

Stellar Atmospheres

Lecture 8



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Many physical processes contribute to opacity

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- **Bound-Bound Transitions** – absorption or emission of radiation from electrons moving between bound energy levels.
- **Bound-Free Transitions** – the energy of the higher level electron state lies in the continuum or is unbound.
- **Free-Free Transitions** – change the motion of an electron from one free state to another.
- **Electron Scattering** – deflection of a photon from its original path by a particle, without changing its wavelength
 - **Rayleigh scattering** – photons scatter off bound electrons. (Varies as λ^{-4})
 - **Thomson scattering** – photons scatter off free electrons (Independent of wavelength)
- **Photodissociation** may occur for molecules

Boltzmann equation & Saha Equation

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- Boltzmann equation:

$$\frac{N_u}{N_l} = \frac{g_u}{g_l} e^{-(E_u - E_l)/kT}$$

Boltzmann constant
 $k = 8.6174 \times 10^{-5} \text{ eV/K}$

$$\log \frac{N_u}{N_l} = \log \frac{g_u}{g_l} - \frac{5040}{T} \chi_{ul} (\text{eV})$$

$\Theta = 5040/T$

- Saha Equation

$$\frac{N_1^+}{N_1} = \frac{2g_1^+}{N_e g_1} \frac{(2\pi m_e kT)^{3/2}}{h^3} e^{-\chi_{ion}/kT}$$

$$\log \frac{N^+}{N^0} = \log \frac{u^+}{u^0} + \log 2 + \frac{5}{2} \log T - \chi_{ion} \Theta - \log P_e - 0.48$$

What can various particles do?



- Free electrons – Thomson scattering
- Atoms and Ions –
 - Bound-bound transitions
 - Bound-free transitions
 - Free-free transitions
- Molecules –
 - BB, BF, FF transitions
 - Photodissociation
- Most continuous opacity is due to hydrogen in one form or another

Summary from the last lecture

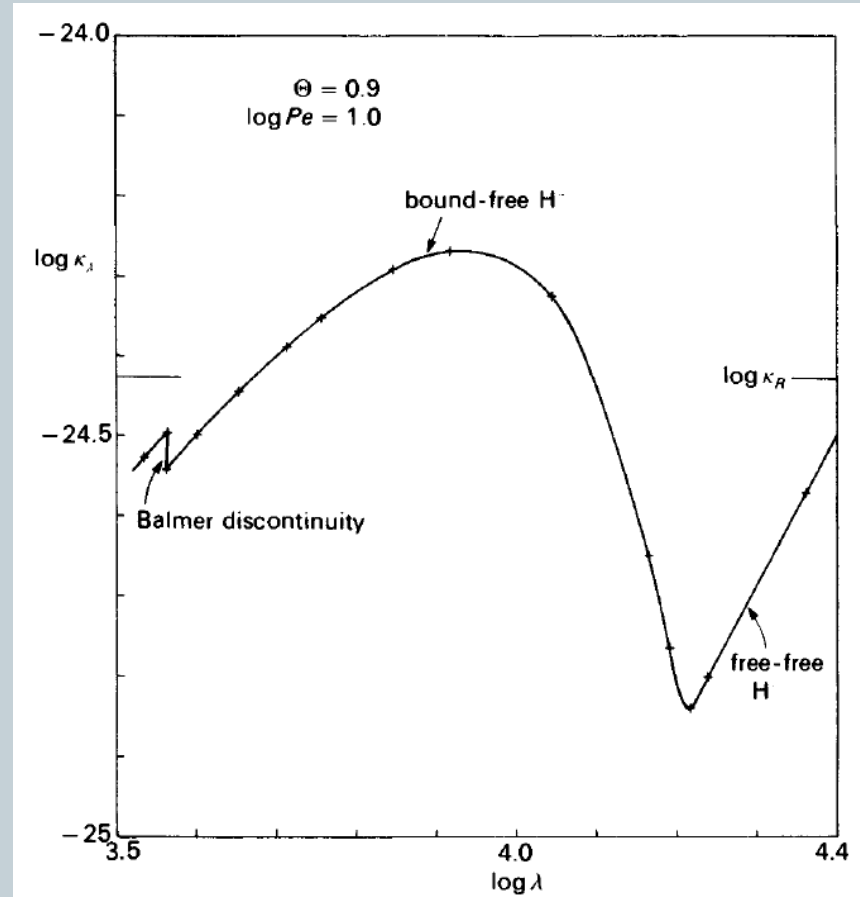
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- **Bound-bound** transitions contribute to the **line absorption**. **Bound-free** and **free-free** transitions (plus scattering) contribute to the **continuous** absorption, mostly by H & He.
- Atomic H absorption coefficient highly T sensitive. For **late-type stars** in the optical and IR, **bound-free** and **free-free** transitions of the **H⁻ ion** dominate the continuous opacity, since the population of atomic H in $n=3$ (Paschen series) is so low.
- For **early-type stars**, **atomic H dominates**, producing strong **jumps** in the opacity at the Lyman, Balmer & Paschen edges.

Hydrogen continuous opacity

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- We have identified H^- (**bound-free**) in the **visual** and H^- (**free-free**) in the **IR** as principal sources of opacity in the Sun.
- The H Balmer continuum shortward of the $3647\text{\AA}$ Balmer jump is an additional contributor.
- What **observational evidence** is there that this is true for the Sun, and what other forms of opacity play a role in other stars?



Other sources of opacity



MORE ON H^-
He ABSORPTION
METALLIC ABSORPTION
SCATTERING
EFFECT OF NONGREYNESS OF THE
TEMPERATURE STRUCTURE

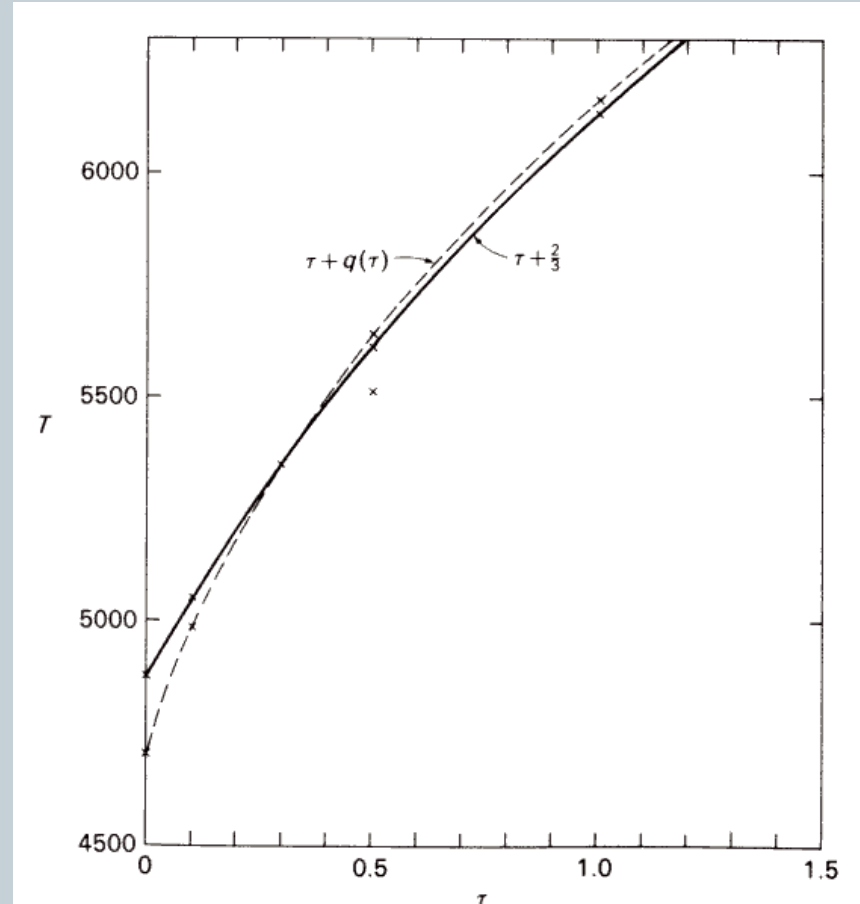
$T(\tau_\lambda)$ from Eddington approximation

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We can use the observed **limb darkening** of the Sun at different λ to derive the depth dependence of the source function, $S_\lambda(\tau_\lambda)$.

Assuming LTE, $S_\lambda(\tau_\lambda) = B_\lambda(T(\tau_\lambda))$ we can obtain the temperature as a function of τ_λ .

Recall from radiative equilibrium (assuming the Eddington approximation), $T(\tau_\lambda)$ can be obtained for a **grey** atmosphere.

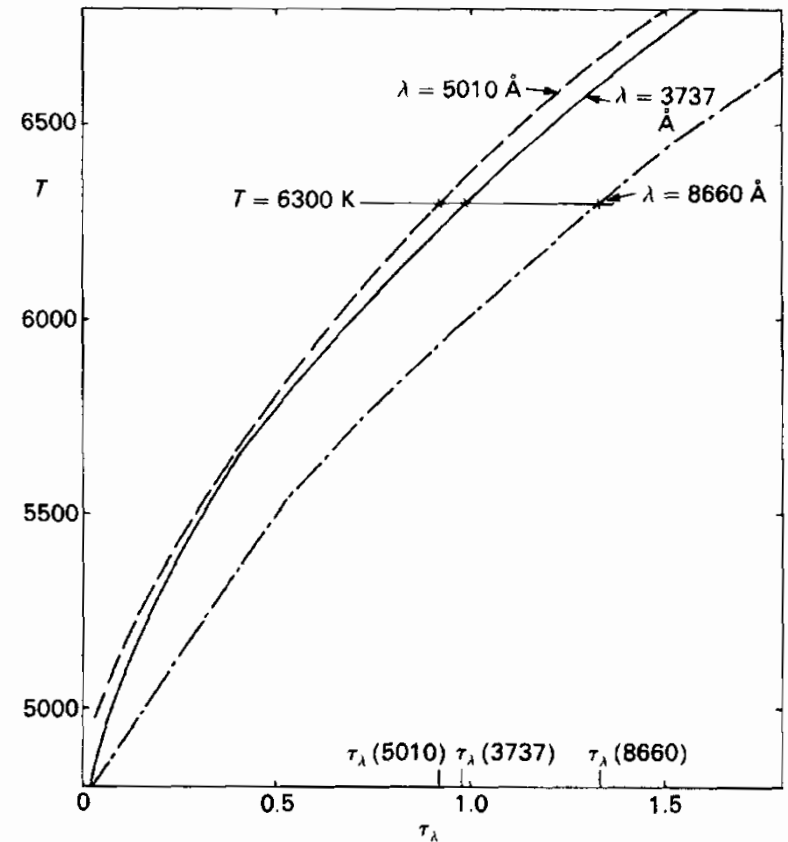


$T(\tau_\lambda)$ from limb darkening

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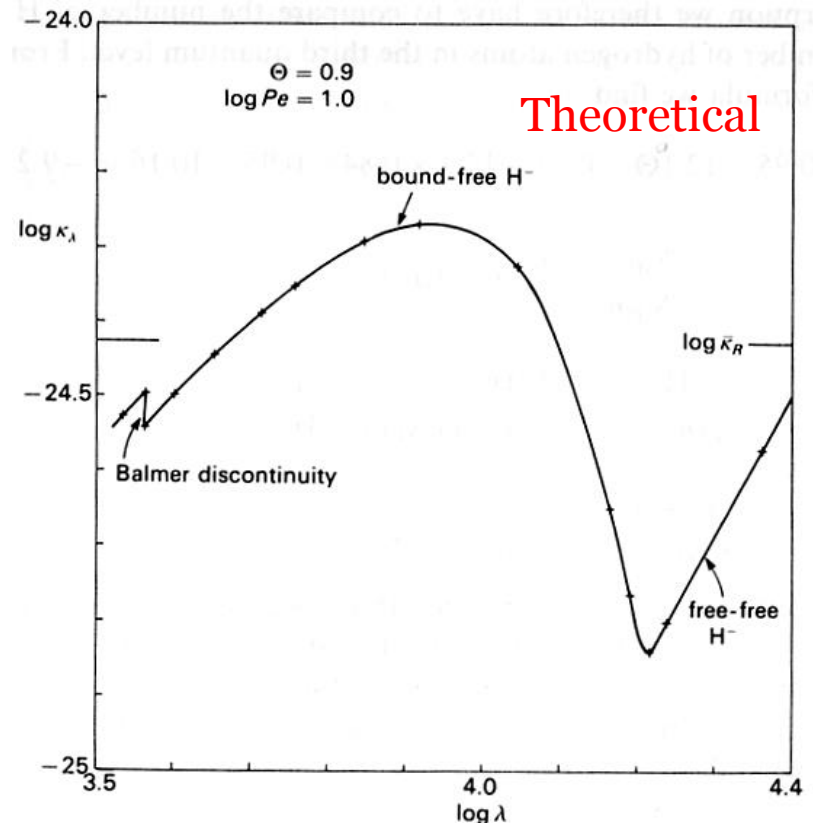
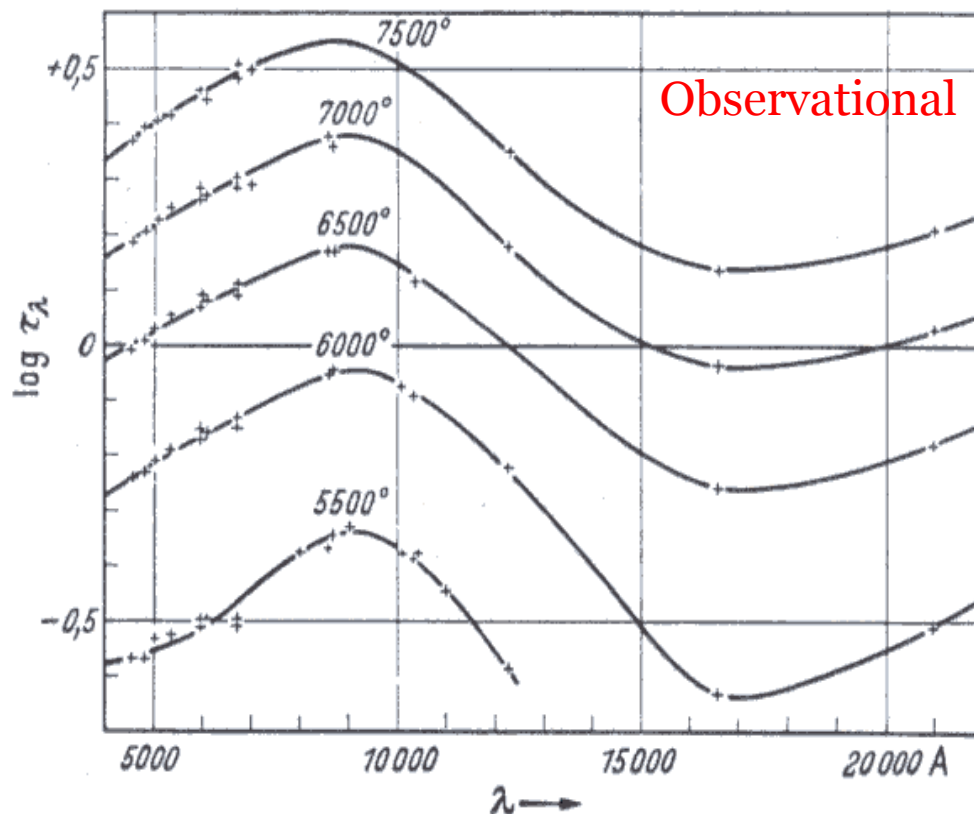
Limb darkening observations of the Sun at different wavelengths (via imaging using suitable filters) to derive $T(\tau_\lambda)$ at various wavelengths (e.g. 3737, 5010 & 8660 Å shown here).

The horizontal line shown at $T=6300\text{K}$ connects points which correspond to the same *geometrical* depth, so it is possible to derive the wavelength dependence of τ_λ .



Confirmation of H^-

The wavelength dependence of τ_λ (and hence κ_λ or α_λ) can be observationally derived for the Sun – the optical and IR dependence **agrees** remarkably well with the theoretical absorption coefficient for b-f and f-f H^-



He opacity?

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Helium is the next most abundant element after H, so **is it important** for the continuous absorption in the Sun or other stars?

Ionization of He to He^+ requires an energy of 24.6eV ($\lambda < 504\text{\AA}$ are needed). Indeed, even the first excited level lies 19.8eV above the ground state, which can contribute only below 600 $\text{\AA}$ where there is very little radiation coming from the Sun. From the Boltzmann formula ($g_1=1$, $g_2=3$):

$$\log(N_{\text{He}}(2s^3S) / N_{\text{He}}(1s^1S)) = 0.48 - 19.8(5040/5777) = -16.8$$

So, only 10^{-17} of the He atoms can contribute to the absorption, and since He is 10% as abundant as H, only one in 10^{-18} atoms are He atoms in the 1st excited state.

Consequently, He opacity plays a negligible role for the Sun. The bound-free absorption from He^- is generally negligible, whilst free-free He^- (with a form similar to free-free H^-) can be significant at long wavelengths in cool stars.

Photoionization (bound-free processes) from He only plays a significant role for the hottest, O-type, stars.

Metal (Iron) opacity

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- If He only plays a role for very hot stars, do any metals contribute to the continuous opacity in cool stars?
- **Iron** ($\text{Fe}/\text{H}=10^{-4}$) is generally the dominant metal continuous opacity source in stellar atmospheres.
- In the Sun, let's consider absorption by **atomic Fe** in the ultraviolet ($2000\text{\AA}$) for which an excitation energy of $\sim 1.7\text{eV}$ is required. The fraction of excited Fe atoms is 4×10^{-2} relative to the ground-state (from Boltzmann formula), whilst the fraction of ionized to neutral Fe is approximately 6 (from Saha equation).
- Accounting for the abundance of Fe, we obtain the fraction of atomic Fe atoms absorbing at $2000\text{\AA}$ relative to the total number of H atoms to be $4 \times 10^{-2} \times 10^{-4} \times 1/6 = 6 \times 10^{-7}$.
- We previously obtained 2×10^{-8} for H^- , so metallic lines **in the UV** are much more important for the absorption than the H^- ion, or the neutral H atom. Even more important is the absorption by the metal atoms in the ground level, which is $< 1570\text{\AA}$ for Fe, $< 1520\text{\AA}$ for Si.

Molecular opacity

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- CN^- , C_2^- , H_2O^- , CH_3 , TiO are important sources of opacity in **late** (K-type) & **very late** (M-type) stars.
- Molecular Hydrogen (H_2) is more common than atomic H in stars cooler than mid-M (brown dwarfs!)
- H_2 does **not** absorb in the **visible** spectrum, so only plays a role in the IR.
- H_2^+ does absorb in the visual, but is less than 10% of H^- . H_2^+ is a significant absorber in the UV for such very cool stars.

Scattering

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In the classical picture of an atom, we can consider the electron as being bound to the atom. Any force trying to remove it will be counteracted by an opposing force. If a force were to pull on the electron and then let go, it would oscillate with eigenfrequencies $\omega=2\pi\nu$.

The **scattering cross-section** for a *classical oscillator* can be written as

$$\sigma_T = \frac{8\pi}{3} \frac{e^4}{m_e^2 c^4} \left[\frac{\nu^4}{(\nu^2 - \nu_0^2)^2 + \gamma^2 \omega^2} \right] \quad \omega=2\pi\nu$$

where ν_0 is the eigenfrequency of an atom and γ is the damping constant.

Thomson & Rayleigh Scattering

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Two cases are of interest:

1. **Thompson (electron) scattering** ($v_o=0, \gamma=0$)
(photons scatters off a free electron, no change in λ , just direction):

$$\sigma_T = \frac{8\pi}{3} \frac{e^4}{m_e^2 c^4} = \frac{8\pi}{3} r_e^2 = 6.65 \times 10^{-25} \text{ cm}^2 / \text{electron}$$

classical electron radius

2. **Rayleigh scattering** by atoms/molecules ($v \ll v_o, \gamma \ll v_o$)

$$\sigma_R(v) \propto \sigma_T v^4 = \sigma_T \lambda^{-4}$$

Electron scattering vs. f-f transition

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- Electron scattering (Thomson scattering) – the path of the photon is altered, but not the energy.
- Free-Free transition – the electron emits or absorbs a photon. A free-free transition **can only occur in the presence of an associated nucleus**. An electron in free space cannot gain the energy of a photon.

Thompson Scattering

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Since an electron is tiny it makes a poor target for an incident photon so the cross-section for Thomson scattering is very small ($\sigma_T = 6.65 \times 10^{-25} \text{ cm}^2$) and has the same value for photons of all wavelengths: **As such electron scattering is the only grey opacity source.**

Although e^- are very abundant in the **Solar** photosphere, the small cross-section makes it **unimportant**.

Electron scattering **is** most effective as a source of opacity at high temperatures. In atmospheres of **OB stars** where most of the gas is completely ionized, other sources of opacity involving bound electrons are excluded. In this regime, α_T **dominates the continuum opacity**.

Rayleigh Scattering

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- Rayleigh scattering by H atoms in **Solar**-type is more relevant than e^- scattering since atoms are much more **common** (recall $N(H) \gg N(H^+)$).
- In **M stars**, H_2 becomes the dominant form for hydrogen, with strong electronic transitions in the UV, so Rayleigh scattering by **molecular** H_2 can be important.
- The cross-section for Rayleigh scattering is much smaller than σ_T and is proportional to λ^{-4} so increases steeply towards the blue. (In the same way the sky appears blue, due to a steep increase in the scattering cross-section of sunlight scattered by molecules in our atmosphere).
- The cross-section is sufficiently small relative to metallic absorption coefficients that Rayleigh scattering only plays a **dominant** role in extended envelopes of **supergiants**.

Total extinction coefficient κ

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- The total extinction coefficient is given by:

$$\kappa_{\nu} = (1 - e^{-h\nu/kT}) \sum_j x_j (\kappa_j^{bb} + \kappa_j^{bf} + \kappa_j^{ff}) + \kappa^s$$

where the sum is over all elements j of number fraction x_j .

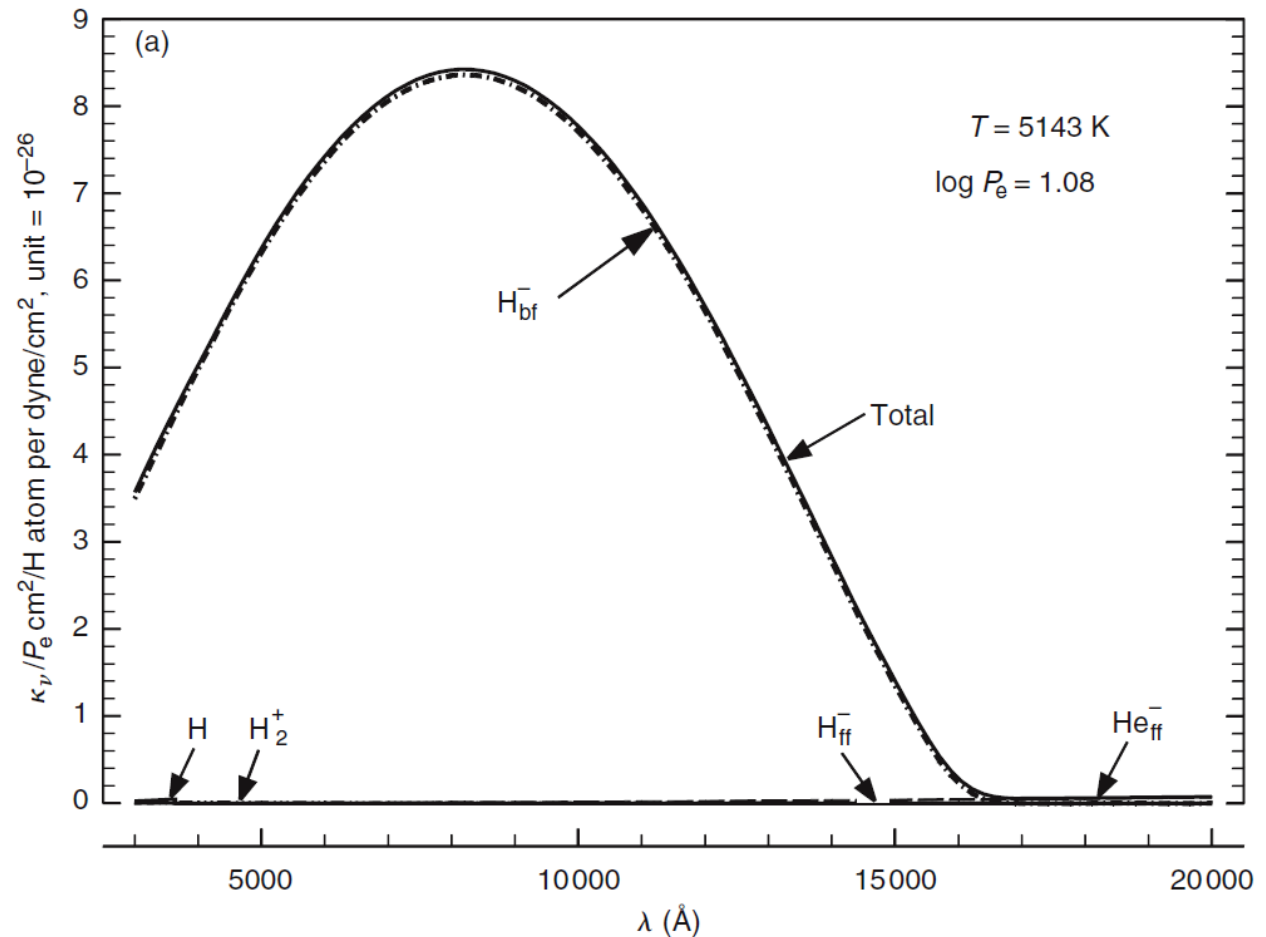
Here the $(e^{-\frac{h\nu}{kT}})$ term accounts for stimulated emission (incident photon stimulates electron to de-excite and emit photon with identical energy, as in a laser).

- What is the total extinction coefficient for different types of star?

G-type (optical depth unity)

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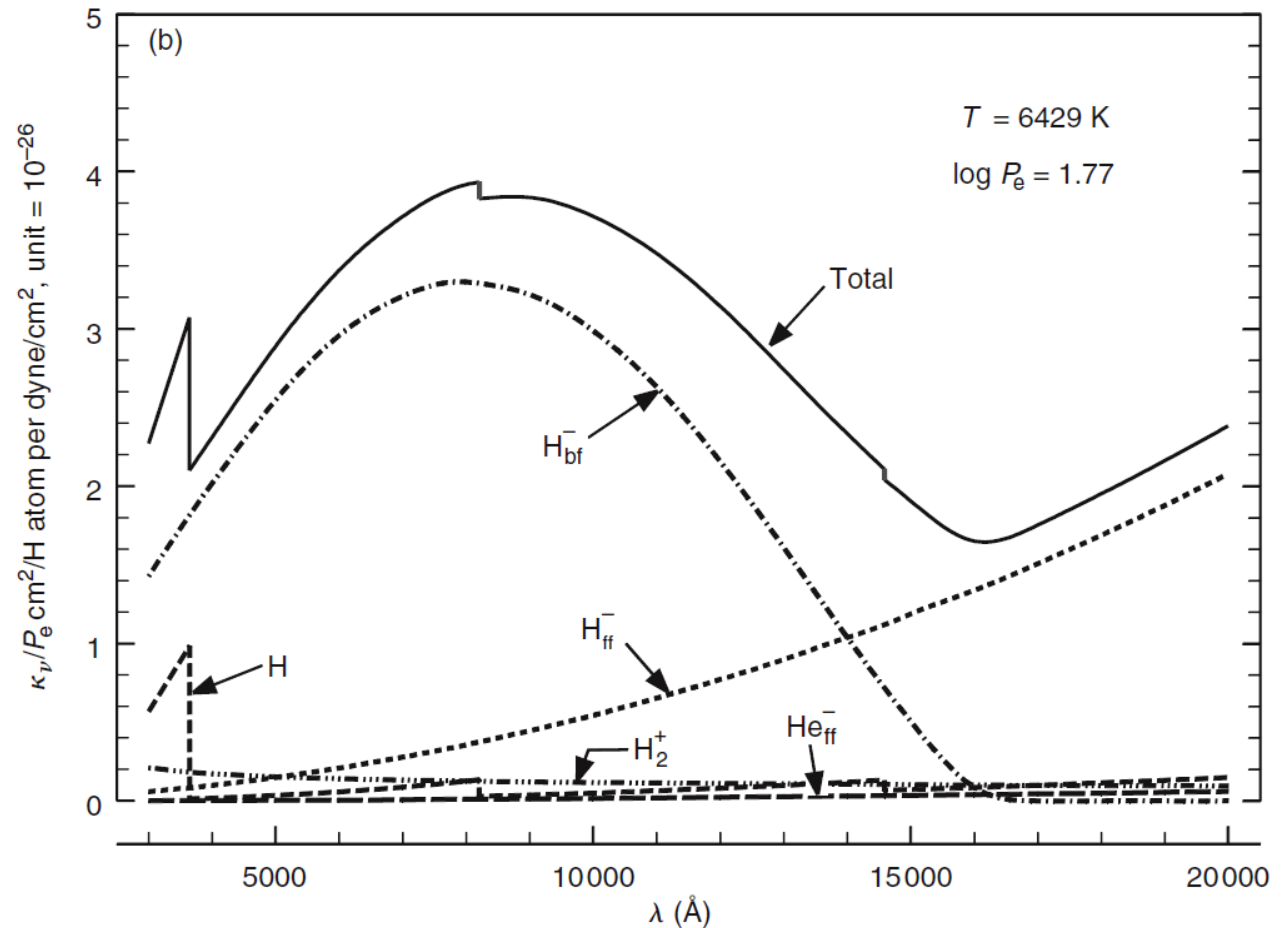
For G stars, the **H⁻ ion (bound-free)** dominates for optical.



F-type (optical depth unity)

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For F stars, the absorption is dominated by the two components of the H^- ion (bound-free) and (free-free), with a contribution from the Balmer continua below $3647\text{\AA}$.

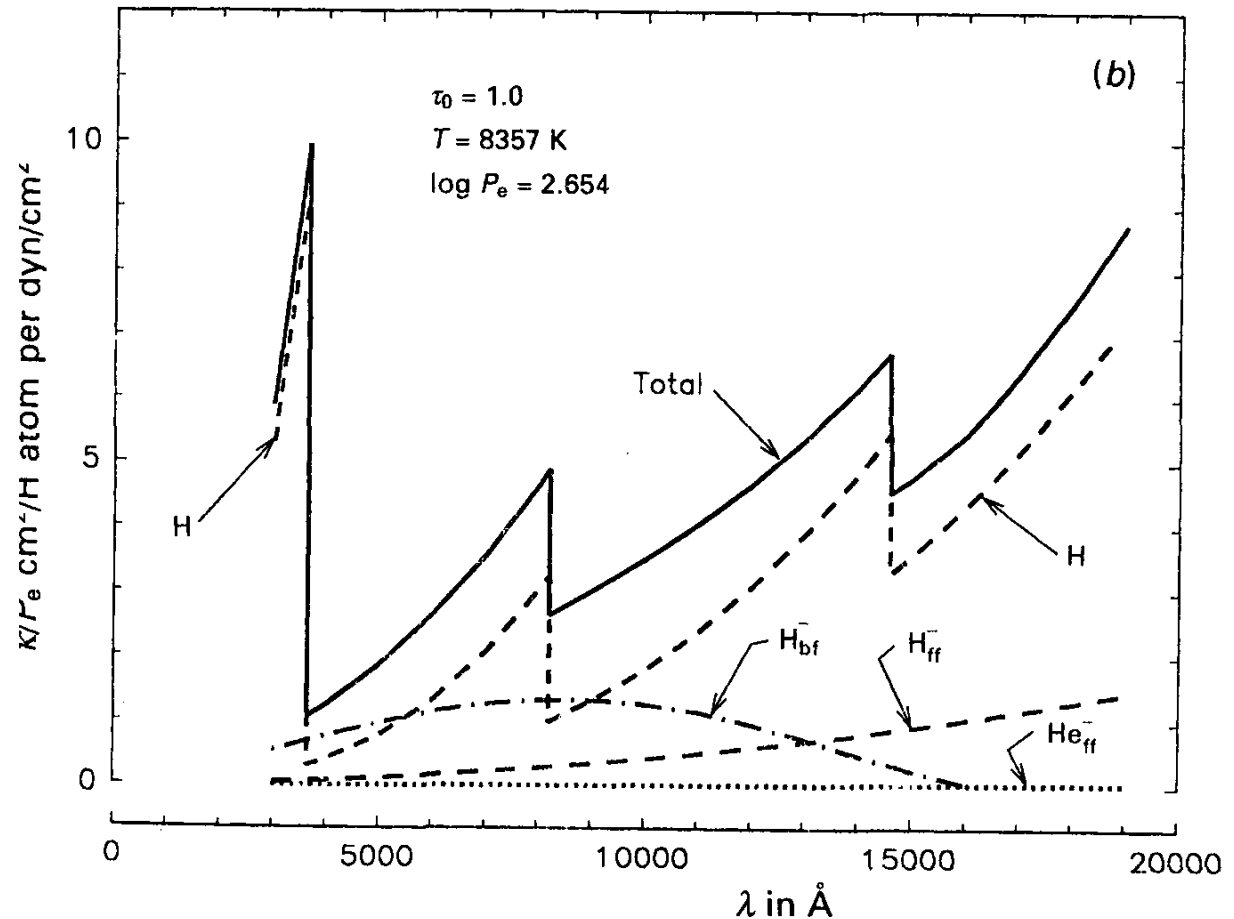


A-type (optical depth unity)

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For a late A star, absorption from the H^- ion is dropping back compared to the cooler cases, while neutral hydrogen has grown with increasing temperature.

H (bound-free)
Balmer, Paschen
and Brackett
continua start to
dominate.

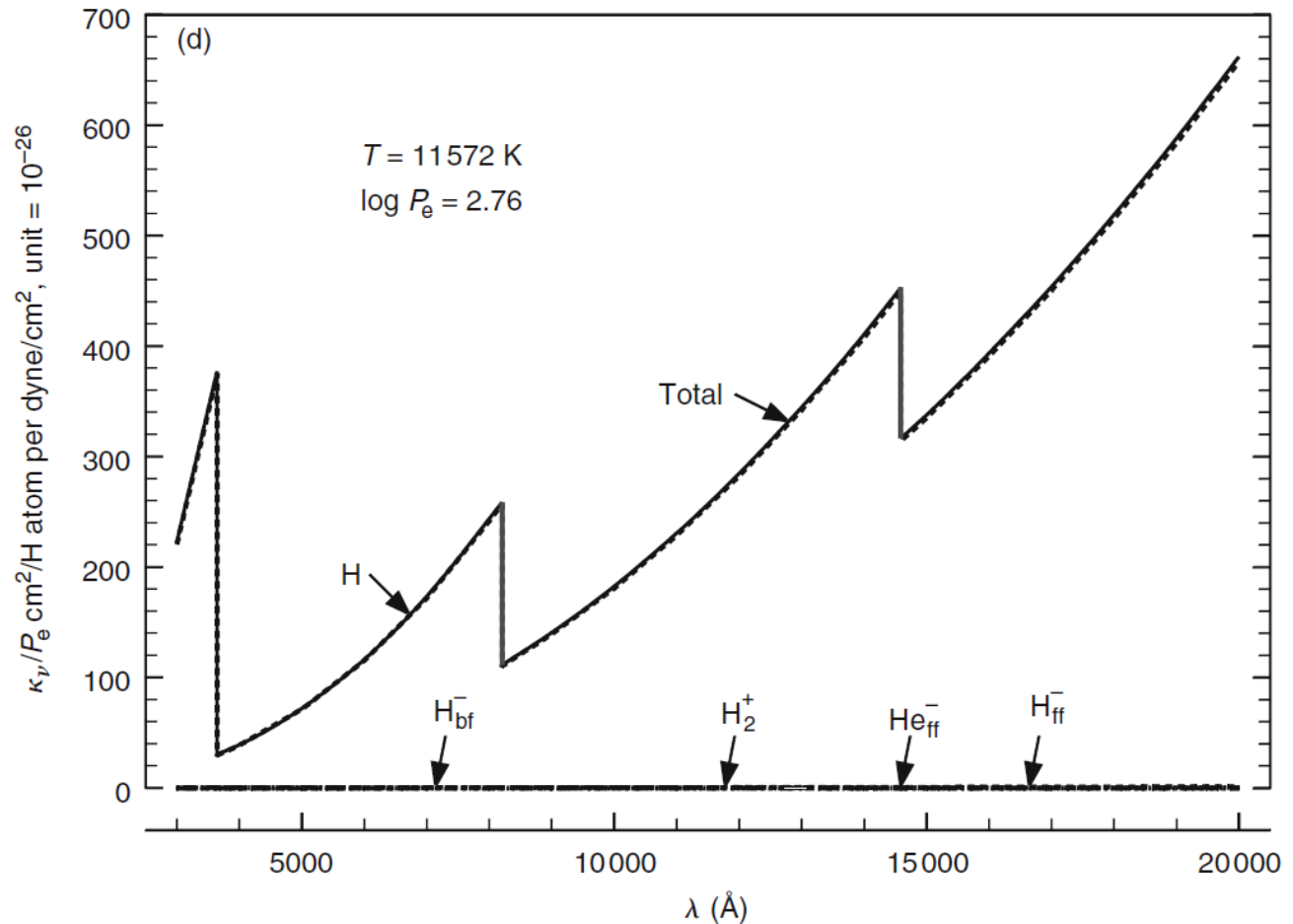


B-type (optical depth unity)

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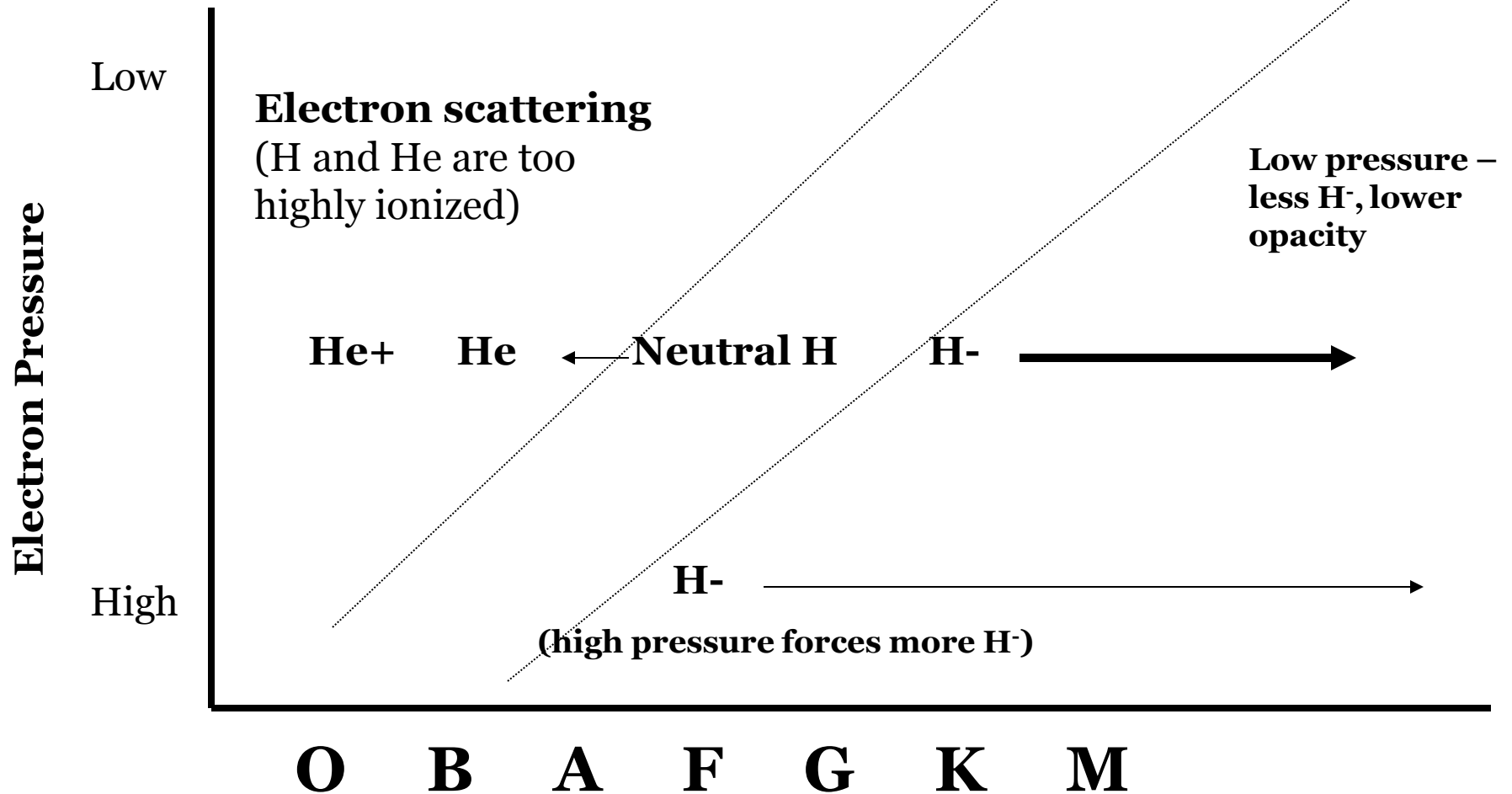
At late B, the
H (bound-free)
Balmer, Paschen &
Brackett continua
completely
dominate.

For O stars
electron scattering
is the primary
opacity source.



Dominant Opacity vs. Spectra Type

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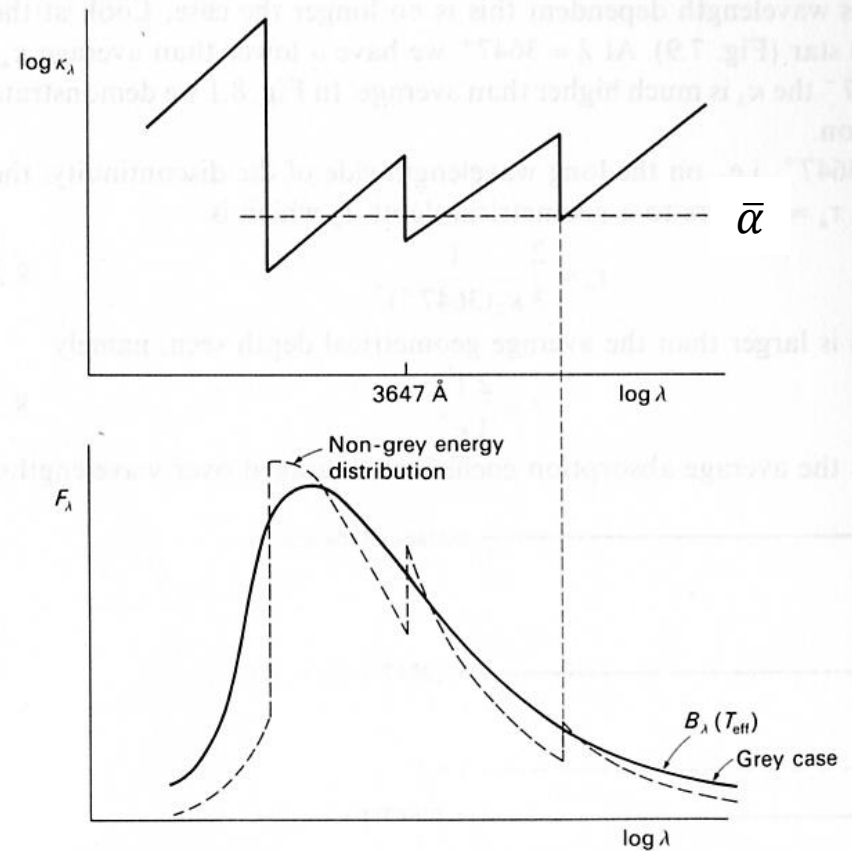
Continuum Energy Distribution

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What is the effect of the λ dependence of α_λ on the emergent spectrum?

Consider the Balmer discontinuity at $3647\text{\AA}$. Immediately **above** the discontinuity (3647^+), the opacity α_λ is **lower** than average (shown as $\bar{\alpha}$), so we probe **deeper** than average into the atmosphere, where S_λ (and F_λ) is **higher** than the grey case, so F_λ exceeds the Planck function.

For 3647^- , the opacity is **higher** than average, so we probe **less deep** into the atmosphere (where T is smaller), and so receive a **lower** F_λ .



Balmer jump

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- In hot stars, $T > 9000\text{K}$, H^- negligible, only H contributes to opacity.

$$\frac{\alpha^+}{\alpha^-} = \frac{\sigma^+(\text{H})}{\sigma^-(\text{H})} \frac{N_{\text{H}}(n=3)}{N_{\text{H}}(n=2)}$$

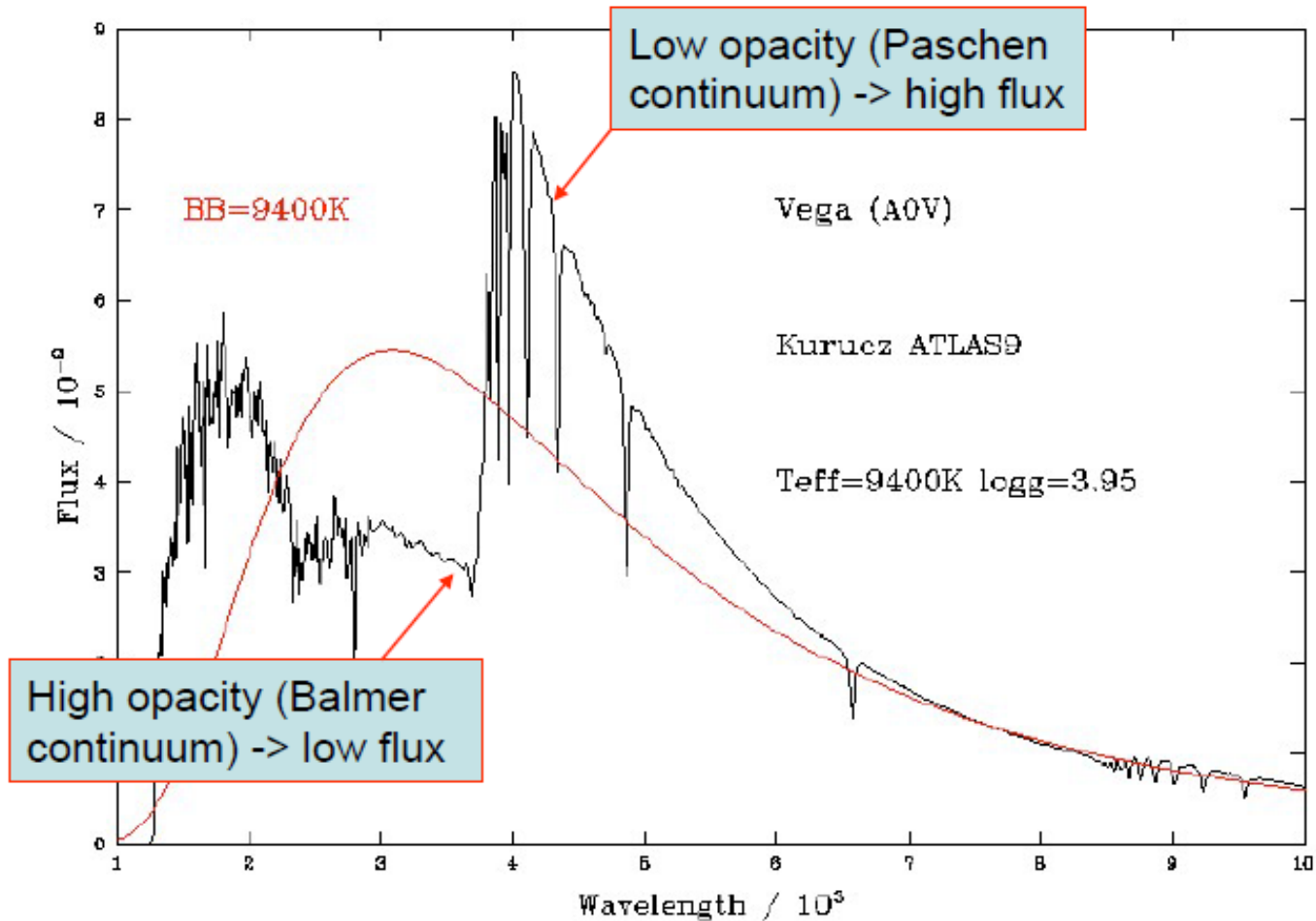
Function of T only

“observed” known From Boltzmann law(T)

Thus we can obtain the temperature.

Balmer jump in Vega

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Summary

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- **Negative H ion** confirmed as dominant Solar optical & IR opacity source from limb darkening.
- He b-f opacity relevant only for very hot stars. **Metal** (Fe) **opacity** contributes to opacity in Solar-type stars in **ultraviolet**.
- **Thompson** (electron) scattering is grey & dominates continuum opacity in **hot stars**. **Rayleigh** scattering most important for **late-type supergiants** in **UV**
- Observed form of e.g. **Balmer jump** in A stars can be understood from the **discontinuity** in continuous **H b-f opacity**.
- Nongreyiness changes the temperature structure.