Stellar Atmospheres

Lecture 11

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The Forgotten Chapters

RADIATION, ASTROPHYSICAL, AND EDDINGTON FLUX

WAVELENGTH DEPENDENCE OF ABSORPTION COEFFICIENTS

Flux, which flux?

RADIATION FLUX
ASTROPHYSICAL FLUX
EDDINGTON FLUX

Radiation Flux

Radiation flux = netto energy going through area ⊥ z-axis.
If there is no azimuthal (ϕ) dependence in I_λ, the **radiation flux** F_λ is

$$F_\lambda = 2\pi \int_{-1}^{1} I(\mu)\, \mu\, d\mu = F^+ - F^- \qquad \mu = \cos\theta$$

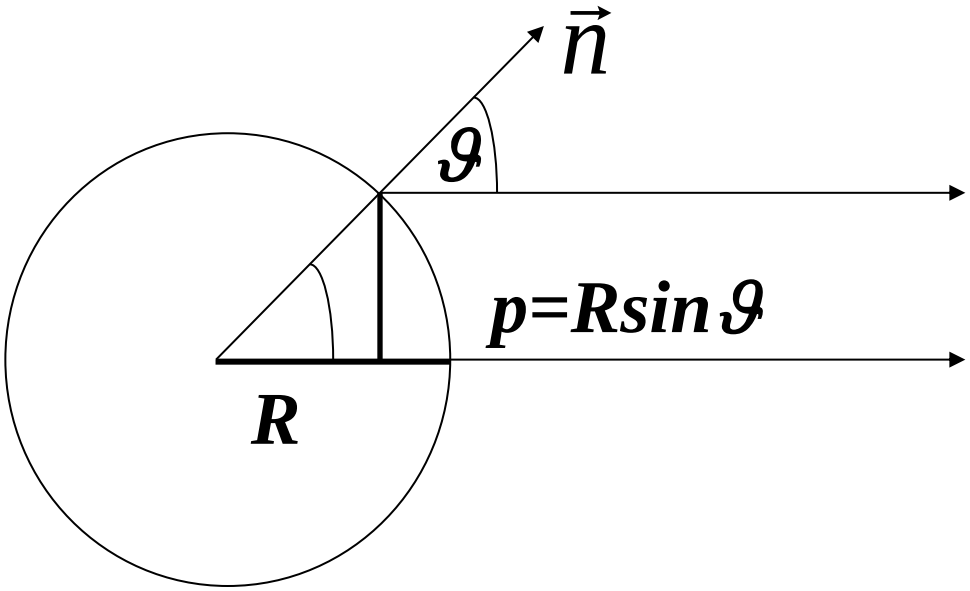
Netto = Outwards – Inwards.

However, there are other definitions of flux.
Don't be confused!

F_λ - astrophysical flux
H_λ - Eddington flux
$F_\lambda = \pi F_\lambda = 4\pi H_\lambda$

Idea behind definition of astrophysical flux

Intensity averaged over stellar disk = astrophysical flux



$$p = R\sin\vartheta$$

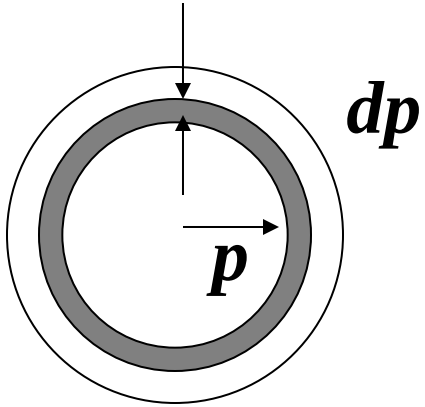
$$p^2 = R^2(1-\mu^2)$$

$$2p\frac{dp}{d\mu} = -2R^2\mu$$

$$pdp = -R^2\mu d\mu$$

Idea behind definition of astrophysical flux

Intensity averaged over stellar disk = astrophysical flux



$$\bar{I}_\nu = \frac{1}{\pi R^2}\int_0^R I_\nu(p)2\pi\, p dp$$

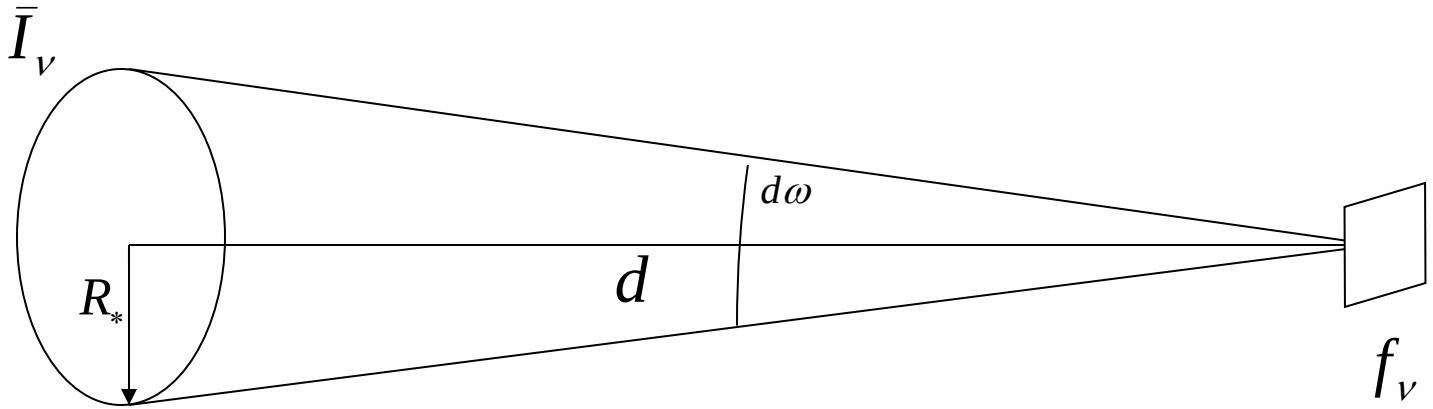
$$= \frac{1}{\pi R^2}\int_0^1 I_\nu(\mu)2\pi R^2 \mu d\mu$$

$$= F_\nu^+ / \pi = \mathrm{F}_\nu^+$$

$\mathrm{F}_\nu^- = 0$ no inward flux at stellar surface

$$\boxed{\bar{I}_\nu = \mathrm{F}_\nu}$$

Flux at location of observer



$$\vec{F}_\nu = \oint_{4\pi} I_\nu(\vec{n})\, \vec{n}\, d\omega$$

Flux at distant observer's detector normal to the line of sight:

$$f_\nu = \bar{I}_\nu d\omega = \bar{I}_\nu \pi R_*^2 / d^2 = \pi \mathrm{F}_\nu \frac{R_*^2}{d^2} = F_\nu \frac{R_*^2}{d^2}$$

Idea behind definition of Eddington flux

In 1-dimensional geometry the n-th moments of intensity are

0-th moment: $$J_\nu = \frac{1}{2}\int_{-1}^{1} I(\mu)\, d\mu$$

1st moment: $$H_\nu = \frac{1}{2}\int_{-1}^{1} I(\mu)\, \mu d\mu$$

2nd moment: $$K_\nu = \frac{1}{2}\int_{-1}^{1} I(\mu)\, \mu^2 d\mu$$

n-th moment: $$= \frac{1}{2}\int_{-1}^{1} I(\mu)\, \mu^n d\mu$$

Mean Intensity, Flux and K-integral

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- The *mean* intensity J_λ is the directional average of the specific intensity (over 4π steradians):

$$J_\lambda = \frac{1}{4\pi}\oint I_\lambda d\omega$$

- Flux F_λ is the projection of the specific intensity in the radial direction (integrated over all solid angles):

$$F_\lambda = \oint I_\lambda \cos\theta\, d\omega$$

- There is also a K-integral which we will use later on:

$$K_\lambda = \frac{1}{4\pi}\oint I_\lambda \cos^2\theta\, d\omega$$

Mean Intensity, Flux and K-integral

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- The *mean* intensity J_λ is the directional average (over 4π steradians) of the specific intensity [0-th moment of intensity]:

$$J_\lambda \equiv \frac{1}{4\pi}\oint I_\lambda d\omega = \frac{2\pi}{4\pi}\int_{-1}^{1} I(\mu)\, d\mu = \frac{1}{2}\int_{-1}^{1} I(\mu)\, d\mu$$

- Eddington flux H_λ, is the directional average (over 4π steradians) of the projection of the specific intensity [1st moment of intensity]:

$$H_\lambda = \oint I_\lambda \cos\theta\, d\omega = \frac{2\pi}{4\pi}\int_{-1}^{1} I(\mu)\, \mu\, d\mu = \frac{1}{2}\int_{-1}^{1} I(\mu)\, \mu\, d\mu$$

- K-integral [2nd moment of intensity]:

$$K_\lambda = \frac{1}{4\pi}\oint I_\lambda \cos^2\theta\, d\omega = \frac{2\pi}{4\pi}\int_{-1}^{1} I(\mu)\, \mu^2\, d\mu = \frac{1}{2}\int_{-1}^{1} I(\mu)\mu^2 d\mu$$

Moments of intensity

- The *mean* intensity J_λ is the directional average (over 4π steradians) of the specific intensity [0-th moment of intensity]:

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- **Eddington** flux $H_{\lambda,,}$ is the directional average (over 4π steradians) of the projection of the specific intensity [1st moment of intensity]:

$$H_\lambda = \frac{1}{4\pi}\oint I_\lambda \cos\theta\, d\omega = \frac{2\pi}{4\pi}\int_{-1}^{1} I(\mu)\,\mu\, d\mu = \frac{1}{2}\int_{-1}^{1} I(\mu)\,\mu\, d\mu$$

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Idea behind definition of Eddington flux

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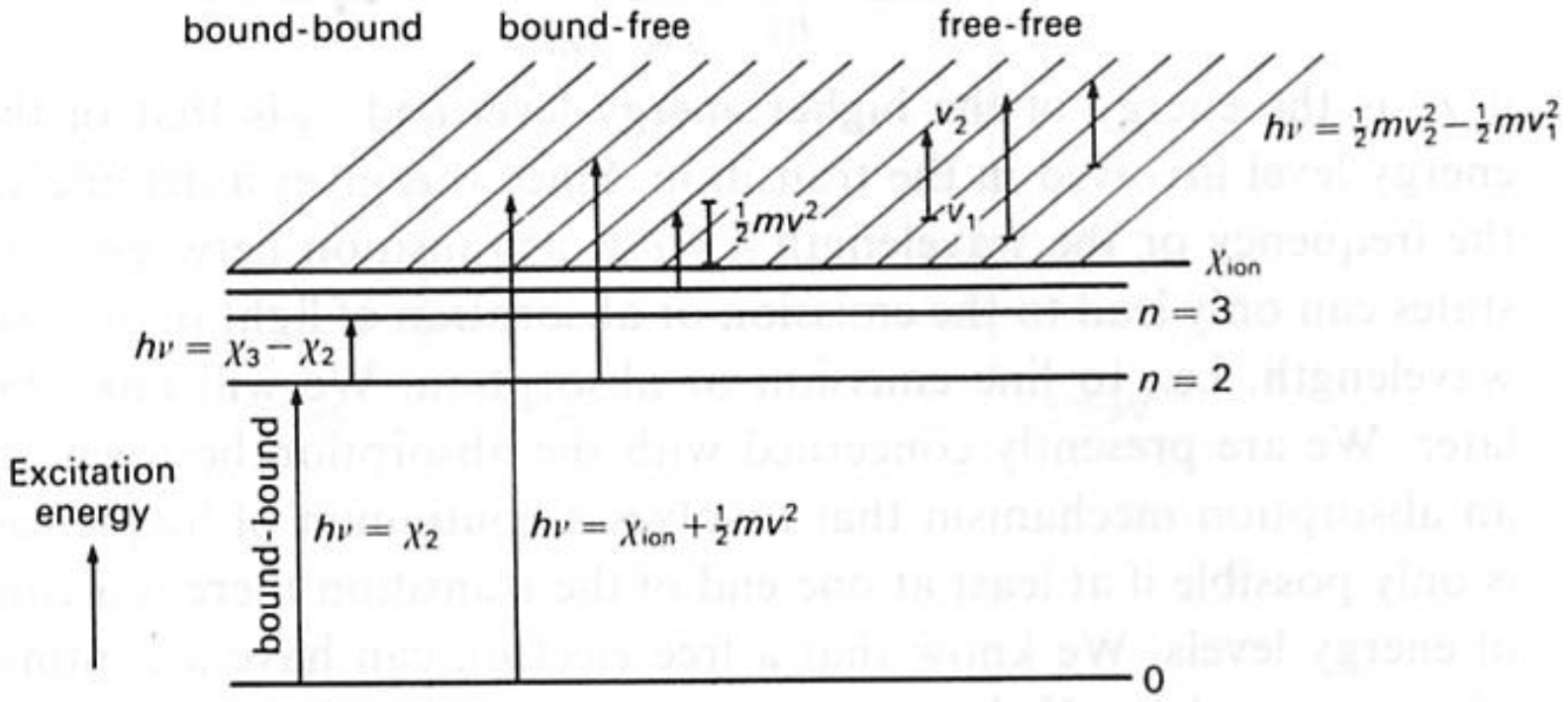
Wavelength dependence of absorption coefficients

bb, bf, ff-processes



1 eV=11600 K =1.602 10^{-12} erg

χ_{ion}=13.6 eV, χ_{ion}/k=158000 K

Bound-free transition

- Photons need an energy great enough to overcome the ionization energy i.e. $h\nu > \chi_{ion} - \chi_n$ or $\lambda < hc/(\chi_{ion} - \chi_n)$. At long wavelengths only energy levels with very large χ_n can contribute to the absorption coefficient $\boldsymbol{\alpha}$, so most continuous opacity is from mainly free-free transitions.
- The contribution of level n will start at $\lambda_n = hc/(\chi_{ion} - \chi_n)$ and continue for shorter λ. There is a **discontinuity** at λ_n because of a sudden change in the number of absorbing atoms, e.g.

  Lyman jump (912A) due to the contribution of **n=1**.
  Balmer jump (3647A) due to the contribution of **n=2**.

- To derive $\boldsymbol{\alpha_\lambda}$, the total absorption at wavelength λ, we have to multiply σ_n by the number of atoms in this state and sum up all states n that contribute at this wavelength. For this we need to use the Boltzmann formula. $\boldsymbol{\alpha_\lambda = \sigma_\lambda n}$

Bound-free absorption coefficient

From Lecture 7: Kramers approximation for continuous cross-section for level *n* of hydrogen:

Gaunt factor ≈ 1

$$\sigma_{bf}(\mathrm{H}) = a_0 \frac{\lambda^3}{n^5} G_{bf} \text{ cm}^2 \text{ per neutral H atom}$$

Huge dependence on n

$a_0 = 1.0449 \times 10^{-26}$ for λ in angstroms

The photoionization threshold is $E_n = h\nu_{nc} = \chi_{ion} - \chi_n$,
so σ_{nc} decreases with ν (increases with λ).
For H, at the threshold, $\sigma_{1c} = 6.3 \times 10^{-18}$ cm^2

The total absorption coefficient for H is:

$$\alpha_{bf}^{H}(\lambda) = \sum_{n > \sqrt{\chi_{ion}/hc}}^{\infty} \sigma_{nc}(\lambda) N_n$$

Example: Lyman continuum

$$\tau_\lambda = \int_0^S \kappa_\lambda \rho ds = \int_0^S \sigma_\lambda n ds$$

Absorption by interstellar medium (ISM) at the Lyman edge:

$$\tau_\lambda = \int_0^S \sigma_{1c}(\lambda)\, N_{ISM} ds = \bar{N}_{ISM}\, \sigma_{1c} S$$

$$\bar{N}_{ISM} \approx 1\mathrm{cm}^3$$

$$\tau_\lambda = 1 \text{ at } \quad S = \frac{1}{N_{ISM}\ \sigma_{1c}} = 1.5 \times 10^{17}\mathrm{cm} = \frac{1}{20}\,\mathrm{pc}$$

Impossible to observe distant objects at λ< 912Å

Wavelength dependence of α(H)

- Consider the H absorption coefficient α (per atom) for T=5040K (Θ=5040/T=1). Let us compare the value of α in the Balmer (n=2) to Lyman (n=1) continua at 912A:

$$\frac{\alpha(Balmer)}{\alpha(Lyman)} = \frac{\sigma_{i2}}{\sigma_{i1}}\frac{N_2}{N_1} = \frac{\sigma_{i2}g_2}{\sigma_{i1}g_1}e^{-(10.2eV/kT)} = \frac{\sigma_{i2}g_2}{\sigma_{i1}g_1}10^{(-10.2\times 5040\,\mathrm{K}/T)}$$

- From above, $\sigma_n \propto n^{-5}$ and $g_n = 2n^2$ so $\dfrac{\alpha(Balmer)}{\alpha(Lyman)} = \dfrac{2^{-5}\times 8}{1\times 2}6.3\times10^{-11} = 8\times10^{-12}$

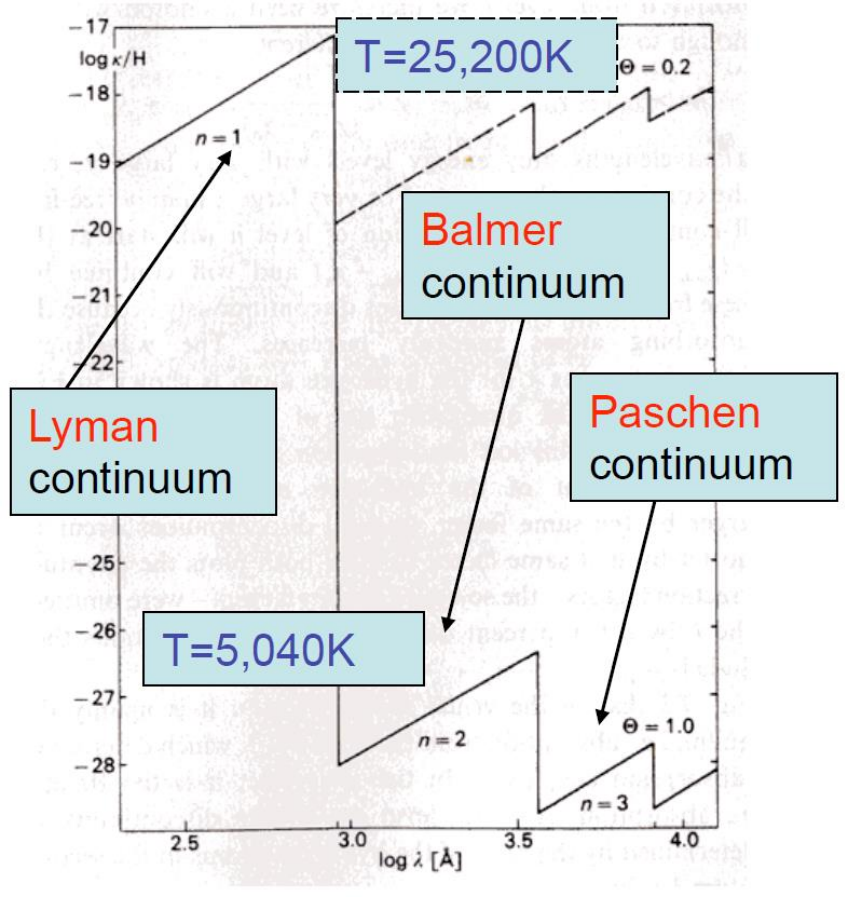
- There is a huge difference in hydrogen absorption coefficient at 912Å (Lyman edge) at T=5040K.
- Similar calculations at T=25 200K (Θ=5040/T=0.2) give

$$\frac{\alpha(Balmer)}{\alpha(Lyman)} = \frac{2^{-5}\times 8}{1\times 2}0.009 = 0.001$$

- Hydrogen absorption coefficient is very T sensitive!

Wavelength dependence of α(H)

- Primarily, the Paschen continuum (absorption from n=3) determines the H absorption coefficient in the visual (3647Å<λ<8205Å).



Balmer jump – why useful?

DETERMINING TEMPERATURES (OR DENSITY)

Balmer jump

- In hot stars, T>9000K, H^- negligible, only H contributes to opacity.

$$\frac{\alpha^+}{\alpha^-} = \frac{\sigma^+(\mathrm{H})}{\sigma^-(\mathrm{H})} \frac{N_H(n=3)}{N_H(n=2)}$$

Function of T only

"observed" known From Boltzmann law(T)

Thus we can obtain the temperature.

In cooler stars (Solar-type)

small

$$\frac{\alpha^+}{\alpha^-} = \frac{\sigma(H^-)N(H^-) + \sigma^+(H)N_H(n=3)}{\sigma(H^-)N(H^-) + \sigma^-(H)N_H(n=2)}$$

$$N(H^-) = N_H(n=1) n_e f(T) \quad \text{Saha eq} \Rightarrow \quad \frac{\alpha^+}{\alpha^-}(n_e, T)$$

One of n_e or T can be determined

$$\text{if } n_e \uparrow \text{ then } \frac{\alpha^+}{\alpha^-} \to 1$$