Stellar Atmospheres
Lecture 6

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Local Thermodynamic Equilibrium (LTE)

MAXWELLIAN VELOCITY DISTRIBUTION
BOLTZMANN EQUATION
SAHA EQUATION

Thermodynamic Equilibrium (TE)

- All particles have Maxwellian distribution in velocities (with the same temperature *T*).
- Atom populations follow Boltzmann law (same T).
- Ionization is described by Saha formula (same T).
- Radiation intensity is given by the Planck function (same T).
- The principle of detailed equilibrium is valid (the number of direct processes = number of inverse processes).

In **Local thermodynamic equilibrium (LTE)**,
1-3 are applied **locally**.

The radiation spectrum can be very far from Planck function.

Local Thermodynamic Equilibrium

- Interaction of radiation and matter is the most important physical process in stellar atmospheres.
- To find $\boldsymbol{I_\lambda}$ we need to know α_λ and ε_λ (or k_λ and j_λ) – absorption and emission coefficients.
- To find α_λ and ε_λ, density ρ, temperature T, and chemical composition X are not enough. We need to know distributions of atoms over levels and ionization states, which depend on radiation $\boldsymbol{I_\lambda}$.
- In LTE, ρ, T, *and* X fully determine α_λ and ε_λ.

LTE

In the study of stellar atmospheres, the assumption of Local Thermodynamic Equilibrium (**LTE**) is described by:

1. Electron and ion velocity distributions are Maxwellian.
2. Excitation equilibrium is given by Boltzmann equation.
3. Ionization equilibrium is given by Saha equation (introduced today).
4. The source function is given by the Planck function

$$S_\lambda = I_\lambda = B_\lambda(T) \qquad \text{i.e. Kirchoff's law} \qquad j_\lambda = \kappa_\lambda B_\lambda(T)$$

Is LTE a valid assumption?

- For LTE to be valid, the photon and particle mean free paths need to be much smaller than the length scale over which these temperature changes significantly.
- Radiation cannot play a role in defining atom populations and ionization state. Collisions should dominate.
- Generally, when collisional processes dominate over radiative processes in the excitation and ionization of atoms, the state of the gas is close to LTE.
- Consequently, **LTE is a good assumption in stellar interiors, but may break down in the atmosphere**. If LTE is no longer valid, all processes need to be calculated in detail via **non-LTE**. This is much more complicated, but needs to be considered in some cases (see later in course).

Mean Free Path

- In the Sun, the characteristic distance over which the temperature varies (the temperature scale height) is ~500km. How does this scale compare with the average distance travelled by an atom before hitting another atom?
- The density of the Solar photosphere is $\rho = 2.5 \times 10^{-7}$ g/cm^3 so the number of H atoms/cm^3 is $n(H) = \rho / m_H = 1.5 \times 10^{17}$ cm^3 where m_H is the mass of the H atom. Two atoms will collide if their centres pass within a radius of 2 Bohr radii ($2a_o$) of each other. The collision cross-section of the atom is $\sigma = \pi(2a_o)^2 = 3.5 \times 10^{-16}$ cm^2.
- The mean free path between collisions is $1/\sigma \times 1/n(H) = 0.02$ cm. i.e. atoms are confined within a limited volume of space in the photosphere at effectively fixed temperature (relative to the temperature scale height).
- In contrast, since the photosphere is the layer visible from Earth, photons must be able to escape freely into space, after ~10^{21} scatterings and re-emissions (thousands years!) from the centre.
- In the upper layers, radiation dominates over collisions → **out of LTE**

Maxwellian velocity distribution

Gas pressure is produced by the motions of the gas particles. The velocities of particles are distributed in a Maxwellian distribution (also called the Maxwell–Boltzmann distribution).

$$\frac{\mathrm{d}N(v)}{N_{\text{total}}} = \left(\frac{2}{\pi}\right)^{1/2}\left(\frac{m}{kT}\right)^{3/2} v^2 \mathrm{e}^{-mv^2/2kT}\,\mathrm{d}v$$

Because the particles produce Doppler shifts, the line of sight velocities have a distribution that is an important special case for spectroscopy:

$$\frac{\mathrm{d}N(v_{\mathrm{R}})}{N_{\text{total}}} = \left(\frac{m}{2\pi kT}\right)^{3/2} \mathrm{e}^{-mv_{\mathrm{R}}^2/2kT}\,\mathrm{d}v_{\mathrm{R}}$$

where v_{R} is the radial (line of sight) velocity component.

Maxwellian velocity distribution

The maximum of the speed distribution occurs at v_1 (the most probable velocity):

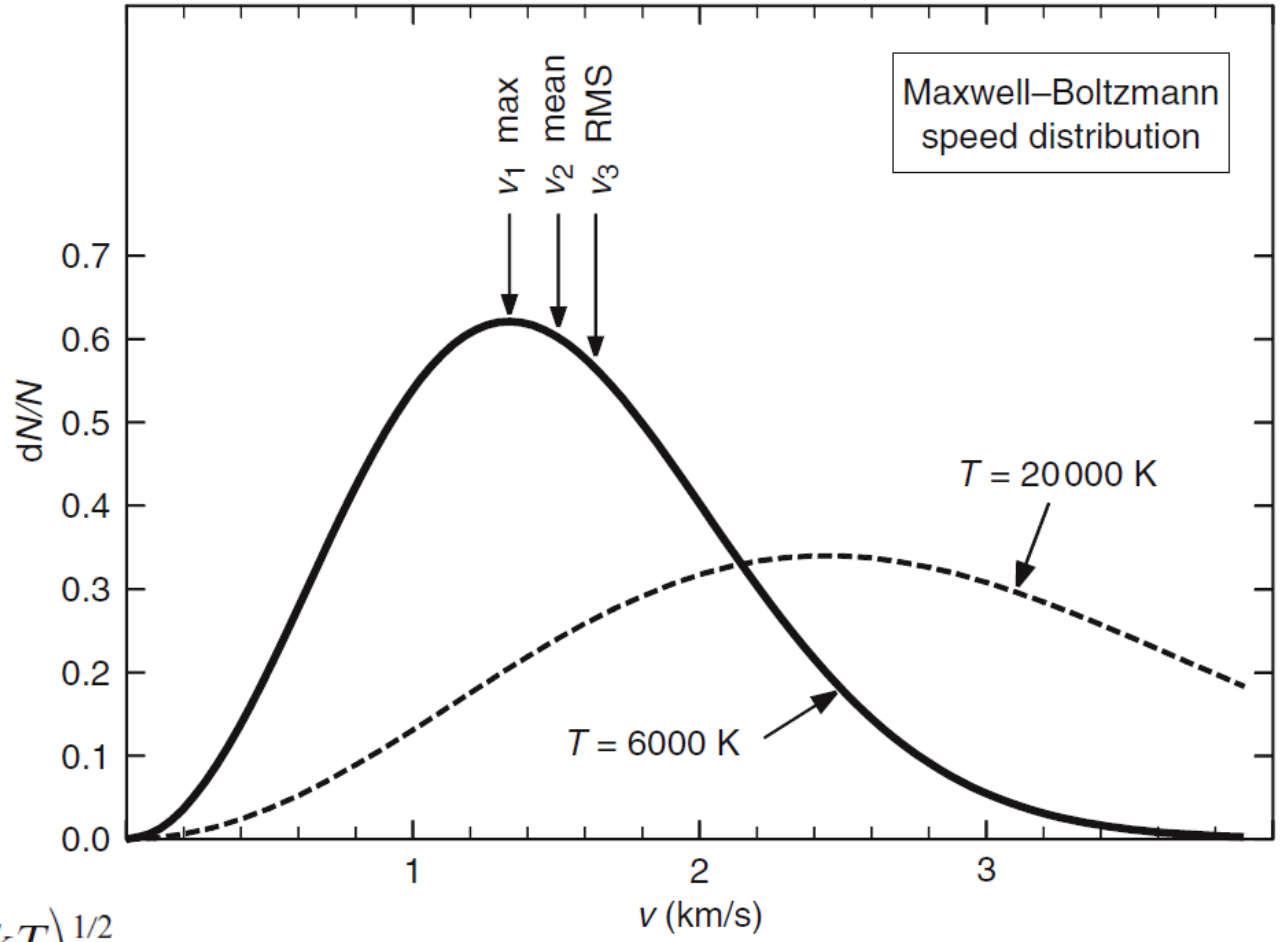
$$v_1 = \left(\frac{2kT}{m}\right)^{1/2}$$

The average velocity, v_2, is

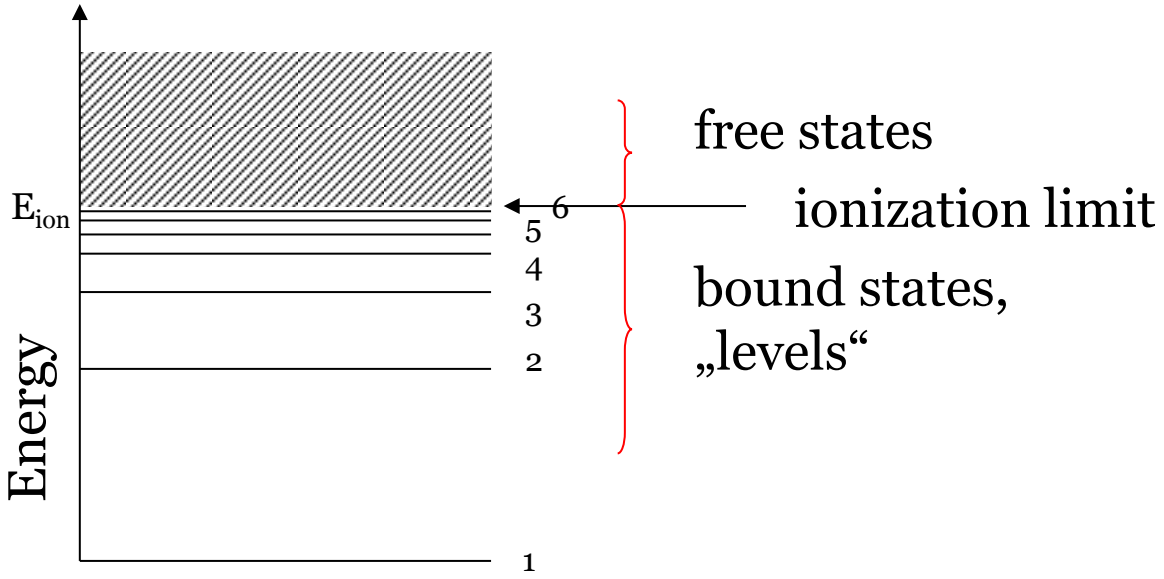
$$v_2 = \left(\frac{8}{\pi}\frac{kT}{m}\right)^{1/2} = 1.128 v_1$$

The root mean square velocity, v_3, is

$$v_3 = \left(\frac{3kT}{m}\right)^{1/2} = 1.225 v_1$$



Boltzmann equation



For excited levels ***u*** and ***l*** of e.g. atomic hydrogen, the **Bolzmann equation** relates their population (occupation) numbers as follows:

$$\frac{N_u}{N_l} = \frac{g_u}{g_l} e^{-(E_u - E_l)/kT}$$

where $\chi_{ul} = E_u - E_l$ is the energy difference between the levels, g_u & g_l are their statistical weights (see next slide), k=8.6174x10^{-5} eV/K is the Boltzmann constant.

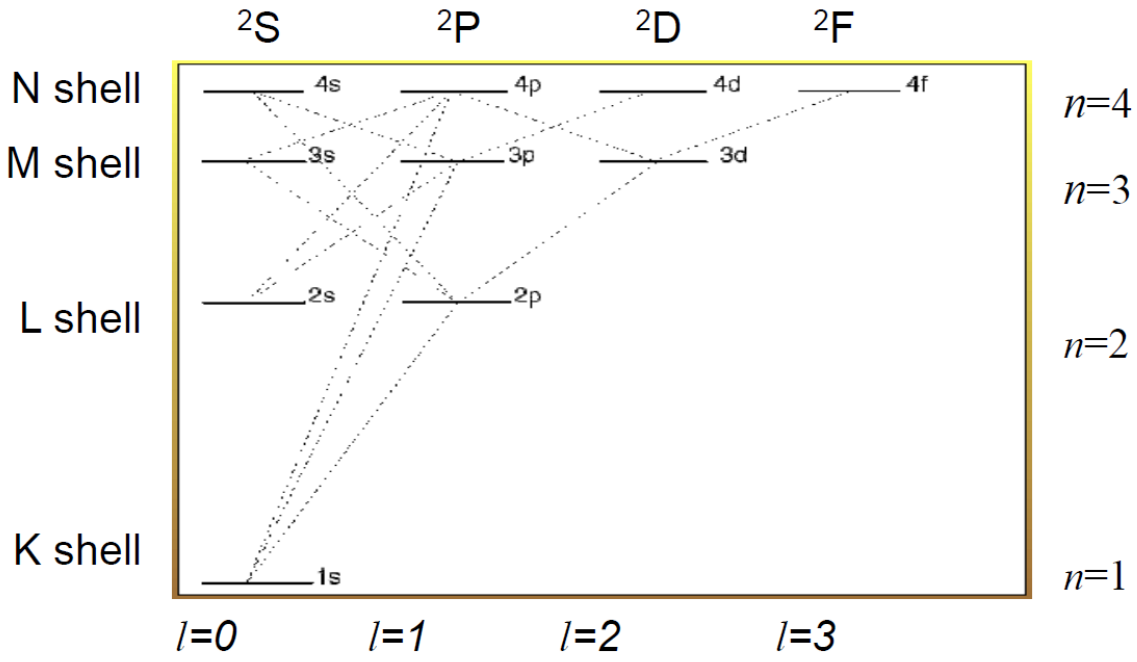
Boltzmann equation may also be written as:

$$\log\frac{N_u}{N_l} = \log\frac{g_u}{g_l} - \frac{5040}{T}\chi_{ul}(eV)$$

$\Theta = 5040/T$

In the "ground state" (n=1) or "first excited state" (n=2) of H more than one quantum state may have the same energy.

The number of these for orbital ***n*** is the **statistical weight, g_n,** (also known as the degeneracy).



Hydrogen

For H, orbital n has a statistical weight of $g_n=2n^2$ – the various permutations for n=1 and n=2 are listed here, with statistical weights $g_1=2$ and $g_2=8$, respectively.

l=0..n-1 azimuthal quantum number
m_l=magnetic quantum number with $-l \leq m_l \leq l$
m_s=electron "spin" angular momentum $\pm 1/2$

Transition energy between levels u and l:

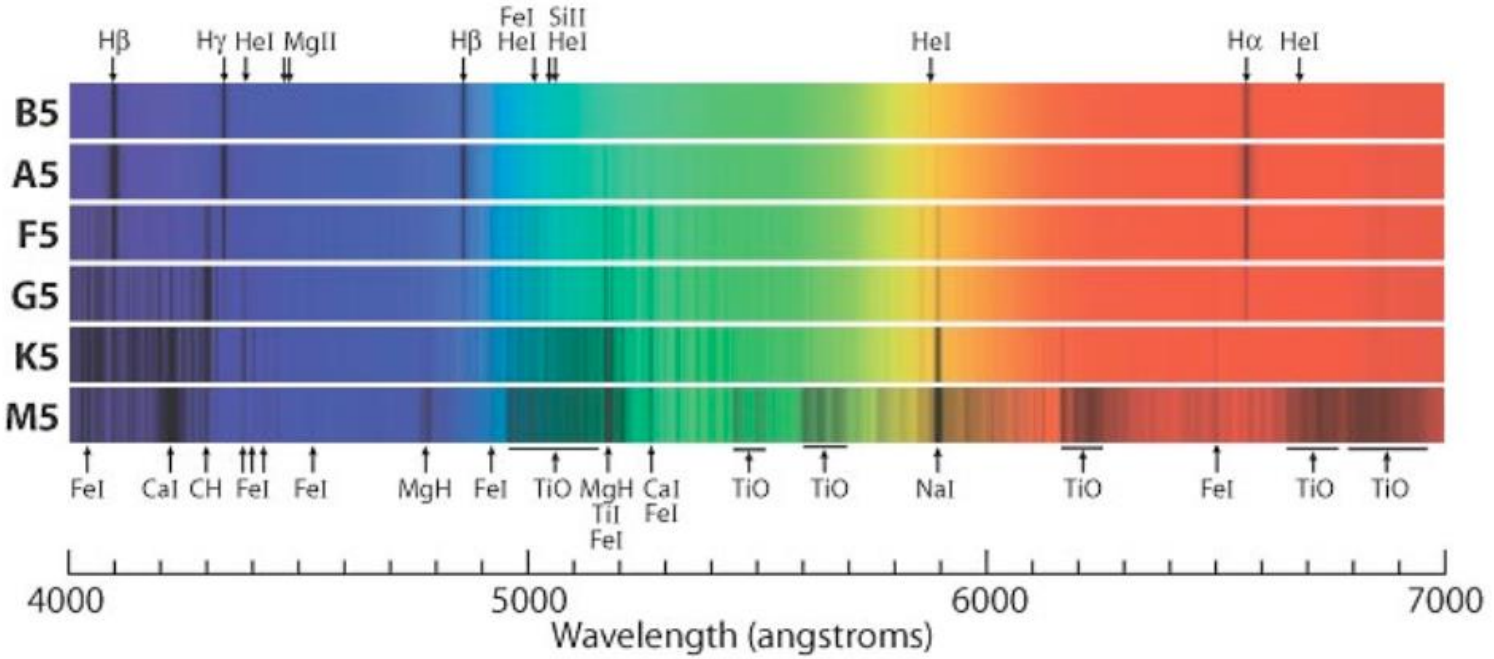
$$\chi_{ul} = C\left(\frac{1}{l^2} - \frac{1}{u^2}\right)$$

where $C=\chi_{ion}$=-13.6 eV

Ground States s_1				Energy E_1
n	ℓ	m_ℓ	m_s	(eV)
1	0	0	$+1/2$	-13.6
1	0	0	$-1/2$	-13.6
First Excited States s_2				Energy E_2
n	ℓ	m_ℓ	m_s	(eV)
2	0	0	$+1/2$	-3.40
2	0	0	$-1/2$	-3.40
2	1	1	$+1/2$	-3.40
2	1	1	$-1/2$	-3.40
2	1	0	$+1/2$	-3.40
2	1	0	$-1/2$	-3.40
2	1	-1	$+1/2$	-3.40
2	1	-1	$-1/2$	-3.40

Balmer lines

An exceptionally high T is required for a significant number of H atoms to have electrons in their 1st excited states. The Balmer lines (involving an upward transition from n=2 orbital) reach a peak strength at spectral class A (≈10000K)



so why do the Balmer lines **diminish** in strength at higher temperatures?

We need Saha equation to answer this question.

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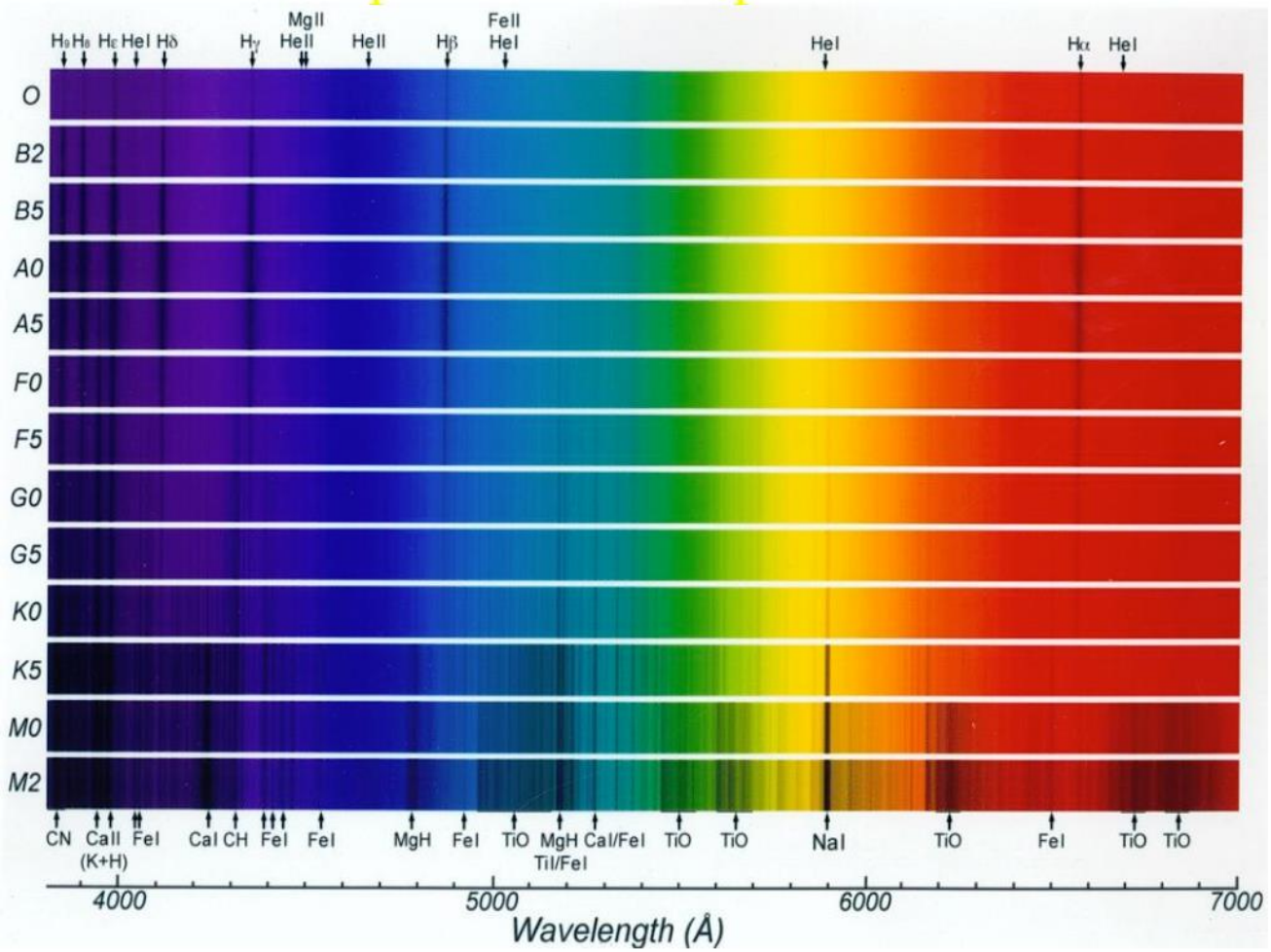
Balmer lines



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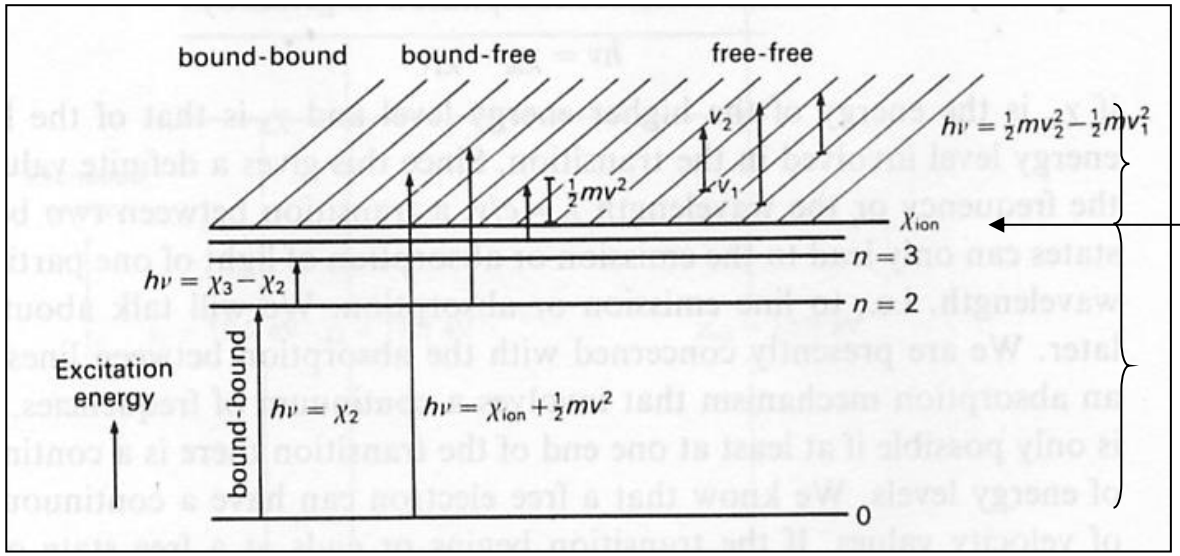
Derivation of the Saha Equation

We can extend the Boltzmann formula to states with positive energies (the upper state is now an ion plus free electron, with energy $\chi_{ion}+1/2m_e v^2$).

Let us consider the simplest case of the lower and upper states being the ground states of the neutral atom (e.g. H^0) and singly ionized ion (H^+).

$$\frac{dN_1^+(v)}{N_l} = \frac{g_1^+ g_e}{g_l} e^{-(\chi_{ion}+\frac{1}{2}m_e v^2)/kT}$$

Here m_e is the mass of the electron, χ_{ion} is the ionization energy, $dN^+_1(v)$ is the number of ions in their ground state with the free electron having a velocity in the interval $(v, v+dv)$, g_1^+ and g_e are the statistical weight of the ground state of the ion & the electron.



ionization limit

The (differential) statistical weight of the electron, g_e, i.e. the number of available states in interval $(v,v+dv)$ is

$$g_e = \frac{1}{N_e}\frac{8\pi m_e^3 v^2 dv}{h^3}$$

Inserting this into Boltzmann's equation reveals

$$\frac{dN_1^+(v)}{N_1} = \frac{g_1^+}{N_e g_1}\frac{8\pi m_e^3}{h^3} e^{-(\chi_{ion}+\frac{1}{2}m_e v^2)/kT} v^2 dv$$

Since we don't care about v, we can integrate over all velocities, substituting $x = v\sqrt{(2 m_e kT)}$ and using

$$\int_0^\infty x^2 e^{-x^2} dx = \frac{\sqrt{\pi}}{4}$$

Finally, we arrive at **the Saha equation** (Meghnad Saha 1920):

$$\frac{N_1^+}{N_1} = \frac{2 g_1^+}{N_e g_1} \frac{(2\pi m_e kT)^{3/2}}{h^3} e^{-\chi_{ion}/kT}$$

This relates the ground state populations of the atom and ion. To derive the ratio of the total number of ions (N^+) to the total number of atoms (N^0) we can use the conventional Boltzmann formula for each level n of the atom and ion, N_n/N_1 and N_n^+/N_1^+ i.e.

$$\frac{N_n}{N_1} = \frac{g_n}{g_1} e^{-\chi_n/kT} \qquad \frac{N_n^+}{N_1^+} = \frac{g_n^+}{g_1^+} e^{-\chi_n^+/kT}$$

Partition function

If N^0 is the sum of *all neutral* particles in their different quantum states:

$$N^0 = N_1^0 + \sum_{n=2}^{\infty} N_n^0 = N_1^0 + \frac{N_1^0}{g_1} \sum_{n=2}^{\infty} g_n \, e^{-\chi_n/kT}$$

We find:

$$N^0 = \frac{N_1^0}{g_1} (g_1 + \sum_{n=2}^{\infty} g_n \, e^{-\chi_n/kT}) = \frac{N_1^0}{g_1} u^0(T)$$

Where we have introduced u^0, the partition function of the atom. This is the weighted sum of the number of ways it can arrange its electrons with the same energy - e.g. all H is in the ground state for the Solar case, so $u^0 \approx 2$ (the ground state statistical weight). Similarly for the ion,

$$N^+ = N_1^0 + \frac{N_1^+}{g_1^+} u^+(T) \qquad u^+(T) = g_1^+ + \sum_{n=2}^{\infty} g_n^+ \, e^{-\chi_n^+/kT}$$

For H^+, $u^+ = 1$, since no electrons left.

Partition function (2)

If we multiply N_1^+/N_1^0 from earlier by N^+/N_1^+ and N_1^0/N^0 we again obtain the **Saha equation** :

$$\frac{N^+ N_e}{N^0} = \frac{2u^+}{u^0}\frac{(2\pi m_e kT)^{3/2}}{h^3} e^{-\chi_{ion}/kT} = 4.83 \times 10^{15} \frac{u^+}{u^0} T^{3/2} e^{-\chi_{ion}/kT}$$

In logarithmic form Saha equation can be written as:

$$\log \frac{N^+}{N^0} = \log \frac{u^+}{u^0} + \log 2 + \frac{5}{2}\log T - \chi_{ion}\Theta - \log P_e - 0.48$$

where χ_{ion} is measured in eV, $\Theta = 5040/T$ and the electron pressure P_e is related to the electron density via the ideal gas law ($P_e = N_e kT$). In stellar atmospheres, P_e lies in the range 1 dyn/cm^2 (cool stars) to 1000 dyn/cm^2 (hot stars).

High temperature favours ionization, high pressure favours recombination.

Note that 1dyn/cm^2=0.1N/m^2 (SI units), so for SI calculations the final constant is -1.48 instead of -0.48

Partition functions (Gray App D2)

Table D.2. *Partition functions, log u(T).*

	θ										
	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	log g_0
H	0.368	0.303	0.301	0.301	0.301	0.301	0.301	0.301	0.301	0.301	0.301
He	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
He^+	0.301	0.301	0.301	0.301	0.301	0.301	0.301	0.301	0.301	0.301	0.301
Li	–	0.987	0.488	0.359	0.320	0.308	0.304	0.302	0.302	0.302	0.301
Be	–	0.328	0.087	0.025	0.007	0.002	0.001	0.000	0.000	0.000	0.000
Be^+	0.541	0.334	0.307	0.302	0.301	0.301	0.301	0.301	0.301	0.301	0.301
B	1.191	0.831	0.786	0.778	0.777	0.777	0.777	0.777	0.777	0.776	0.778
B^+	0.435	0.051	0.006	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
C	1.163	1.037	0.994	0.975	0.964	0.958	0.954	0.951	0.950	0.948	0.954
C^+	0.853	0.782	0.775	0.774	0.773	0.772	0.771	0.770	0.769	0.767	0.778
C^{++}	0.143	0.010	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
N	1.060	0.729	0.645	0.616	0.606	0.603	0.602	0.602	0.602	0.602	0.602
N^+	1.073	0.993	0.965	0.953	0.946	0.942	0.939	0.937	0.934	0.932	0.954
O	1.095	0.991	0.964	0.953	0.947	0.944	0.941	0.939	0.937	0.935	0.954
O^+	0.895	0.655	0.614	0.604	0.602	0.602	0.602	0.602	0.602	0.602	0.602
F	0.788	0.772	0.768	0.765	0.762	0.759	0.756	0.753	0.750	0.747	0.778
F^+	1.034	0.968	0.949	0.940	0.935	0.930	0.926	0.923	0.919	0.915	0.954
Ne	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Ne^+	0.771	0.766	0.760	0.754	0.748	0.743	0.737	0.732	0.727	0.723	0.778
Na	4.316	1.043	0.493	0.357	0.320	0.309	0.307	0.306	0.306	0.306	0.301
Na^+	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Mg	2.839	0.478	0.110	0.027	0.007	0.002	0.001	0.001	0.001	0.000	0.000

$\Theta = 5040/T$

Partition functions (Gray, old edition)

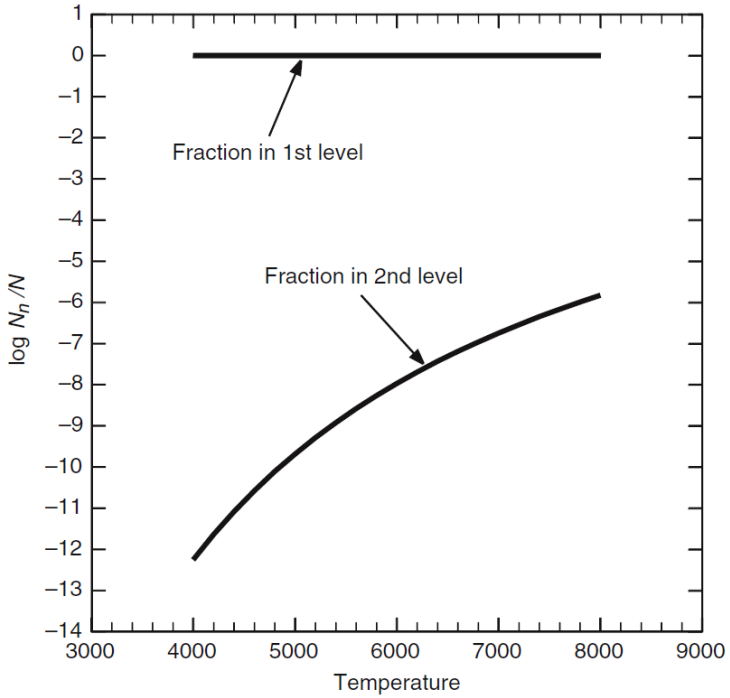
$$\log u(T) = c_0 + c_1 \log \Theta + c_2 \log^2 \Theta + c_3 \log^3 \Theta + c_4 \log^4 \Theta$$

$\Theta = 5040/T$

Element		c_0	c_1	c_2	c_3	c_4
H	1	0.30103	−0.00001			
He	2	0.00000	0.00000			
He^+		0.30103	0.00000			
Li	3	0.31804	−0.20616	0.91456	−1.66121	1.04195
Be	4	0.00801	−0.17135	0.62921	−0.58945	
Be^+		0.30389	−0.00819			
B	5	0.78028	−0.01622			
B^+		0.00349	−0.01035			
C	6	0.96752	−0.09452	0.08055		
C^+		0.77239	−0.02540			
N	7	0.60683	−0.08674	0.30565	−0.28114	
N^+		0.94968	−0.06463	−0.01291		
O	8	0.05033	−0.05703			
O^+		0.60405	−0.03025	0.04525		
F	9	0.76284	−0.03582	−0.05619		
Ne	10	0.00000	0.00000			
Ne^+		0.74847	−0.06562	−0.07088		
Na	11	0.30955	−0.17778	1.10594	−2.42847	1.70721
Mg	12	0.00556	−0.12840	0.81506	−1.79635	1.26292
Mg^+		0.30257	−0.00451			
Al	13	0.76786	−0.05207	0.14713	−0.21376	
Al^+		0.00334	−0.00995			
Si	14	0.97896	−0.19208	0.04753		
Si^+		0.75647	−0.05490	−0.10126		
P	15	0.64618	−0.31132	0.68633	−0.47505	
P^+		0.93588	−0.18848	0.08921	−0.22447	
S	16	0.95254	−0.15166	0.02340		
S^+		0.61971	−0.17465	0.48283	−0.39157	
Cl	17	0.74465	−0.07389	−0.06965		
Cl^+		0.92728	−0.15913	−0.01983		
K	19	0.34419	−0.48157	1.92563	−3.17826	1.83211
Ca	20	0.07460	−0.75759	2.58494	−3.53170	1.65240
Ca^+		0.34383	−0.41472	1.01550	0.31930	
Sc	21	1.08209	−0.77814	1.78504	−1.39179	
Sc^+		1.35894	−0.51812	0.15634		
Ti	22	1.47343	−0.97220	1.47986	−0.93275	
Ti^+		1.74561	−0.51230	0.27621		
V	23	1.68359	−0.82055	0.92361	−0.78342	
V^+		1.64112	−0.74045	0.49148		
Cr	24	1.02332	−1.02540	2.02181	−1.32723	
Cr^+		0.85381	−0.71166	2.18621	−0.97590	−2.72893
Mn	25	0.80810	−0.39108	1.74756	−3.13517	1.93514
Mn^+		0.88861	−0.36398	1.39674	−1.86424	−2.32389
Fe	26	1.44701	−0.67040	1.01267	−0.81428	
Fe^+		1.63506	−0.47118	0.57918	−0.12293	
Co	27	1.52929	−0.71430	0.37210	−0.23278	
Ni	28	1.49063	−0.33662	0.08553	−0.19277	
Ni^+		1.03800	−0.69572	0.53893	0.28861	

Degree of ionization of H in stars

- We can use Saha to study the degree of ionization of H in general in stellar photospheres. The fraction of ionized hydrogen to the total is defined below. We find that H switches from mostly neutral below 7000K to mostly ionized above 11000K for typical N_e. This allows us to understand why hydrogen lines are strongest in A-type stars, with temperatures of 7500-10000K.



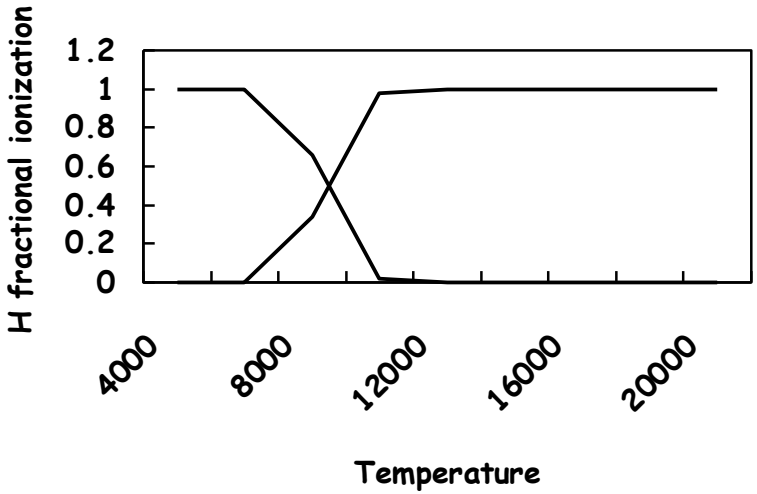
$$\frac{H^+}{H} = \frac{H^+}{H^0 + H^+} = \frac{H^+/H^0}{1 + H^+/H^0}$$

$$\frac{N^+ N_e}{N^0} = 2.4 \times 10^{15}\, T^{3/2} e^{-158000/T}$$

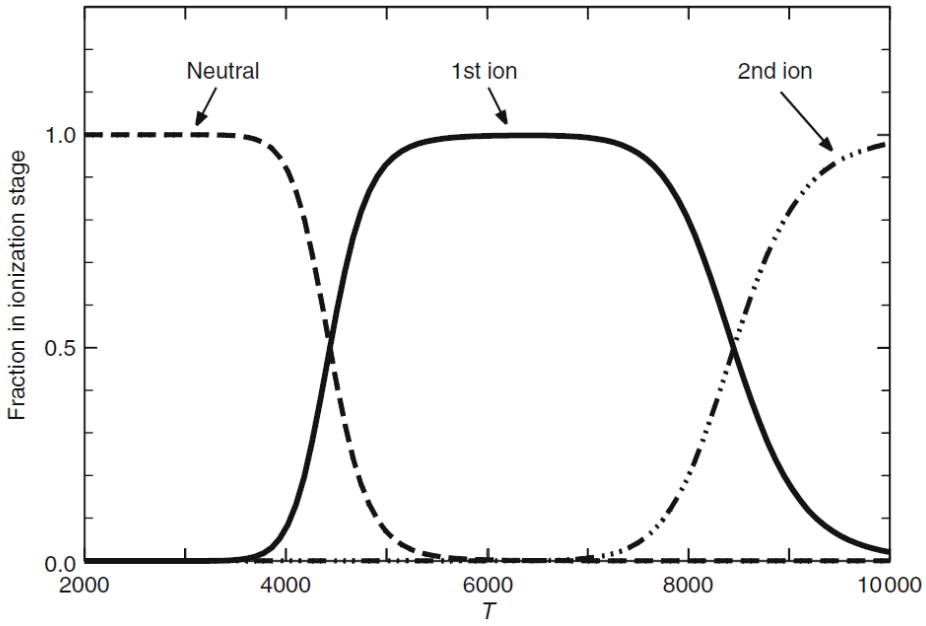
Using 1eV per particle, the hydrogen is heated from 0 to 10^4 K. Supplying 13.6 eV more, the temperature increases only up to 2x 10^4 K. Ionization is an extremely energy consuming process. Ionization happens within a very small temperature interval.

Degree of ionization in stars

Hydrogen:



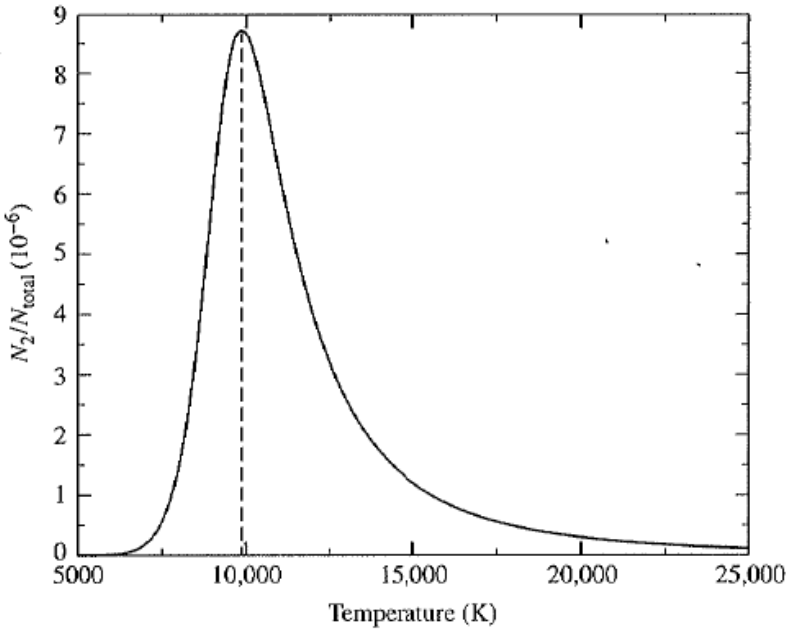
Iron:



As temperature increases, ionization occurs rather abruptly. In a stellar photosphere, elements exist mainly in just two ionization stages.

Strong Balmer lines in A stars – why?

From today's first example, a very high T was required to populate level $n=2$ of H relative to the ground state. We can now use the Boltzmann & Saha equations to measure H($n=2$)/H(total) as a function of T. For increasing T, the n=2 population increases due to the Boltzmann equation, reaching a maximum value around 10,000K (equivalent to A spectral type) and then reduces as H becomes mostly ionized. This is why A stars have strong Balmer lines.



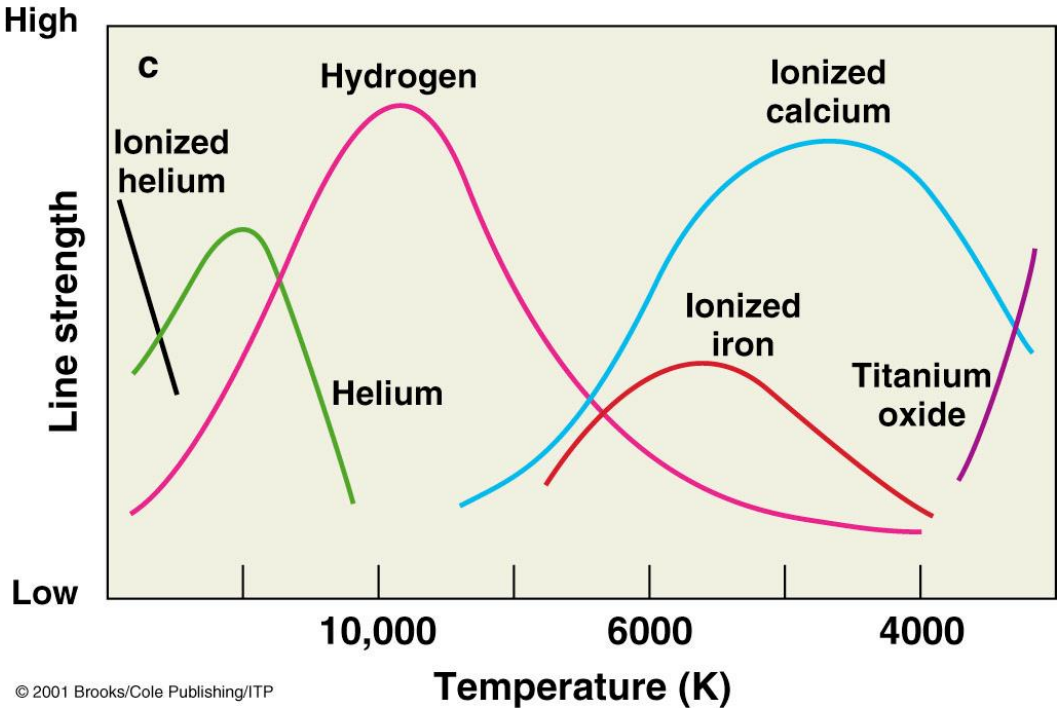
Note: He in stellar atmospheres complicates this calculation since ionized He provides excess electrons with which H ions can recombine, so it takes higher temperatures to achieve the same degree of ionization.

Strong lines in Solar photosphere

λ	Element	W(Å)	Name	λ	Element	W(Å)	Name
3581.21	Fe I	2.14	N	4920.51	Fe I	0.43	
3719.95	Fe I	1.66		4957.61	Fe I	0.45	
3734.87	Fe I	3.03	M	5167.33	Mg I	0.65	b_4
3749.50	Fe I	1.91		5172.70	Mg I	1.26	b_2
3758.24	Fe I	1.65		5183.62	Mg I	1.58	b_1
3770.63	H_{11}	1.86		5232.95	Fe I	0.35	
3797.90	H_{10}	3.46		5269.55	Fe I	0.41	
3820.44	Fe I	1.71	L	5324.19	Fe I	0.32	
3825.89	Fe I	1.52		5238.05	Fe I	0.38	
3832.31	Mg I	1.68		5528.42	Mg I	0.29	
3835.39	H_9	2.36		5889.97	Na I	0.63	D_2
3838.30	Mg I	1.92		5895.94	Na I	0.56	D_1
3859.92	Fe I	1.55		6122.23	Ca I	0.22	
3889.05	H_8	2.35		6162.18	Ca I	0.22	
3933.68	Ca II	20.25	K	6562.81	H_α	4.02	C
3968.49	Ca II	15.47	H	6867.19	O_2	tell	B
4045.82	Fe I	1.17		7593.70	O_2	tell	A
4101.75	H_δ	3.13	h	8194.84	Na I	0.30	
4226.74	Ca I	1.48	g	8498.06	Ca II	1.46	
4310 ± 10	—	—	G	8542.14	Ca II	3.67	
4340.48	H_γ	2.86		8662.17	Ca II	2.60	
4383.56	Fe I	1.01		8688.64	Fe I	0.27	
4861.34	H_β	3.68		8736.04	Mg I	0.29	
4891.50	Fe I	0.31					

Ca II in the Sun

- The photosphere of the Sun has only two calcium atoms for every million H atoms, yet the Ca II **H and K lines** (produced by the ground state of singly ionized calcium, Ca^{+}) are ***stronger*** than the Balmer lines of H (produced by the 1st excited state of neutral H). Why?



Saha-Boltzmann applied to Ca

From the Saha equation we have found that **H** is essentially neutral in the Solar photosphere, yet from the Boltzmann formula **$H(n=2)/H(n=1)=5\times10^{-9}$.** i.e. very little H is available to produce Balmer absorption lines.

For Calcium, χ_{ion}=6.1eV, and partition functions may be determined from tables via

$$\log u\,(T) = c_0 + c_1 \log \Theta + c_2 \log^2 \Theta + c_3 \log^3 \Theta + c_4 \log^4 \Theta$$

For Θ=5040/T=0.872, the partition function of neutral Ca

$$\log u^0\,(T) = 0.075 - 0.757 \log \Theta + 2.58 \log^2 \Theta + 3.53 \log^3 \Theta - 1.65 \log^4 \Theta = 0.13$$

i.e. **$u^0=1.3$**. Similarly, **$u^+=2.3$**.

$$\log\frac{Ca^+}{Ca^0} = \log\frac{2.3}{1.3} + \log 2 + 9.40 - 5.34 - 1.18 - 0.48 = +2.95$$

Essentially *all* Calcium is singly ionized. N(Ca^+) in the first excited state relative to the ground state (g_1=2, g_2=4, χ=3.12eV) is 1/265 from Boltzmann eqn, so nearly all Calcium in the Sun's photosphere is in the ground state of Ca^+.

Combining these results:

$N(Ca^+_{g.s.})/N(H_{n=2}) = N(Ca^+_{g.s.})/N(Ca) \times N(Ca)/N(H) \times N(H)/N(H_{n=2}) = 400$

There are 400 times more Ca^+ ions with electrons in the ground-state (which produce the Ca II H&K lines) than there are neutral H atoms in the first excited state (which produce Balmer lines).

The Ca II lines in the Sun are so strong due to T dependence of excitation and ionization (not high Ca/H abundance).

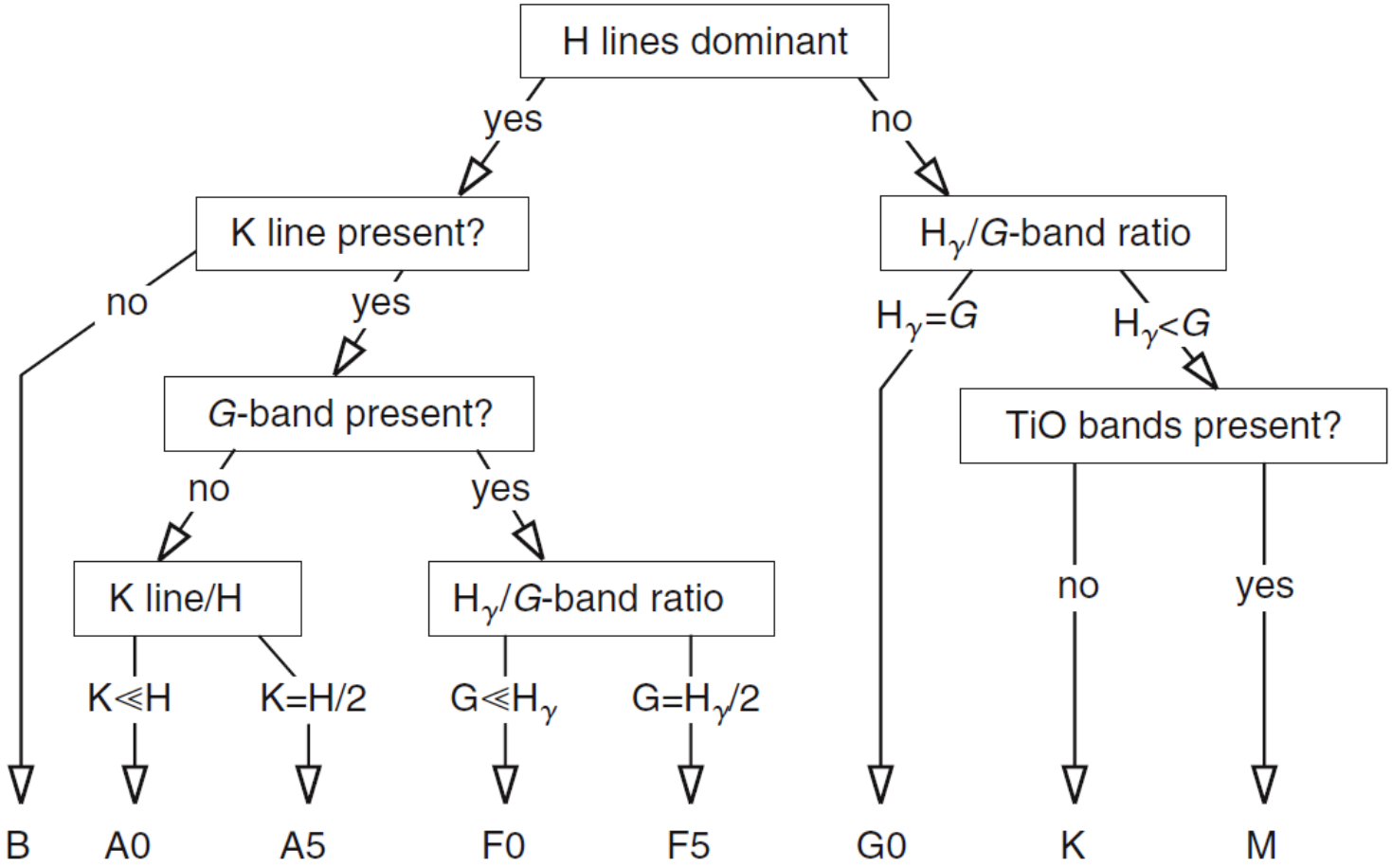
Ionization Potentials

Atom		Stage of ionization												
		I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII	XIV
1	H	13.598 44													
2	He	~~24.587 41~~	54.417 78												
3	Li	5.391 72	75.640 18	122.454											
4	Be	9.322 63	18.211 16	153.897	217.713										
5	B	8.298 03	25.154 84	37.931	259.366	340.22									
6	C	11.260 30	24.383 32	47.888	64.492	392.08	489.98								
7	N	14.534 14	29.601 3	47.449	77.472	97.89	552.06	667.03							
8	O	13.618 06	35.117 30	54.936	77.413	113.90	138.12	739.29	871.41						
9	F	17.422 82	34.970 82	62.708	87.140	114.24	157.17	185.19	953.91	1 103.1					
10	Ne	21.564 54	40.963 28	63.45	97.12	126.21	157.93	207.28	239.10	1 195.8	1 362.2				
11	Na	5.139 08	47.286 4	71.620	98.91	138.40	172.18	208.50	264.25	299.9	1 465.1	1 648.7			
12	Mg	7.646 24	15.035 28	80.144	109.265	141.27	186.76	225.02	265.96	328.1	367.5	1 761.8	1 963		
13	Al	5.985 77	18.828 56	28.448	119.99	153.83	190.49	241.76	284.66	330.1	398.8	442.0	2 086	2 304	
14	Si	8.151 69	16.345 85	33.493	45.142	166.77	205.27	246.49	303.54	351.1	401.4	476.4	523	2 438	2 673
15	P	10.486 69	19.769 4	30.203	51.444	65.03	220.42	263.57	309.60	372.1	424.4	479.5	561	612	2 817
16	S	10.360 01	23.337 9	34.79	47.222	72.59	88.05	280.95	328.75	379.6	447.5	504.8	564	652	707
17	Cl	12.967 64	23.814	39.61	53.465	67.8	97.03	114.20	348.28	400.1	455.6	529.3	592	657	750
18	Ar	15.759 62	27.629 67	40.74	59.81	75.02	91.01	124.32	143.46	422.5	478.7	539.0	618	686	756
19	K	~~4.340 66~~	31.63	45.806	60.91	82.66	99.4	117.56	154.88	175.8	503.8	564.7	629	715	787
20	Ca	6.113 16	11.871 72	50.913	67.27	84.50	108.78	127.2	147.24	188.5	211.3	591.9	657	727	818
21	Sc	~~6.561 44~~	12.799 67	24.757	73.489	91.65	111.68	138.0	158.1	180.0	225.2	249.8	688	757	831
22	Ti	6.828 2	13.575 5	27.492	43.267	99.30	119.53	140.8	170.4	192.1	215.9	265.1	292	788	863
23	V	6.746 3	14.66	29.311	46.71	65.28	128.1	150.6	173.4	205.8	230.5	255.1	308	336	896
24	Cr	6.766 64	16.485 7	30.96	49.16	69.46	90.64	161.18	184.7	209.3	244.4	270.7	298	355	384
25	Mn	7.434 02	15.639 99	33.668	51.2	72.4	95.6	119.20	194.5	221.8	248.3	286.0	314	344	404
26	Fe	7.902 4	16.187 8	30.652	54.8	75.0	99.1	124.98	151.06	233.6	262.1	290.2	331	361	392
27	Co	7.881 0	17.083	33.50	51.3	79.5	103	131	160	186.2	276.2	305	336	379	411
28	Ni	7.639 8	18.168 84	35.19	54.9	75.5	108	134	164	193	224.6	321	352	384	430
29	Cu	7.726 38	20.292 40	36.841	55.2	79.9	103	139	167	199	232	266	369	401	435
30	Zn	9.394 05	17.964 40	39.723	59.4	82.6	108	136	175	203	238	274	311	412	454

More from Saha

- Another observational effect that can be understood using the Saha equation is that supergiants and giants have *lower temperatures* than dwarfs of the same spectral type.
- Spectral classes are defined by line ratios of different ions, e.g. He II 4542A / He I 4471 for O stars. At higher temperatures the fraction of He II will increase relative to He I, so the above ratio will increase.
- However, supergiants have lower surface gravities (or pressure) than main-sequence stars, so from Saha equation a lower P_e at the same temperature will give a higher ion fraction, N^+/N^0
- Assuming a given spectral class corresponds to a fixed ratio N^+/N^0, a star with a lower pressure can have a lower $T_{\rm eff}$ for the same ratio and spectral class

Spectral classification



Summary

- LTE = Maxwell + Boltzmann + Saha.
- Boltzmann equation describes degree of excitation of an atom or ion, e.g. $N(\mathrm{H}_{n=2})/N(\mathrm{H}_{n=1})$.
- Saha equation describes degree of ionization of successive ions, e.g. $N(\mathrm{He}^{+})/N(\mathrm{He}^{0})$ or $N(\mathrm{He}^{2+})/N(\mathrm{He}^{+})$.
- The Partition function is the weighted sum of the number of ways an atom or ion can arrange its electrons with the same energy
- Ionization is an extremely energy consuming process. Ionization happens within a very small temperature interval.
- Saha-Boltzmann explains the spectral type (or temperature) dependence of lines in stellar atmospheres, e.g. Strongest Balmer series at spectral type A and strong CaII lines in Solar-type stars.