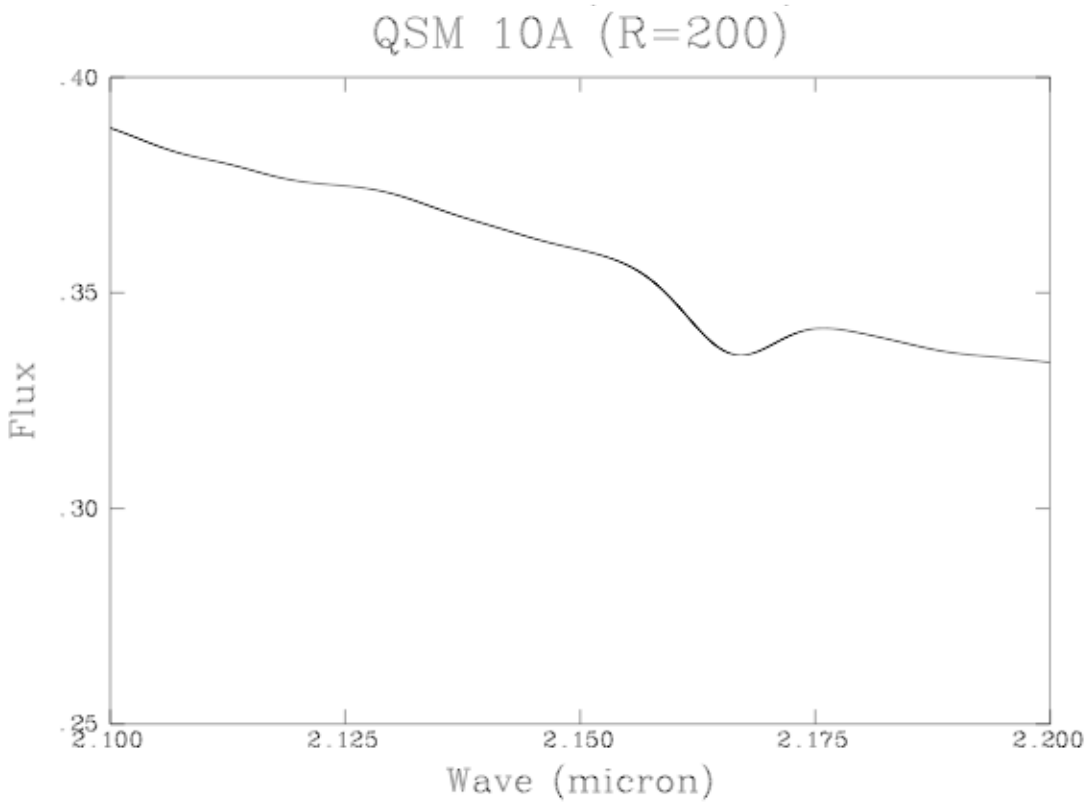
- Note that the broadening because of microturbulence does not depend on the mass of an atom!

Instrumental Broadening

Any spectrograph used to observe a star has a finite resolution ($R=\lambda/\Delta\lambda$), regardless of the sharpness of the spectral line. For low resolution data (necessary when observing faint objects), this may affect the observed line profile more than everything else.

High (R=20,000),medium (R=2,000) and low (R=200) resolution Solar spectra at 2microns.

Faint stars with intrinsically narrow lines are generally broadened the most by the spectrograph!



Summary

- Final profile is a convolution of all the key broadening processes
- Convolution of Lorentzian profiles: $\Gamma_{total}=\Sigma\Gamma_i$
- Convolution of Lorentzian and Doppler broadening yields a Voigt profile
- Pressure/collisional broadening via linear Stark broadening (only for hydrogenic ions), quadratic Stark broadening (interaction with electrons – hot stars) or Van der Waals broadening (interaction between neutral atoms – cool stars)
- Inglis-Teller relation allows estimate of N_e from overlapping Balmer lines in hot stars
- Non-pressure broadening mechanisms include microscopic (thermal Doppler), macroscopic (rotational Doppler), turbulent, Zeeman, instrumental.
- Line profiles typically have characteristic Voigt profiles – Gaussian (thermal) cores and Lorenzian (pressure) wings