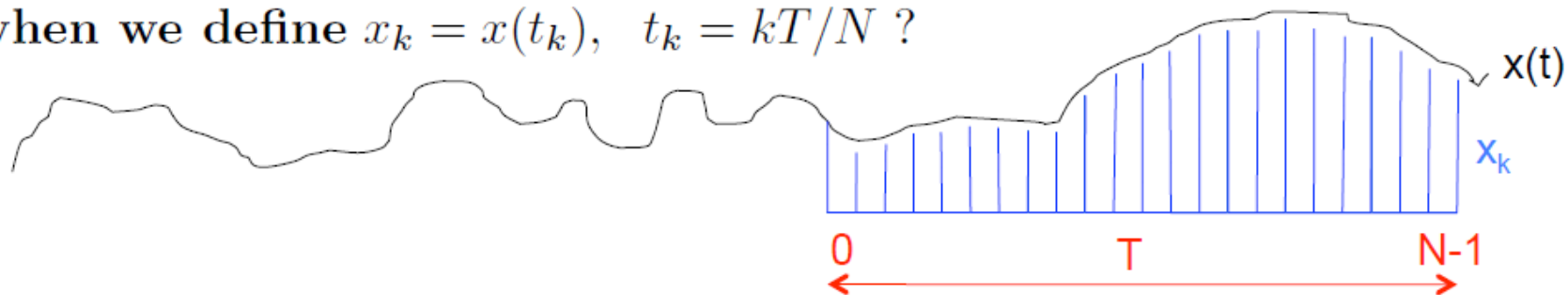


Continuous FT \Leftrightarrow Discrete FT

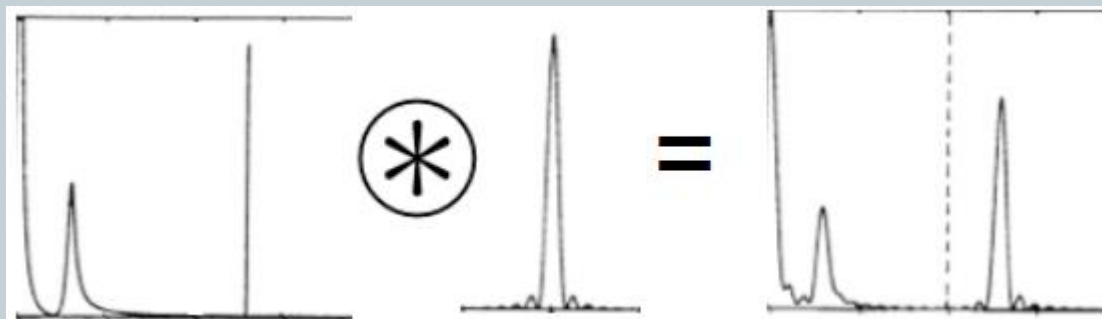
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- How can we connect continuous and discrete FT?

What is the relation of this 'ideal case' with the discrete Fourier transform when we define $x_k = x(t_k)$, $t_k = kT/N$?



- We use the convolution theorem: "the transform of the product is the convolution of the transforms".



Continuous FT \Leftrightarrow Discrete FT

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- We have:

- Time-series $x(t)$

- Window function $w(t)$:

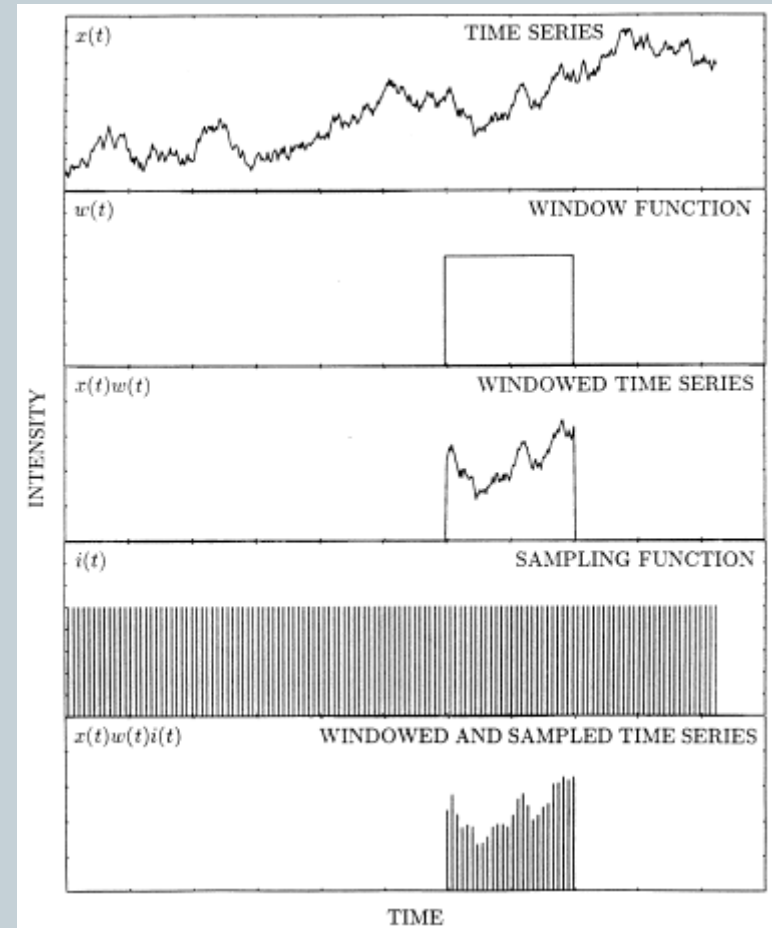
$$w(t) = \begin{cases} 1, & 0 \leq t < T \\ 0, & \text{otherwise} \end{cases}$$

- Sampling function $i(t)$:

$$i(t) = \sum_{-\infty}^{+\infty} \delta\left(t - \frac{kT}{N}\right)$$

- Now we multiply:

$$x_k = x(t) w(t) i(t)$$



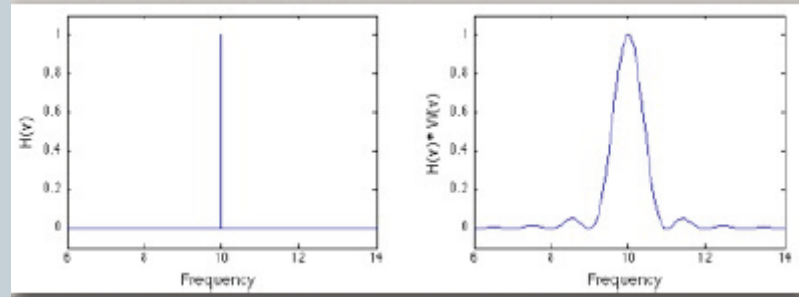
Continuous FT \Leftrightarrow Discrete FT

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- The power spectrum of $W(\nu)$:

$$|W(\nu)|^2 \equiv \left| \int_{-\infty}^{\infty} w(t) e^{2\pi\nu it} dt \right|^2 = \left| \frac{\sin \pi\nu T}{\pi\nu} \right|^2$$

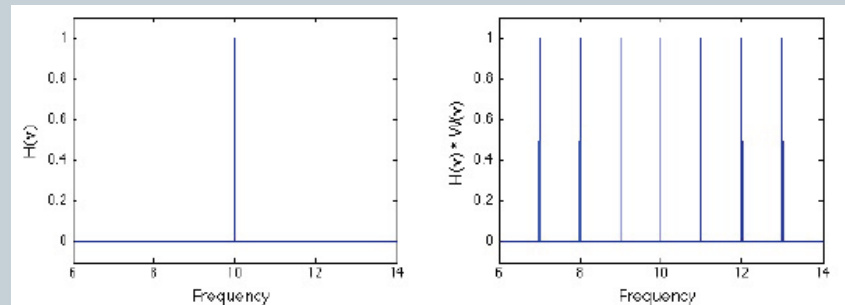
- Broadening of peaks



- The Fourier transform of $i(t)$:

$$I(\nu) \equiv \int_{-\infty}^{\infty} i(t) e^{2\pi\nu it} dt = \frac{N}{T} \sum_{l=-\infty}^{\infty} \delta\left(\nu - l \frac{N}{T}\right)$$

- Infinite series of δ functions, with spacing $N/T = 2 \nu_{Ny}$



Aliasing

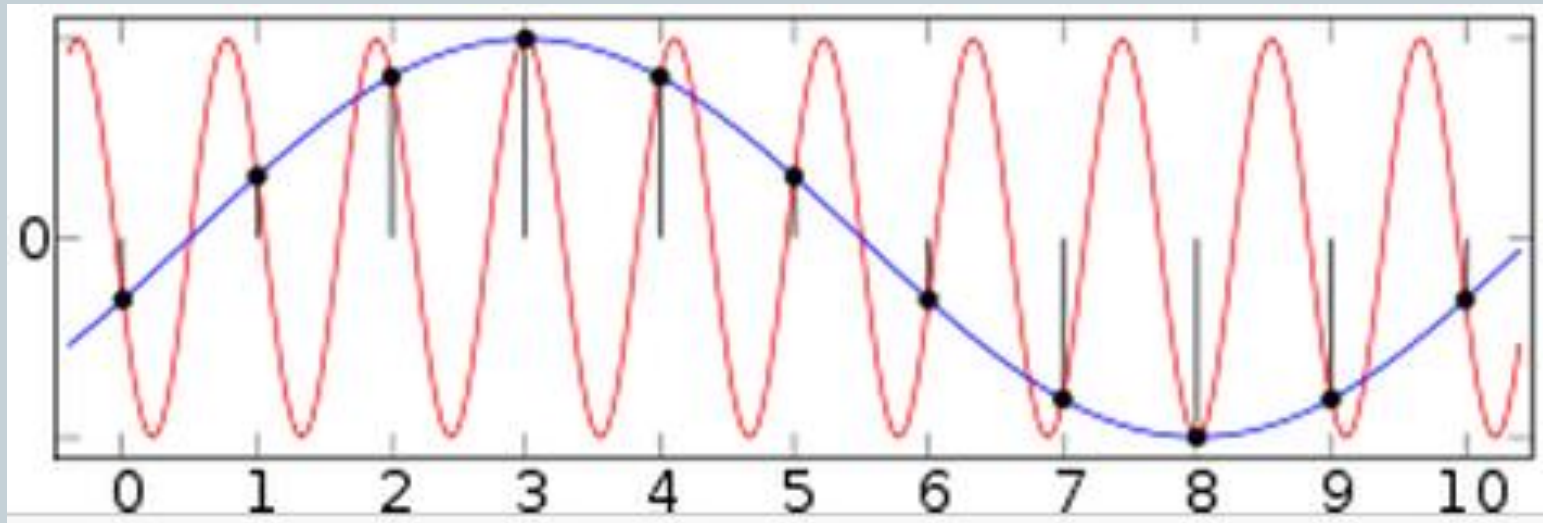
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- FT is symmetric in frequency for a real signal;
- Alias repeats it every $2\nu_{Ny}$;
- The power spectrum of the convolved function $|a(\nu) * I(\nu)|^2$ is reflected around the Nyquist frequency ν_{Ny} .
- This causes features with a frequency exceeding ν_{Ny} by ν_x (so, located at $\nu = \nu_{Ny} + \nu_x$) to also appear at a frequency $(\nu_{Ny} - \nu_x)$ — a phenomenon known as aliasing; the reflected feature is called the alias of the original one.

Aliasing

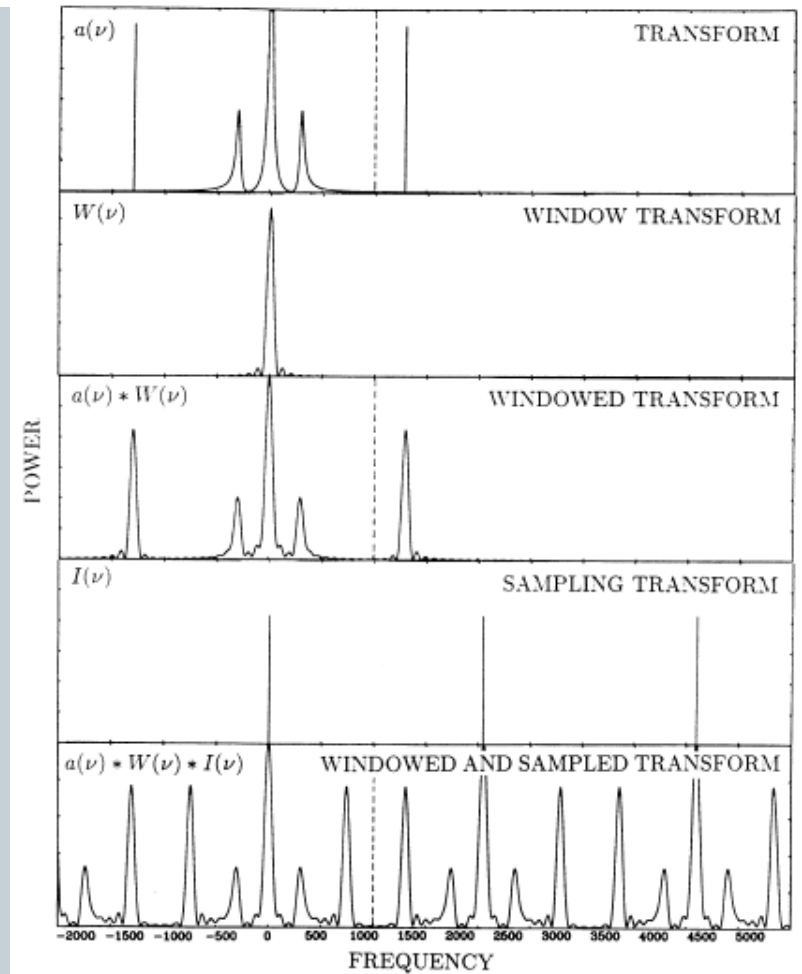
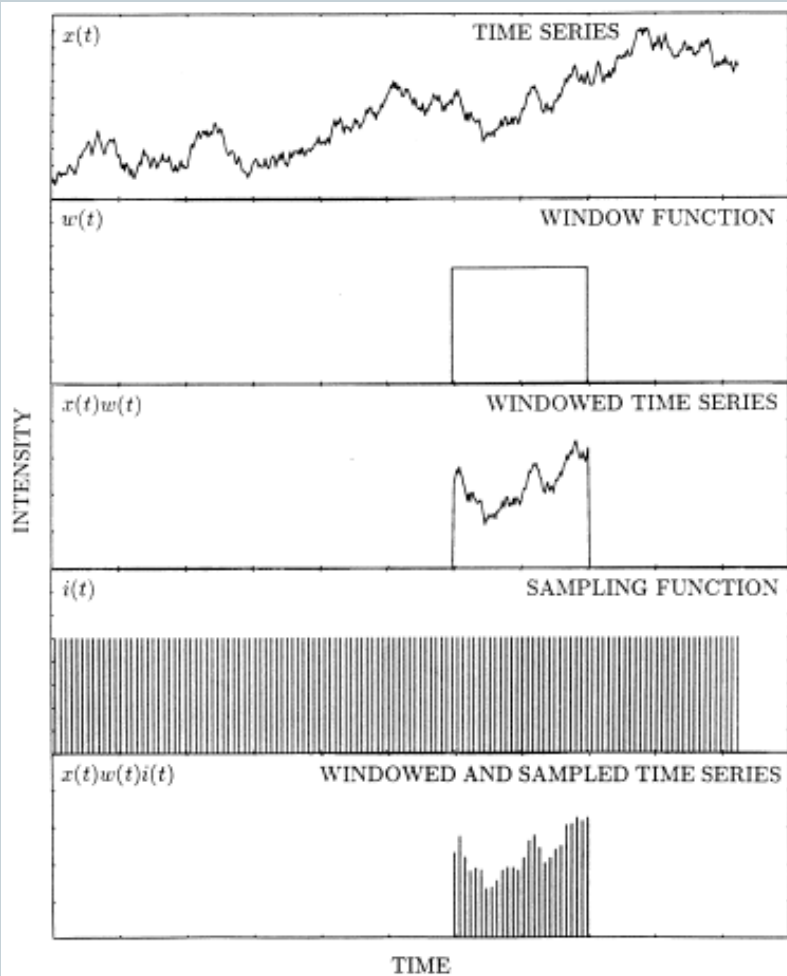
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- Two different sinusoids fit the same set of samples:



Continuous FT \Leftrightarrow Discrete FT

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Summary of discrete FT effects

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- **WINDOW:** broadening & sidebands
- **SAMPLING:** aliasing

Is aliasing a problem?

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- Not such a big problem for high-energy astronomy;
- In practice, we do not really discretely sample the data, but rather bin the data up;
- That means that before discrete sampling we convolve the $x(t)$ with the bin width.
- So, in the frequency domain, we multiply $a(\nu)$ with

$$B(\nu) = \frac{\sin\left(\frac{\pi\nu N}{T}\right)}{\pi\nu N/T}$$

- Suppression of high frequencies.

Is windowing a problem?

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- **Yes:**
 - It broadens delta peaks;
 - It flattens the slopes of noise components (sidelobes);
 - For steep spectra the "leakage" can be severe.
- **Solution: The longer the observation, the better.**