Stellar atmospheres

Compulsory Home Exercises. Problem Set 2.

Return by Wednesday, October 16, 2019 (before the lecture).

Please, write down every step in your line of thinking and state assumptions etc.
A sole answer is not enough.

1. *(Problem from the 1st lecture: A B5V star in the LMC –distance 50kpc– has V=13.5 mag, B-V=-0.07 mag. What is its bolometric luminosity, relative to the Sun?).* What is its stellar radius? At what wavelength does its radiation peak (assuming Wien's law)?

2. Let's approximate the directional (μ) dependence of the specific intensity using the first-order Taylor expansion,

$$I(\tau, \mu) = I_0(\tau) + I_1(\tau)\,\mu$$

where I_0 and I_1 depend on the vertical optical depth τ but not on μ.
Assuming a gray atmosphere,

   a. Derive expressions for the flux F, mean intensity J, and radiation pressure $P_{\rm rad}$, in terms of I_0 and I_1.

   b. Show that the radiation field obeys the Eddington approximation and

$$P_{rad} = \frac{4\pi}{3c} J$$

   c. Within the Eddington approximation, the solution of the radiative transfer equation is

$$S = \frac{3}{4\pi}\left(\tau + \frac{2}{3}\right) F$$

   Use this solution to find an expression for I_0 as a function of I_1 and τ.

   d. Show that if $\tau \gg 1$ then $I_0 \gg I_1$ (this result justifies the use of the first-order Taylor expansion at large optical depths, showing that the radiation field becomes isotropic).

3. Assuming a grey atmosphere and using the Eddington approximation and an expression for the source function S as a function of vertical optical depth τ and flux F (Lecture 5),

   a. Calculate the upward specific intensity $I(\tau, \mu)$ as a function of F, τ_v, and direction μ>0, using the formal solution to RTE (Lecture 4)

$$I_\lambda(\tau_{\lambda,v}, \mu > 0) = \int_{\tau_{\lambda,v}}^{\infty} S_\lambda e^{(\tau_{\lambda,v} - t)/\mu} \frac{dt}{\mu},$$

b. Using your expression for the upward specific intensity $I(\tau, \mu)$, evaluate the upward component of the radiative flux,

$$F^+ = 2\pi \int_0^1 I(\mu)\, \mu\, d\mu$$

c. Find the corresponding downward component of the flux, F^-.