ASTROPHYSICS OF INTERACTING BINARY STARS

Lecture 4

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More about Accretion Disks

- In the most well-studied model, the disk is assumed to be physically *thin* and optically *thick*. This allows the maximum amount of heat to radiate away from the surface of the disk before matter falls into the accreting star.
- We also assume a stationary (**steady**) disk - the physical quantities in a disk do not change with time.
- Important, stationarity does not mean that there is no flow, for instance in radial direction in a disk. The only requirement is that this flow proceeds in such a manner that the physical quantities, like the surface density Σ and the radial velocity V_R remain unchanged. In other words: stationarity means that the time derivatives in the equations vanish.

The angular momentum problem

How does the accreting matter lose its angular momentum?

- Angular momentum is strictly conserved!
- Gas must shed its angular momentum for it to be actually accreted.
- Suppose that there is some kind of "viscosity" in the disk
  - Different annuli of the disk rub against each other and exchange angular momentum
  - Results in most of the matter moving inwards and eventually accreting
  - Angular momentum carried outwards by a small amount of material
- Process producing this "viscosity" might also be dissipative… could turn gravitational potential energy into heat (and eventually radiation)

Viscous accretion disks

- Disk self-gravitation is negligible so material in differential or Keplerian rotation with angular velocity $\Omega_K(R)$.
- Consider two consecutive rings of the accretion disk.
- If ν is the kinematic viscosity for rings of gas rotating, the torque exerted by the outer ring on the inner ring will be

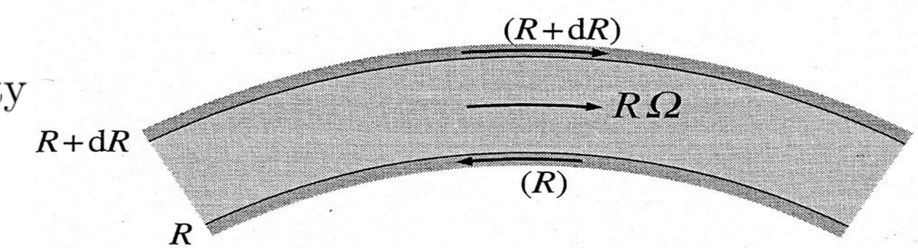
$$Q(R) = 2\pi R\, h\rho\nu R^2\, \frac{d\Omega}{dt}$$

h = thickness of disk at radius R

ν = coefficient of (kinematic) viscosity

ρ = density of disk at radius R

Ω = angular velocity of disk orbit



$\Sigma = h\rho$ is the surface density with h (scale height) measured in the z direction.

Viscous accretion disks

- Each ring has two plane faces of area $4\pi RdR$, so the radiative dissipation from the disk per unit area is

$$D(R) = \frac{Q}{4\pi R}\frac{d\Omega}{dt} = \frac{1}{2}\nu\Sigma\left(R\frac{d\Omega}{dt}\right)^2$$

- Evaluating for circular Keplerian orbits:

$$D(R) = \frac{9}{8}\nu\Sigma\frac{QM}{R^3}$$

Viscous accretion disks

- From a consideration of radial mass and angular momentum flow in the disk, it can be shown (Frank, King & Raine) that

$$\nu\Sigma = \frac{\dot{M}}{3\pi}\left[1 - \left(\frac{R_*}{R}\right)^{1/2}\right]$$

- We then have:

$$D(R) = \frac{3GM\dot{M}}{8\pi R^3}\left[1 - \left(\frac{R_*}{R}\right)^{1/2}\right]$$

and hence the radiation energy flux through the disk faces is **independent of viscosity**

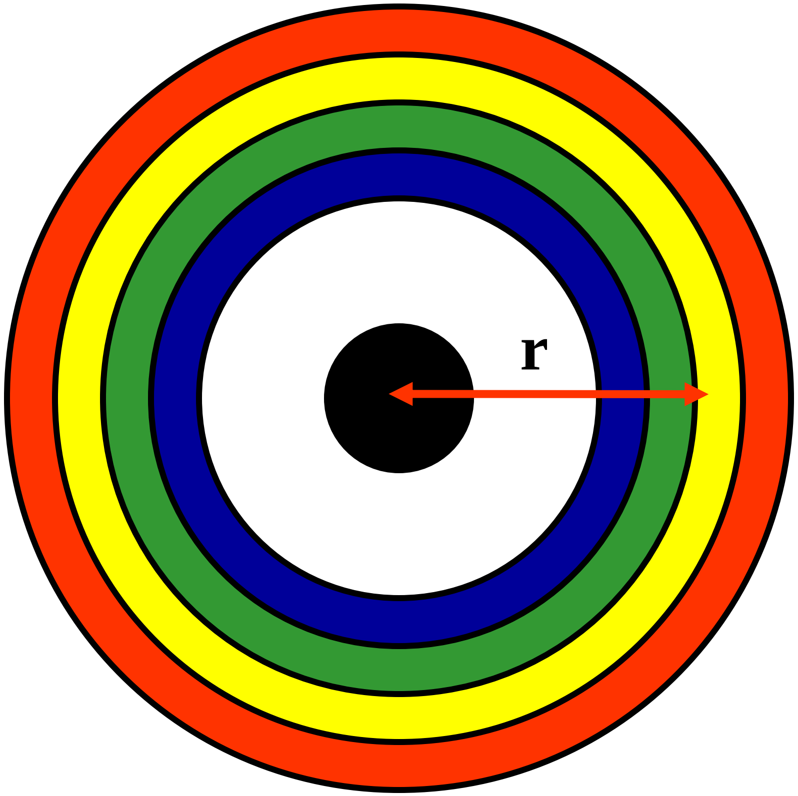
Viscous accretion disks

- The total disk luminosity is

$$L_{\text{disk}} = \int_{R_*}^{\infty} D(R) 2\pi R dR = \frac{1}{2}\frac{GM\dot{M}}{R_*},$$

i.e., half the gravitational energy released in accreting the gas to radius R_*. The remaining gravitational energy goes into rotational energy, which may be either dissipated in a boundary layer or sucked into a black hole.

Accretion Disk Temperature Structure



- If the accretion disk is optically thick, it can be considered as rings or annuli of blackbody emission.
- Dissipation rate, *D(R)* is

$$D(R) = \frac{3GM\dot{M}}{8\pi R^3}\left[1 - \left(\frac{R_*}{R}\right)^{1/2}\right]$$

= blackbody flux

$= \sigma T(R)^4$

Accretion Disk Temperature Structure

- Thus temperature as a function of radius *T(R)*:

$$T(R) = \left\{ \frac{3GM\dot{M}}{8\pi R^3 \sigma} \left[1 - \left(\frac{R_*}{R} \right)^{1/2} \right] \right\}^{1/4}$$

- and if $T_* = \left(\frac{3GM\dot{M}}{8\pi R_*^3 \sigma} \right)^{1/4}$

- Then for $R \gg R_*$

$$T(R) = T_* \left(R / R_* \right)^{-3/4}$$

Accretion Disk Temperature Structure

- In dwarf novae in outburst and long-period novalikes, this simple $R^{-3/4}$ radial temperature profile is indeed observed.
- In quiescent dwarf novae a much flatter profile is observed. This is thought to be because the disk does not achieve a steady state in quiescence.

Accretion Disk Spectrum

- Integrating the blackbody spectrum over radius gives the predicted spectrum of an optically thick, geometrically thin, steady-state accretion disk

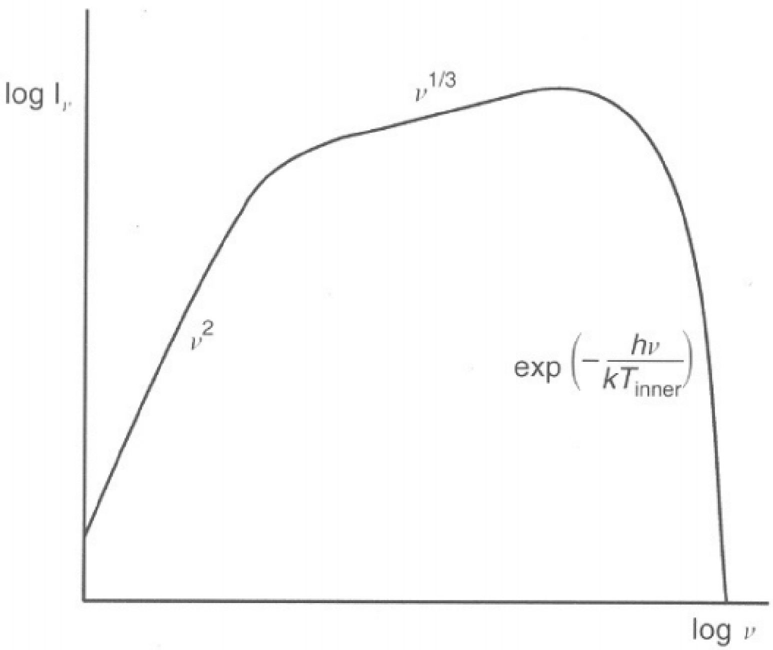
$$S_\nu \propto \int_{R_{\rm in}}^{R_{\rm out}} B_\nu[T(R)] 2\pi R dR,$$

- We now have the shape of the spectrum over most of its range:

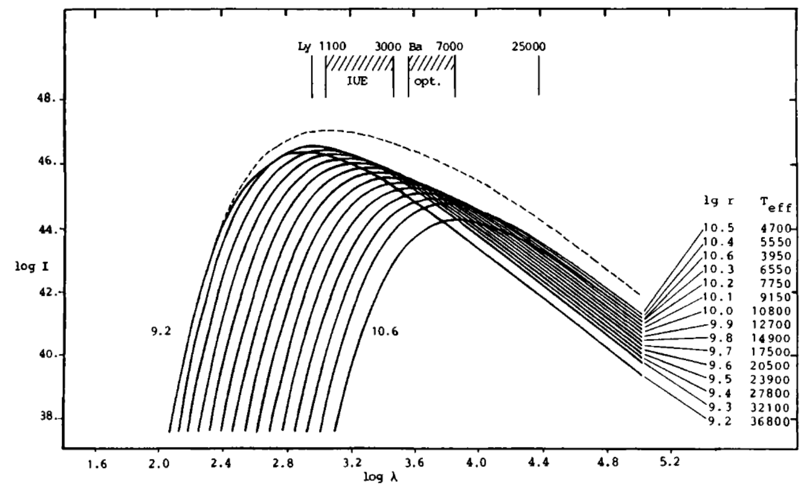
$$S_\nu \propto \nu^{1/3}$$

Accretion Disk Spectrum

Theoretical Spectrum of an AD



Contributions of BB annuli to the total intensity distribution of an AD



Accretion Disk Spectrum

- Note, though, that the low- and high-frequency ends will have a different form:
  - From the outer edge of the disc we will see the Rayleigh-Jeans tail of T_{outer}

$$S_\nu \propto \nu^2$$

  - From the inner edge, an exponential cut-off

$$S_\nu \propto e^{-h\nu/kT_{inner}}$$

Accretion Disk Thickness

- Assume that there is no motion in the z-direction, the hydrostatic equilibrium equation is

$$\frac{1}{\rho}\frac{\partial P}{\partial z} = \frac{\partial}{\partial z}\left[\frac{GM}{(R^2+z^2)^{1/2}}\right]$$

- For a thin disk ($z \ll R$) this becomes

$$\frac{1}{\rho}\frac{\partial P}{\partial z} = -\frac{GMz}{R^3}$$

- We can relate P and ρ via the sound speed in the gas $dP = c_s^2\, d\rho$, and then integrate to find that the density in the disk falls off exponentially with height

$$\rho(z) = \rho_0 \exp\left(-\frac{z^2}{2h^2}\right)$$

with a height scale factor given by

$$h^2 \cong \frac{c_S^2 R^3}{GM}$$

Accretion Disk Thickness (cont.)

- The scale height can be re-written in terms of the rotational velocity:

$$V_{rot} = \sqrt{\frac{GM}{R}} \Longrightarrow h^2 \cong \frac{c_S^2 R^2}{V_{rot}^2}$$

- If we have $R \gg h$ we must have $\boldsymbol{V_{rot}^2} \gg \boldsymbol{c_S^2}$ and so the rotation of the disk is highly supersonic.

- Lets re-write the scale height again:

$$\frac{h}{R} \cong \frac{c_S}{V_{rot}}$$

Viscous accretion disks

- Normal "molecular/atomic" viscosity fails to provide required angular momentum transport by many orders of magnitude!
- **What gives rise to viscosity?**
- Source of <u>anomalous viscosity</u> was a major puzzle in accretion disk studies!
- Long suspected to be due to some kind of turbulence in the gas… Shakura & Sunyaev (1973) introduced the "α-disk" parameterization : $\nu = \alpha c_s h$

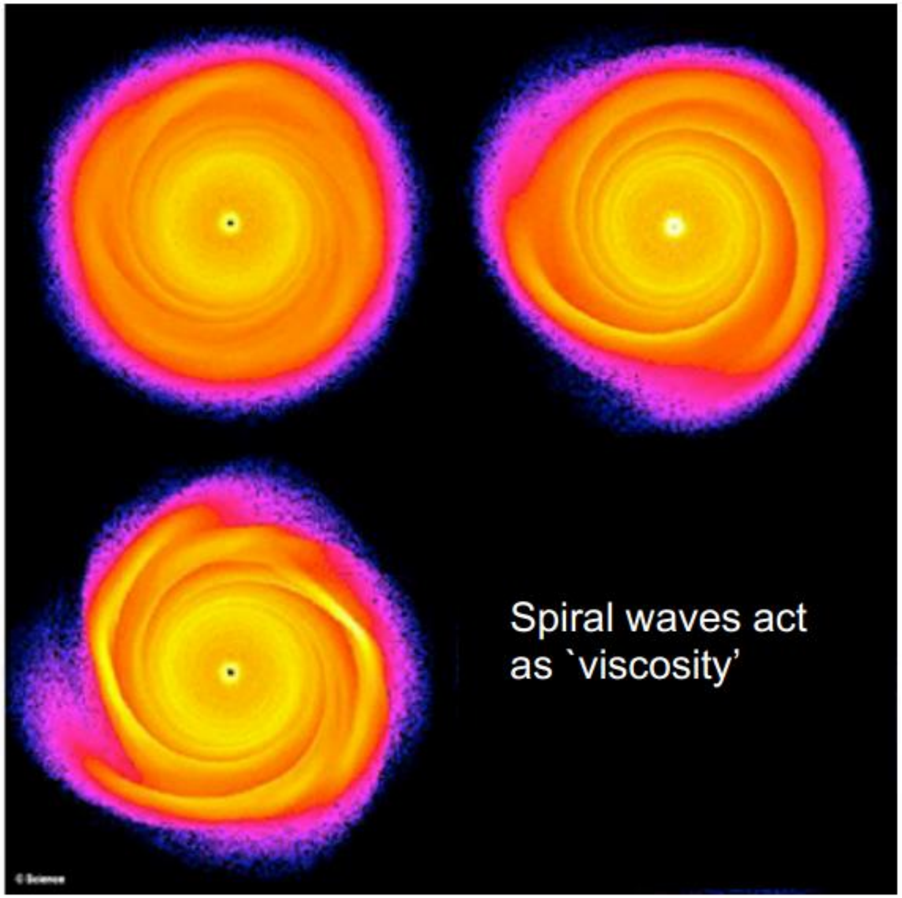
where c_S is the sound speed, *h* is the disk scale height (a function of radius), and α is a dimensionless constant.

- Typical models of disks have $\alpha \sim 0.01 - 0.1$

Viscous accretion disks

- For specified α, one can completely solve for the structure of a (steady-state or time-dependent) accretion disk. However, the assumption that α is constant with radius, with time, or from one accretion disk to another is nothing more than **an assumption**.
- 20 years of accretion disk studies were based on this "alpha-prescription"…
- While the notion of "turbulent viscosity" is intuitively appealing, detailed studies suggest that hydrodynamic mechanisms alone will not produce sustained turbulence in differentially rotating disks.

Gravitational (in)stability



Rice & Armitage

The magnetorotational instability

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A POWERFUL LOCAL SHEAR INSTABILITY IN WEAKLY MAGNETIZED DISKS.
I. LINEAR ANALYSIS

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ABSTRACT

In this paper and a companion work, we show that a broad class of astrophysical accretion disk is dynamically unstable to axisymmetric disturbances in the presence of a weak magnetic field. Because of the ubiquity of magnetic fields, this result bears upon gaseous differentially rotating systems quite generally. This work presents a linear analysis of the instability. (The companion work presents the results of nonlinear numerical simulations.) The instability is local and extremely powerful. The maximal growth rate is of order the angular rotation velocity and is *independent* of the strength of the magnetic field, provided only that the energy density in the field is less than the thermal energy density. Unstable axisymmetric disturbances require the presence of a poloidal field component, and are indifferent to the presence of a toroidal component. The instability also requires that the angular velocity be decreasing outward. In the absence of a powerful dissipation process, there are no other requirements for instability. Fluid motions associated with the instability directly generate both poloidal and toroidal field components. We discuss the physical interpretation of the instability in detail. Conditions under which saturation occurs are suggested. The *nonemergence* of the classical Rayleigh criterion for shear instability in the limit of vanishing field strength is noted and explained. The instability is sensitive neither to disk boundary conditions nor to the constituative fluid properties. Its existence precludes the possibility of internal (noncompressive) wave propagation in a disk. If present in astrophysical disks, the instability, which has the character of an interchange, is very likely to lead to generic and efficient angular momentum transport, thereby resolving an outstanding theoretical puzzle.

Subject headings: accretion — hydrodynamics — hydromagnetics — instabilities

1. INTRODUCTION

A long-standing challenge to the theory of accretion disks has been to show from first principles a mechanism capable of generating a turbulent viscosity, since the angular momentum transport resulting from the action of ordinary molecular viscosity is extremely inefficient (Pringle 1981). In this work and a companion paper (Hawley & Balbus 1991, hereafter II), we show that accretion disks are subject to a very powerful shearing instability mediated by a *weak* magnetic field of any plausible astrophysical strength. We suggest that this instability is of some relevance to understanding the origin of turbulent viscosity in accretion disks.

It is of course widely appreciated that magnetic fields can play an important role in accretion disk dynamics (e.g., Blandford 1989). In their seminal paper, Shakura & Sunyaev (1973) noted that magnetic turbulance could act as a viscous couple, but argued that nonlinear perturbations would be required to disrupt laminar flow. Magnetic fields have also been invoked, for example, as a source

The magnetorotational instability

- Major breakthrough in 1991… Steve Balbus and John Hawley (re)-discovered a powerful magneto-hydrodynamic (MHD) instability
  - Called magnetorotational instability (MRI)
  - MRI will be effective at driving turbulence
  - Turbulence transports angular momentum in just the right way needed for accretion
- Rough idea: MHD instabilities in a differentially rotating, magnetized disk drive turbulence, which in turn produces viscosity.

Structure of the standard α-disk

Astron. & Astrophys. 24, 337–355 (1973)

Black Holes in Binary Systems. Observational Appearance

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Summary. The outward transfer of the angular momentum of the accreting matter leads to the formation of a disk around the black hole. The structure and radiation spectrum of the disk depend, mainly on the rate of matter inflow $\dot{M}$ into the disk at its external boundary. The dependence on the efficiency of mechanisms of angular momentum transport (connected with the magnetic field and turbulence) is weaker. If $\dot{M} = 10^{-9} - 3 \cdot 10^{-8} \frac{M_\odot}{\text{year}}$ the disk around the black hole is a powerful source of X-ray radiation with $h\nu \sim 1 - 10$ keV

saturated by broad recombination and resonance emission lines. Variability, connected with the character of the motion of the black hole, with gas flows in a binary system and with eclipses, is possible. Under certain conditions, the hard radiation can evaporate the gas. This can counteract the matter inflow into the disk and lead to autoregulation of the accretion.

If $\dot{M} \gg 3 \cdot 10^{-8} \frac{M_\odot}{\text{year}}$ the luminosity of the disk around the black hole is stabilized at the critical level of

Properties of the thin, Steady-State accretion disk

- Thickness: $\frac{h}{R} \cong \frac{c_S}{V_{rot}}$
- Surface density (kg/m²): $\Sigma = \int_{-\infty}^{+\infty} \rho \, dz = \sqrt{2\pi}\rho_0 h$
- Viscosity (alpha model – hides uncertain physics): $\nu \equiv \alpha c_S h$
- Temperature: $T(R) = T_*(R/R_*)^{-3/4}$
- The radial velocity is highly subsonic:

$$V_R = \frac{\dot{M}}{2\pi R\Sigma} = \frac{3\nu}{2R\left[1-\left(\frac{R_*}{R}\right)^{1/2}\right]} \sim \frac{\nu}{R} \sim \alpha c_S \frac{h}{R} \ll c_S$$

$$\nu\Sigma = \frac{\dot{M}}{3\pi}\left[1-\left(\frac{R_*}{R}\right)^{1/2}\right]$$

Accretion disk typical timescales:

- Dynamical timescale – the timescale on which inhomogeneities on the disk surface rotate, or hydrostatic equilibrium in the vertical direction is established:

$$t_{dyn} \sim \frac{R}{V_{rot}} \sim \Omega_K^{-1}$$

- Viscous timescale, the timescale on which matter diffuses through the disk under the effect of viscous torques:

$$t_{vysc} \sim \frac{R}{V_R} \sim \frac{R^2}{\nu}$$

- Thermal timescale, the timescale for re-adjustment to thermal equilibrium: $t_{th} \sim \frac{\Sigma c_S^2}{D(R)} \sim (h/R)^2 t_{vysc}$

Steady accretion disks: confrontation with observation

- Inner regions:
  - closely related to the compact star.
- Outer regions:
  - radiating predominantly in optical and IR.

- To study the outer regions of the disks observationally, we require that the light in one or more of these parts of the spectrum is dominated by the disk contribution.