Stellar Atmospheres
Lecture 13

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Spectral line formation

EINSTEIN COEFFICIENTS

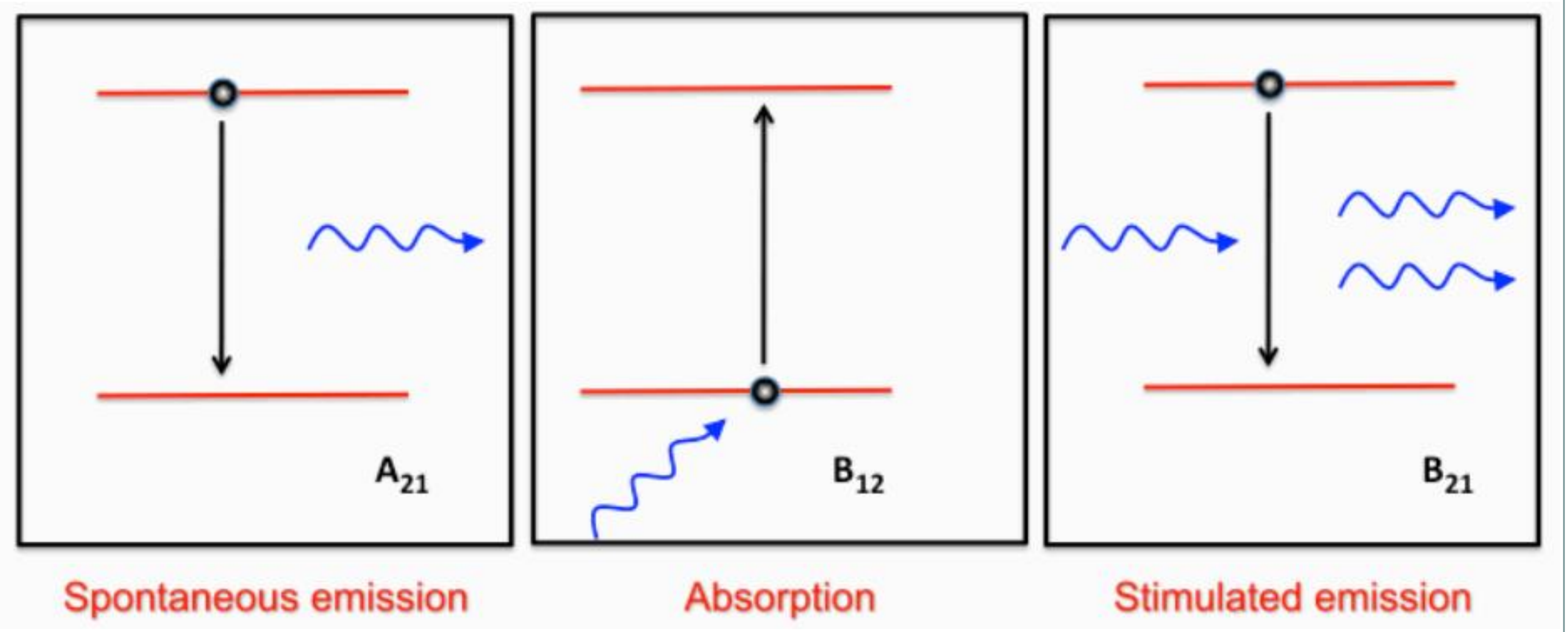
Bound-Bound (free-free) transitions

There are 3 basic kinds of line processes associated with bound-bound transitions of atoms or ions:

1. **Direct Absorption**, in which the absorbed photon induces a bound electron to go into a higher energy level.
2. **Spontaneous Emission**, in which an electron in a higher energy level spontaneously decays to lower level, emitting the energy difference as a photon.
3. **Stimulated Emission**, in which an incoming photon induces an electron in a higher energy level to decay to a lower level, emitting in effect a second photon that is nearly identical in energy (and even phase) to the original photon.

The probability that the atom will emit (or absorb) its quantum of energy is described by **Einstein probability coefficients**, written as B_{ij} , A_{ji}, and B_{ji}.

Einstein coefficients



Einstein coefficients concern the probability that a particle spontaneously emits a photon, the probability to absorb a photon, and the probability to emit a photon under the influence of another incoming photon. Einstein's coefficients are valid for all radiation fields.

Spontaneous emission

Consider an upper level, *u*, and a lower level, *l*, separated by an energy $h\nu_0$.

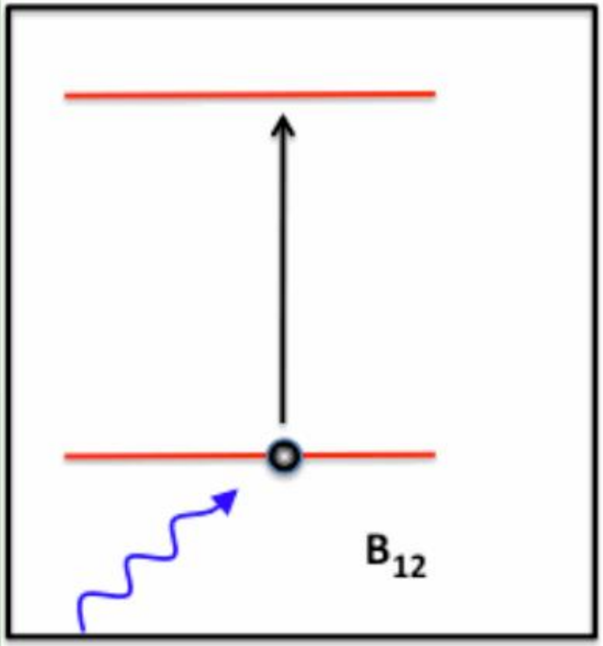
- The probability that the atom will spontaneously emit its quantum of energy within a time dt and in a solid angle dω is $A_{ul}\, dt\, d\omega$.
- The proportionality constant, A_{ul}, is the Einstein probability coefficient for spontaneous emission [s^{-1}].
- Occurs independently of the radiation field.
- Emits **isotropically**.

For Hα, A_{32}=4.4x10^7 s^{-1}. If at time t_0=0 there are $N_i(0)$ atoms in level *u*, then at time t the population is $N_u(t)=N_u(0)\exp(-A_{ul}t)$. Lifetime = $1/A_{ul}$

Absorption

Consider an upper level, *u*, and a lower level, *l*, separated by an energy $h\nu_0$.

- Photons with energies *close* to $h\nu_0$ cause transitions from levels *l* to *u*.
- The probability per unit time for this process will evidently be proportional to the mean intensity J_ν at the frequency ν_0.
- $B_{lu}\ J$: transition probability of absorption per unit time.
- The proportionality constant B_{lu} is one of the Einstein *B*-coefficients.

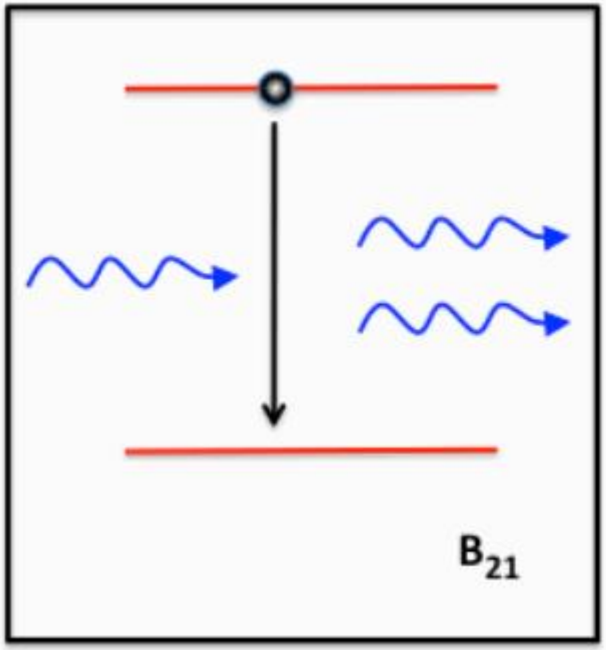


Absorption

Stimulated emission

Planck's law does not follow from considering only spontaneous emission and absorption. Must also include stimulated emission, which like absorption is proportional to the mean intensity J.

- The system goes from an upper level u to a lower level l stimulated by the presence of a radiation field ($h\nu$ corresponding to the energy difference between levels u and l).
- The energy of the emitted photon is the same as of the incoming photon (also direction and phase are the same).
- $B_{ul}\,J$: transition probability of stimulated emission per unit time.
- The proportionality constant B_{ul} is a second Einstein B-coefficient.
- The process of stimulated emission is sometimes referred to as a process of *negative absorption*.
- Stimulated emission occurs into the same state (frequency, direction, polarization) as the photon that stimulated the emission.



Stimulated emission

Relation between Einstein coefficients

Einstein's Coefficients are not independent. To find a relation between them, let's assume strict Thermodynamic Equilibrium (TE), and, for simplicity, adopt a 2-level approximation.

In TE, each process is in equilibrium with its inverse, i.e., within one line there is no netto destruction or creation of photons (detailed balance)

$$n_1 B_{12} J_\nu = n_2 A_{21} + n_2 B_{21} J_\nu$$

Transitions 1→2 equal to 2→1
n_1, n_2: number density of e^- in levels 1,2

$$J_\nu = \frac{A_{21}/B_{21}}{\left(\frac{n_1}{n_2}\right)\left(\frac{B_{12}}{B_{21}}\right) - 1}$$

Thermodynamic equilibrium:
Boltzmann, $J=B_\nu(T)$

$$\frac{n_1}{n_2} = \frac{g_1}{g_2} e^{h\nu_{21}/kT}$$

Relation between Einstein coefficients

TE: blackbody, $J = B_\nu(T)$

$$B_\nu(T) = \frac{A_{21}/B_{21}}{\left(\frac{g_1 B_{12}}{g_2 B_{21}}\right) e^{h\nu_{21}/kT} - 1}$$

Comparison with Planck blackbody radiation:

$$B_\nu(T) = \frac{A_{21}}{B_{21}}\left(\frac{g_1 B_{12}}{g_2 B_{21}} e^{\frac{h\nu_{21}}{kT}} - 1\right)^{-1} = \frac{2h\nu_{21}^3}{c^2}\left(e^{\frac{h\nu_{21}}{kT}} - 1\right)^{-1}$$

$$\frac{A_{21}}{B_{21}} = \frac{2h\nu_{21}^3}{c^2} \quad \rightarrow \quad A_{21} = B_{21}\frac{2h\nu_{21}^3}{c^2}$$

$$\frac{g_1 B_{12}}{g_2 B_{21}} = 1 \quad \rightarrow \quad g_1 B_{12} = g_2 B_{21}$$

Einstein coefficients

Thus, if one of the Einstein Coefficients is known then two other can be calculated.

Important: The Einstein's coefficients are atomic constants. Although the above relations were derived under the conditions of TE, these relations hold in any non-TE state.

Total amount of absorbed photons per unit time at a given frequency is

$$n_1 B_{12} J_\nu - n_2 B_{21} J_\nu = n_1 B_{12} J_\nu \left(1 - \frac{n_2 B_{21}}{n_1 B_{12}}\right) = n_1 B_{12} J_\nu \underbrace{\left(1 - \frac{g_1 n_2}{g_2 n_1}\right)}$$

Thus, to take into account negative absorption (stimulated emission), one must multiply the number of absorbed photons by or, assuming LTE (Bolzmann), by

$$\left(1 - e^{-h\nu_{12}/kT}\right)$$

Comparison of induced and spontaneous emission

Home work:

- When (temperatures, wavelengths) is spontaneous or induced emission stronger?

Assume LTE (blackbody)

Lifetime of atom in excited state

In the absence of collisions and of any other transitions than the ***ul*** one, the mean lifetime of particles in state ***u*** is **Lifetime = 1/A_{ul}**

If at time $t_0=0$ there are $N_u(0)$ atoms in level u, then at time t the population is

$$N_u(t)=N_u(0)\exp(-A_{ul}t).$$

Typical value of A_{ij} is 10^7- 10^8 s^{-1} (for Hα, $A_{32}=4.4\text{x}10^7$ s^{-1}), so lifetime is ~10^{-8} s.

However, not all transitions are allowed, some are strictly forbidden!

In practice, strictly forbidden means **very low probability of occurrence** ➔ **Metastable states** at which a lifetime is much longer than of the ordinary excited states but shorter than of the ground state.

Lifetimes at metastable states can reach several hours!

Einstein A-coefficients for Hydrogen

k \ i	1	2	3	4	5	6	7
2	$4,67 \cdot 10^8$	—	—	—	—	—	—
3	$5,54 \cdot 10^7$	$4,39 \cdot 10^7$	—	—	—	—	—
4	$1,27 \cdot 10^7$	$8,37 \cdot 10^6$	$8,94 \cdot 10^6$	—	—	—	—
5	$4,10 \cdot 10^6$	$2,52 \cdot 10^6$	$2,19 \cdot 10^6$	$2,68 \cdot 10^6$	—	—	—
6	$1,64 \cdot 10^6$	$9,68 \cdot 10^5$	$7,74 \cdot 10^5$	$7,67 \cdot 10^5$	$1,02 \cdot 10^6$	—	—
7	$7,53 \cdot 10^5$	$4,37 \cdot 10^5$	$3,34 \cdot 10^5$	$3,03 \cdot 10^5$	$3,24 \cdot 10^5$	$4,50 \cdot 10^5$	—
8	$3,85 \cdot 10^5$	$2,20 \cdot 10^5$	$1,64 \cdot 10^5$	$1,42 \cdot 10^5$	$1,38 \cdot 10^5$	$1,55 \cdot 10^5$	$2,26 \cdot 10^5$

Line profiles

All the spectral lines are not monochromatic but have a finite width and a particular profile. Width and shape of a line depend directly in atomic transitions and plasma environment

Energy levels are **not** infinitely sharp.

An unavoidable source of broadening is due to the Heisenberg uncertainty principle – $dE\, dt \geq h/2\pi$,
dt being the timescale of decay (finite lifetime of energy levels).

In each spectral lines, photons of different frequencies (but close to central frequency ν_0) can be absorbed. Let us call $\varphi(\nu)$ the probability that the transition occurs by emitting or absorbing a photon with energy $h\nu$ (emission or absorption line, $\int \varphi(\nu) d\nu \equiv 1$).

This natural broadening has the form of a *Lorentzian function.*

We will discuss it on Wednesday.