Stellar Atmospheres
Lecture 2

VITALY NEUSTROEV

ASTRONOMY RESEARCH UNIT
UNIVERSITY OF OULU

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Basics about radiative transfer

RADIATION TERMS
SPECIFIC AND MEAN INTENSITY
FLUX & LUMINOSITY
BLACK BODY RADIATION (PLANCK FUNCTION)
EFFECTIVE TEMPERATURE
(STEFAN-BOLTZMANN LAW)
BRIGHTNESS AND COLOR TEMPERATURES

Radiation Terms and Definitions

We must have a carefully defined terminology to properly describe light and its interaction with the material in a stellar atmosphere.

Let's start with Specific and mean Intensity, and Flux.

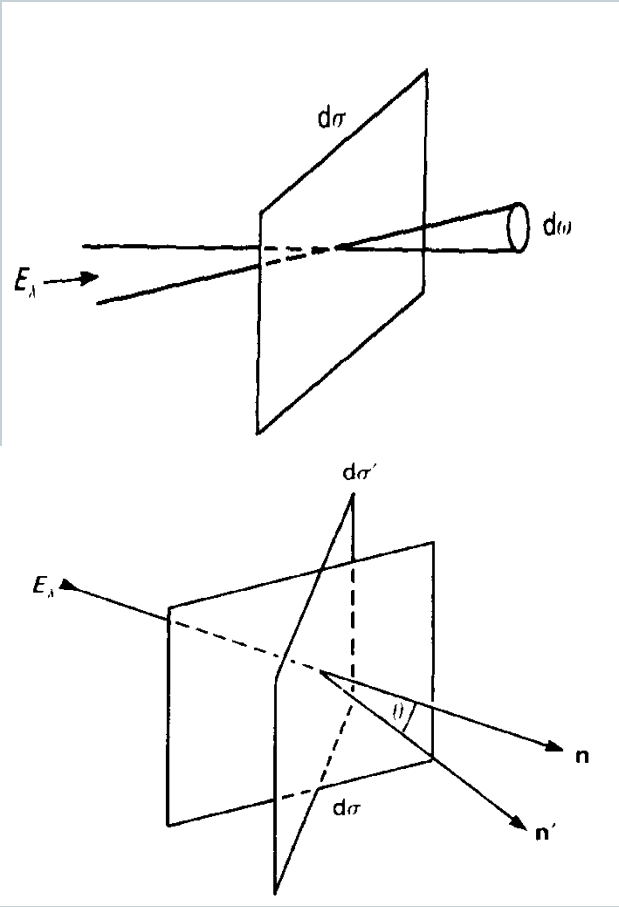
Specific Intensity (1)

Consider light passing through a *perpendicular* surface area dσ in a narrow cone of opening solid angle dω. The amount of energy E_λ passing through this area is given by

$$E_\lambda = I_\lambda \, d\lambda \, d\sigma \, d\omega \, dt$$

Now consider the energy passing through a surface area dσ at an angle θ with respect to the normal of this surface area, the effective beam width is reduced by cos(θ):

$$E_\lambda = I_\lambda \cos\theta \, d\lambda \, d\sigma \, d\omega \, dt$$



Specific Intensity (1)

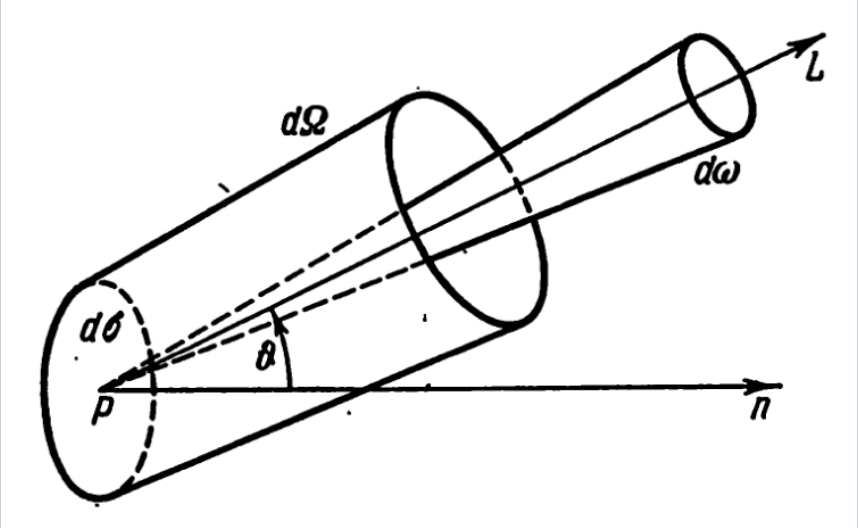
Consider light passing through a *perpendicular* surface area dσ in a narrow cone of opening solid angle dω. The amount of energy E_λ passing through this area per second is given by

$$E_\lambda = I_\lambda \, d\lambda \, d\sigma \, d\omega \, dt$$

Now consider the energy passing through a surface area dσ at an angle θ with respect to the normal of this surface area, the effective beam width is reduced by cos(θ):

$$E_\lambda = I_\lambda \cos\theta \, d\lambda \, d\sigma \, d\omega \, dt$$

Intensity



Possibly this plot is more clear.

Specific Intensity (2)

$$I_\lambda = \frac{E_\lambda}{\cos\theta \, d\lambda \, d\sigma \, d\omega \, dt}$$

The (specific) intensity I_λ is then a measure of brightness with units of erg/(s cm² rad² Å). In model atmosphere calculations, I_λ is obtained from the "transfer equation" – which we shall introduce later on.

I_λ is ***independent of distance*** from the source, and can only be measured directly **if we resolve the radiating surface** (e.g. Sun, nebulae, planets).

Bolometric Intensity

- One can alternatively define intensity in frequency units such that

$$I_\lambda d\lambda = I_\nu d\nu$$

- Note that $I_\lambda \neq I_\nu$! The two spectral distributions have different shapes for the same spectrum. The Solar spectrum has a maximum in the green in I_λ (5175Å), but for I_ν the maximum is in the far-red (8800Å).

$c=\lambda\nu$, $d\nu/d\lambda = -c/\lambda^2$

so equal intervals of λ correspond to different intervals of ν across the spectrum.

Why?

- Integrated (bolometric) intensity is

$$I = \int_0^\infty I_\lambda \, d\lambda = \int_0^\infty I_\nu \, d\nu$$

Mean Intensity and Flux

- The *mean* intensity J_λ is the directional average of the specific intensity (over 4π steradians):

$$J_\lambda = \frac{1}{4\pi} \oint I_\lambda \, d\omega$$

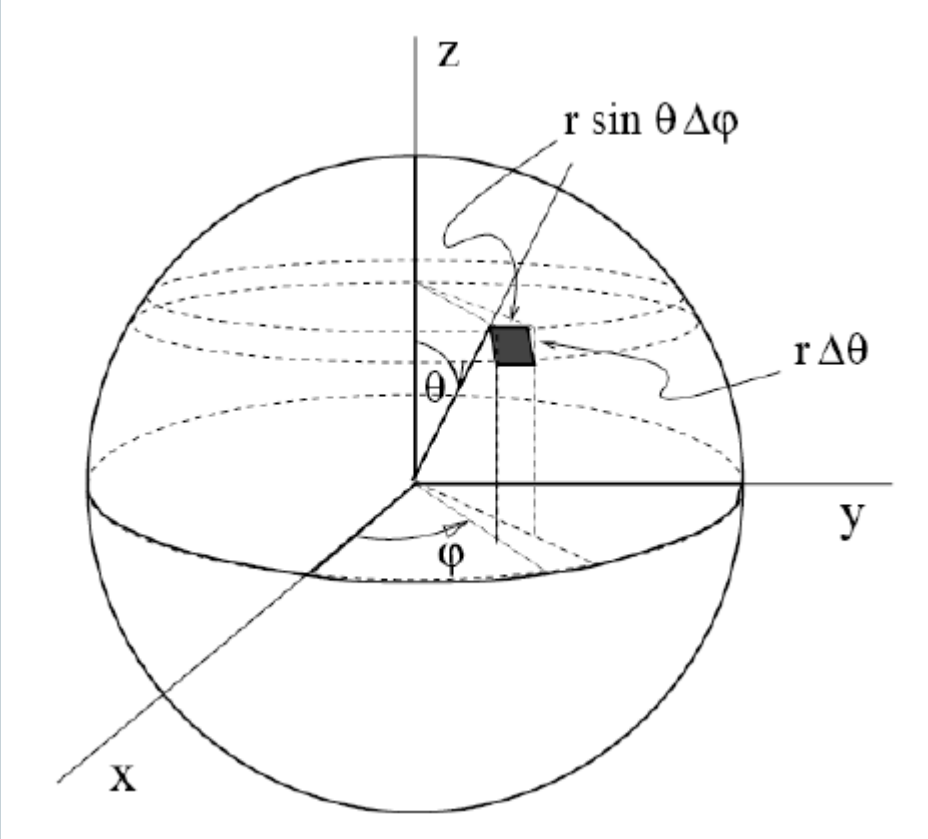
Integrated over the whole unit sphere centered on the point of interest.

- Flux F_λ, is the projection of the specific intensity in the radial direction (integrated over all solid angles):

$$F_\lambda = \oint I_\lambda \cos\theta \, d\omega$$

The amount of energy going through 1 cm² per second per 1Å into the solid angle dω in the direction inclined by the angle θ to the normal of the area.

Solid angle in spherical coordinates



Put the center at the given point of a stellar atmosphere, with the polar axis along the star's radius.

Area of a patch on a unit radius sphere limited by (θ, $\theta+\Delta\theta$) and (φ, $\varphi+\Delta\varphi$) is

$$\Delta\omega = \sin\theta\,\Delta\theta\,\Delta\varphi$$

Flux

Expressing **dω** by means of **θ** and **φ**,

$$F_\lambda = \oint I_\lambda \cos\theta \, d\omega = \int_0^{2\pi} d\varphi \int_0^{\pi} I_\lambda \cos\theta \sin\theta \, d\theta$$

If no flux enters the surface & if there is no azimuthal dependence for I_λ then

$$F_\lambda = \oint I_\lambda \cos\theta \, d\omega = 2\pi \int_0^{\pi} I_\lambda \cos\theta \sin\theta \, d\theta$$

In the plane-parallel or spherical case, we do not find any dependence of I_λ on the longitude **φ**

$$F_\lambda = -2\pi \int_0^{\pi} I_\lambda \cos\theta \, d(\cos\theta)$$

Meaning of flux:

Radiation flux = **netto** energy going through area ⊥ z-axis
Decomposition into two half-spaces:

$$F = -2\pi \int_0^{\pi} I_\lambda \cos\theta \, d(\cos\theta) = 2\pi \int_{-1}^{1} I(\mu)\, \mu \, d\mu \qquad \mu = \cos\theta$$

$$= 2\pi \int_0^1 I(\mu)\, \mu \, d\mu + 2\pi \int_{-1}^{0} I(\mu)\, \mu \, d\mu$$

$$= 2\pi \int_0^1 I(\mu)\, \mu \, d\mu - 2\pi \int_0^1 I(-\mu)\mu \, d\mu = \boldsymbol{F^+ - F^-}$$

Netto = Outwards - Inwards

Special case: isotropic radiation field: $F=0$

Intensity and Flux

I is independent of distance from the source, and can only be measured directly if we resolve the radiating surface. In contrast, ***F*** obeys the inverse square law and is all that may be measured for most stars.

Mean Intensity, Flux and K-integral

- The *mean* intensity J_λ is the directional average of the specific intensity (over 4π steradians):

$$J_\lambda = \frac{1}{4\pi} \oint I_\lambda d\omega$$

- Flux F_λ, is the projection of the specific intensity in the radial direction (integrated over all solid angles):

$$F_\lambda = \oint I_\lambda \cos\theta \, d\omega$$

- There is also a K-integral which we will use later on:

$$K_\lambda = \frac{1}{4\pi} \oint I_\lambda \cos^2\theta \, d\omega$$

Surface Flux and Luminosity

- Integral over frequency at outer boundary (Surface Flux):

$$F_s = \int_0^\infty F_\lambda \, d\lambda$$

- Multiplied by stellar surface area yields the Luminosity, total energy radiated away by the star

$$L = 4\pi R^2 F_s$$

- The total energy arriving above the Earth's atmosphere is its observed flux, ***f***, corrected for the distance to the star, neglecting interstellar absorption:

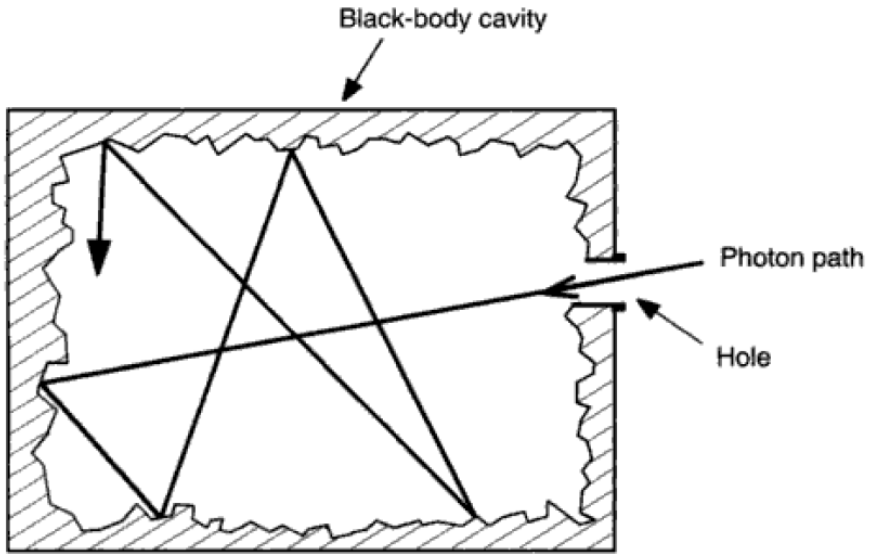
$$L = 4\pi d^2 f$$

- Thus,

$$F_s = f(d/R)^2$$

The Black Body

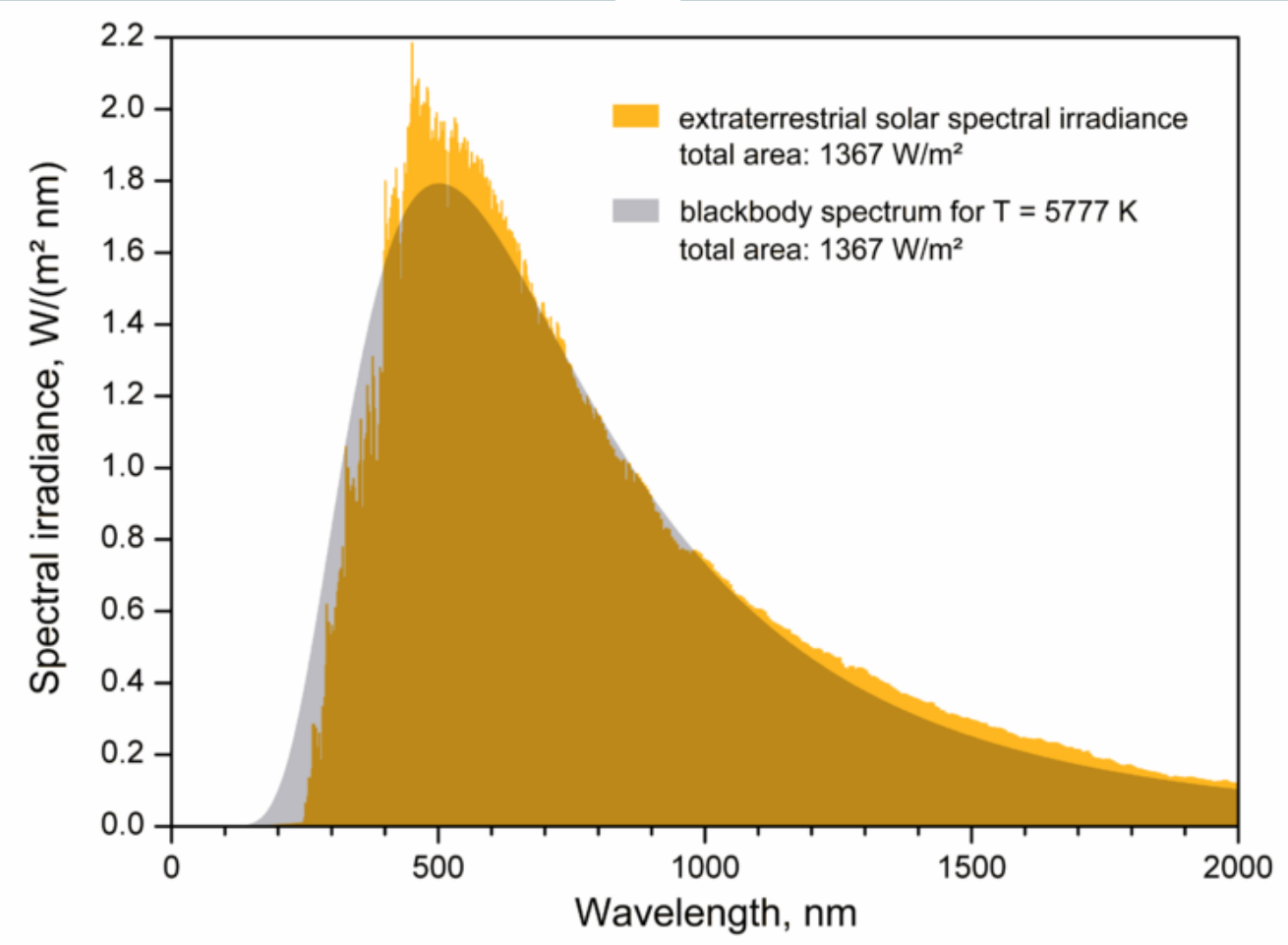
Imagine a box which is completely closed except for a small hole. Any light entering the box will have a very small likely hood of escaping & will eventually be absorbed by the gas or walls. For constant temperature walls, this is in thermodynamic equilibrium.



If this box is heated the walls will emit photons, filling the inside with radiation. A small fraction of the radiation will leak out of the hole, but so little that the gas within it remains in equilibrium. **The emitted radiation is that of a black-body.**

Stars share properties of the black-body emitter, in the sense that a negligibly small fraction of the radiation escapes from each.

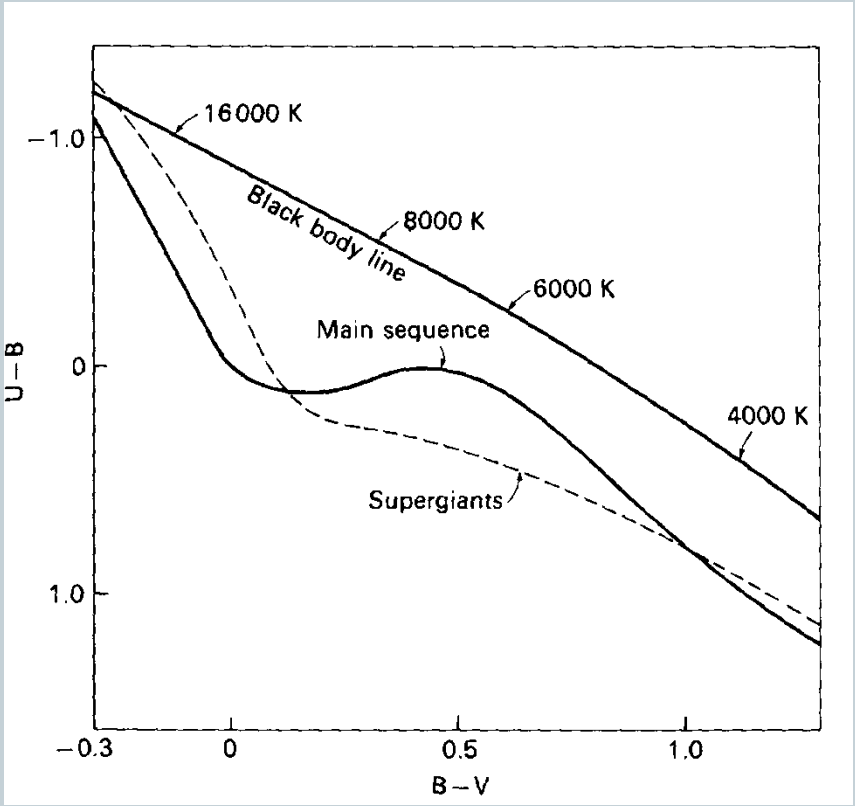
Are stars black bodies?



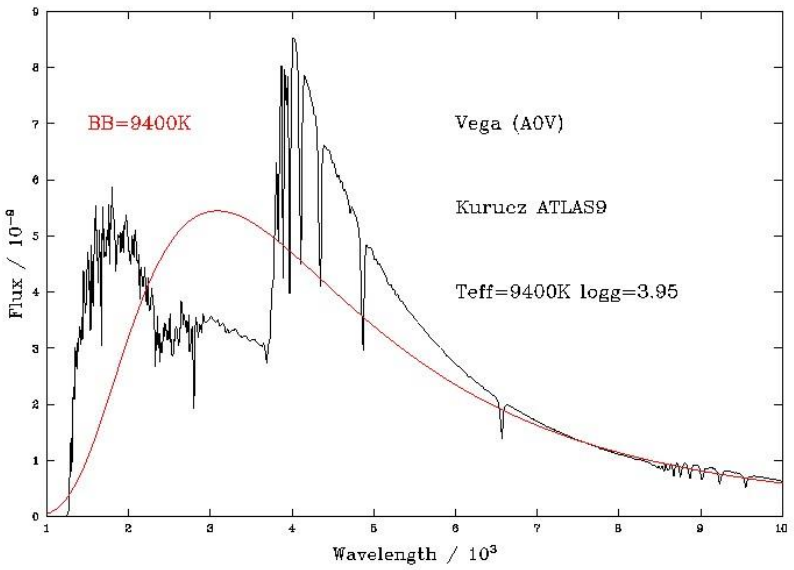
Not really

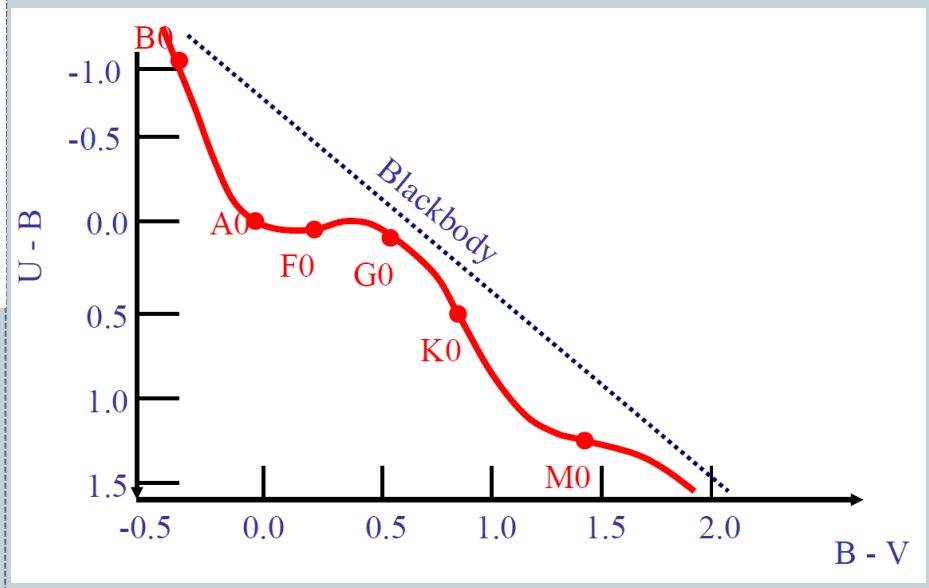
Stars do differ from black bodies

The observed flux distributions of real stars deviate from black body curves, as indicated here for the UBV colors of dwarfs and supergiants. This difference is due to sources of continuous and line opacity in the stellar photospheres and will be discussed in this course.



Stars do differ from black bodies





Stefan – Boltzmann Law

Blackbody radiation is continuous and isotropic whose intensity varies only with wavelength and temperature.

Following empirical (Josef Stefan in 1879) and theoretical (Ludwig Boltzmann in 1884) studies of black bodies, there is a well known relation between Flux and Temperature known as Stefan-Boltzmann law:

$$F=\sigma T^4$$

with $\sigma=5.6705\text{x}10^{-5}$ erg/cm^2/s/K^4

(Note that Bohm-Vitense refers to "astronomical flux", F/π, as `flux').

Effective temperatures of stars

- The Stefan-Boltzmann law, $F=\sigma T^4$, or alternatively $L = 4\pi R^2 F = \sigma T^4$ defines the "effective temperature" of a star, i.e. the temperature which a black body would need to radiate the same amount of energy as the star.

- T_{eff} is 5777K for the Sun.

Planck formula

The black body intensity is defined (following discovery by Max Planck in 1900) as either

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1} \quad \text{or} \quad B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}$$

where c=2.99x10^{10} cm, h=6.57x20^{-27} erg s, k=1.38x10^{-16} erg/s.

Note that: $B_\nu(T)\mathrm{d}\nu = B_\lambda(T)\mathrm{d}\lambda \Rightarrow B_\lambda = B_\nu\left|\frac{\mathrm{d}\nu}{\mathrm{d}\lambda}\right| = B_\nu\frac{c}{\lambda^2}$

Let us compute the bolometric flux:

$$F = \pi\int_0^\infty B_\nu(T)\mathrm{d}\nu = \pi\int_0^\infty \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}\mathrm{d}\nu = \pi\frac{2h}{c^2}\left(\frac{kT}{h}\right)^4\int_0^\infty \frac{x^3}{e^x-1}\mathrm{d}x = \pi\frac{2h}{c^2}\left(\frac{kT}{h}\right)^4\frac{\pi^4}{15} = \sigma_{SB}T^4$$

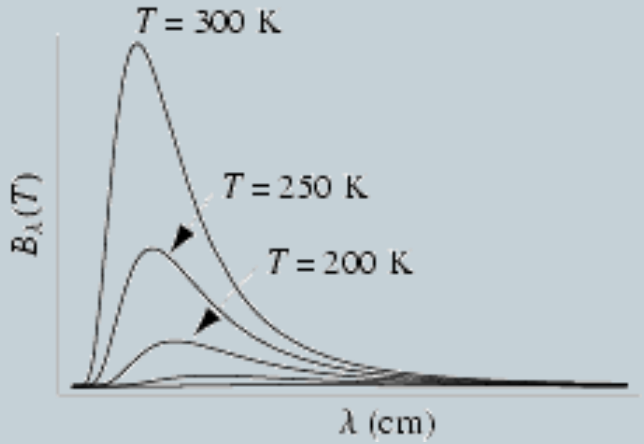
$$\sigma_{SB} = 2\frac{\pi^5k^4}{15c^2h^3} = 5.67\ 10^{-5}\ \text{erg cm}^{-2}\text{s}^{-1}\ \text{K}^{-4} - \text{Stefan-Boltzmann constant}$$

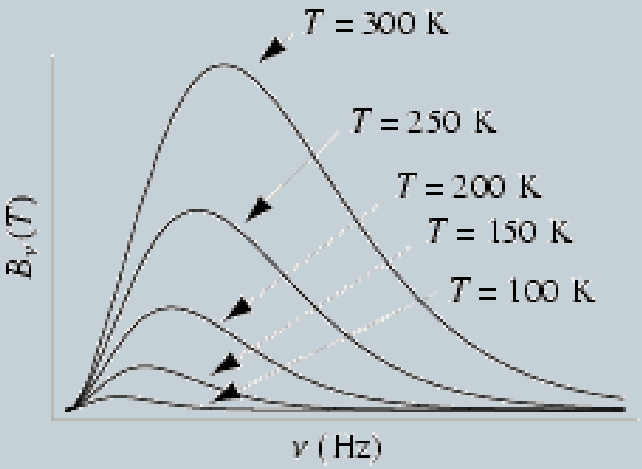
Planck function is monotonic with temperature: $\frac{\partial B_\nu(T)}{\partial T} = \frac{2h^2\nu^4}{c^2kT^2}\frac{e^{h\nu/kT}}{\left(e^{h\nu/kT}-1\right)^2} > 0$

Properties of the Planck law

- *For increasing temperatures, the black body intensity increases for all wavelengths.* The maximum in the energy distribution shifts to shorter λ (longer ν) for higher temperatures.
- $\lambda_{max}\, T = 2.98978 \times 10^7$ Å K

  is *Wien's displacement law* for the maximum I_λ providing an estimate of the peak emission (λ_{max} =5175 Å for the Sun).





EXAMPLE

Rayleigh-Jeans and Wien approximations

At long wavelengths $\lambda >> \lambda_{max}$ (small frequencies $\nu << \nu_{max}$) the Planck formulae

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1} \qquad B_\lambda(T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}$$

can be approximated by the Rayleigh-Jeans law

$$B_\nu(T) \approx 2\frac{\nu^2}{c^2}kT\,, \quad B_\lambda(T) \approx 2ckT\lambda^{-4}$$

At short wavelengths $\lambda \leq \lambda_{max}$ (large frequencies $\nu \geq \nu_{max}$), the Wien law is a good approximation

$$B_\nu(T) \approx 2\frac{h\nu^3}{c^2}e^{-\frac{h\nu}{kT}}\,, \quad B_\lambda(T) \approx 2\frac{hc^2}{\lambda^5}e^{-\frac{hc}{\lambda kT}}$$

Color and brightness temperatures

Define brightness temperature as $I_\nu = B_\nu(T_b)$

In radio band we get

$$I_\nu = 2\frac{\nu^2}{c^2}kT_b \quad , \text{ so that } T_b = \frac{c^2}{2\nu^2 k}I_\nu \text{ for } h\nu << kT$$

Colour temperature T_c is obtained by "fitting" the observed spectrum with the Planck function ignoring normalization. It gives correctly the temperature of the black body source of unknown absolute scale of the intensity.

Summary

- The Stefan-Boltzmann law ($F=\sigma T^4$) defines a star's effective temperature , i.e. the temperature which a black body would need in order to radiate the same amount of energy as the star.
- The specific (I_λ or I_ν) is distance independent.
- $\lambda I_\lambda = \nu I_\nu$
- The flux (F_λ or F_ν) is the angular integral of the projection of the specific intensity in the radial direction and obeys the inverse square law.
- Planck function is monotonic in temperature. Maxima of B_ν and B_λ are reached at different wavelengths.
- We defined color and brightness temperatures.