

# ASTROPHYSICS OF INTERACTING BINARY STARS

**Lecture 3**

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# Accretion disks: Boundary Layer

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- The required Keplerian angular velocity  $\Omega_K$  cannot be maintained at the inner edge of the disk if it is to join smoothly to a **non-magnetic** accreting star spinning at below the break-up velocity  $\Omega_K(R)$ .
- The region over which gas moving at Keplerian velocities in the disk is decelerated to match the star angular velocity  $\omega_*$  is called the **boundary layer** (BL).
- If the star spins more slowly than the break-up value, the BL must release a large amount of energy as the accreting matter comes to rest at the stellar surface. Some of this is used to spin up the star, but there remains an amount to be dissipated.

# Accretion disks: Boundary Layer

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- From conservation of energy and angular momentum it can be shown (Kley 1991) that the energy released in the BL is

$$L_{BL} = \frac{GM_*\dot{M}}{2R} \left(1 - \frac{\omega_*}{\Omega_K}\right)^2 = L_d \left(1 - \frac{\omega_*}{\Omega_K}\right)^2$$

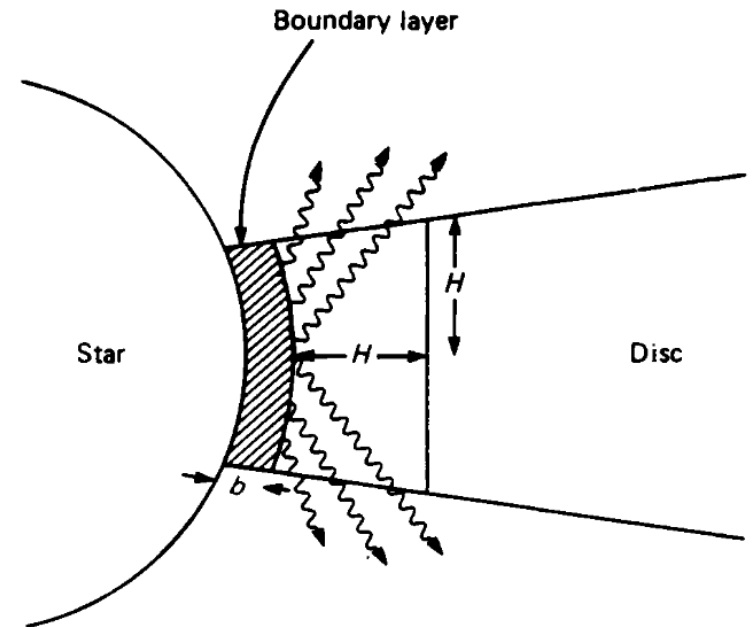
where  $L_d$  is the total accretion disk luminosity.

- For  $\omega_* \ll \Omega_K$  this is one-half of the total accretion luminosity.

# Accretion disks: Boundary Layer

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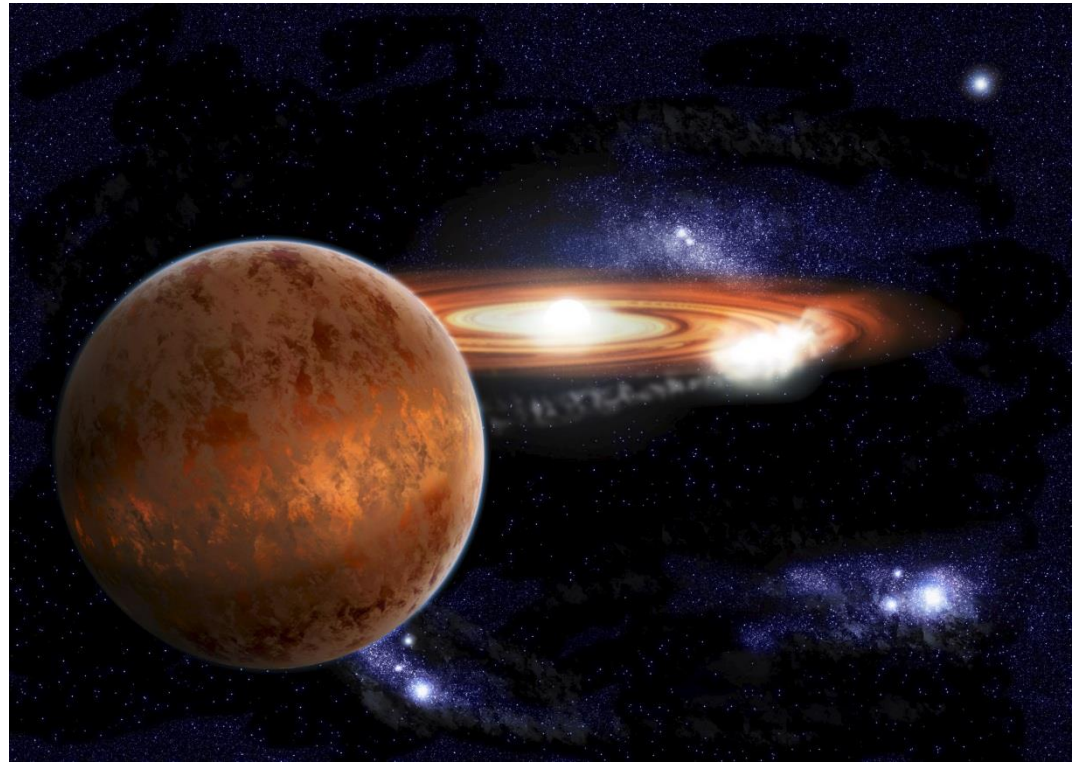
- Where the luminosity of the BL appears in the spectrum depends on its optical thickness.
- The bulk of the radiation from an optically thick BL should be emitted in the soft X-ray and EUV regions.
- An optically thin BL should radiate in hard X-rays, with energies  $\sim 20$  keV.



# Accretion disks: Bright spot

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The gas stream impacts onto the outer rim of an accretion disk at supersonic speeds, creating a shock-heated area that may radiate as much or more energy at optical wavelengths as all the other components (primary, secondary, disk) combined.



# Accretion disks: Bright spot

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- If the bulk of the stream flow impacts at the rim bright spot, its luminosity will be given approximately by the energy released on allowing mass to fall at a rate  $\dot{M}_2$  from infinity to a distance  $r_d$  from the primary:

$$L_{BS} \approx \frac{GM_1\dot{M}_2}{r_d}$$

This is an upper limit:

(i) the fall is from  $L_1$  not  $\infty$       (ii) the stream meets the rotating disk edge obliquely

- Compare with the luminosity of the accretion disk, through which mass is flowing at a rate  $\dot{M}_d$ :

$$L_d \approx \frac{GM_1\dot{M}_d}{2R_1}$$

- Sometimes  $L_{BS} > L_d \rightarrow \dot{M}_d < \dot{M}_2$

# Accretion disks: Bright spot

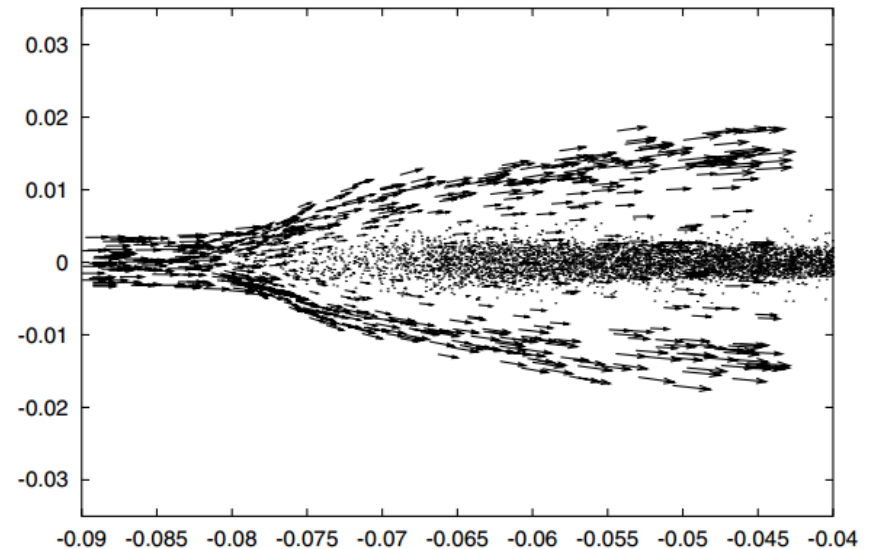
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- The location of the bright spot is obviously determined by the intersection of the stream trajectory with the outer edge of the disk.
- Can be used to measure  $q$  and the disk radius (position of the bright spot can be derived from eclipse observations).

# Stream-disk overflow

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- Part of the stream can flow over the rim of the disk and continue approximately along the single particle trajectory over the face of the disk until it impacts the disk at a later time.



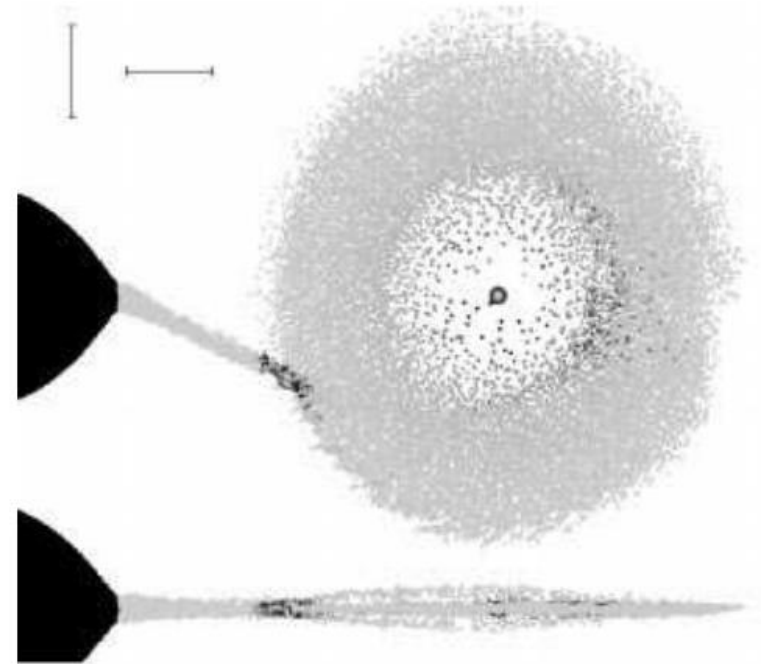
Stream-disk impact region  
(from Kunze, Speith and Hessman, 2001)



# Stream-disk overflow

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- The ultimate impact point for stream material continuing over the face of the disk is in the vicinity of the point of closest approach to the primary, creating there a **second** bright spot at a position  $(r, \alpha) \approx (r_{\min}, 148^\circ)$ .  
(Lubow, 1989)



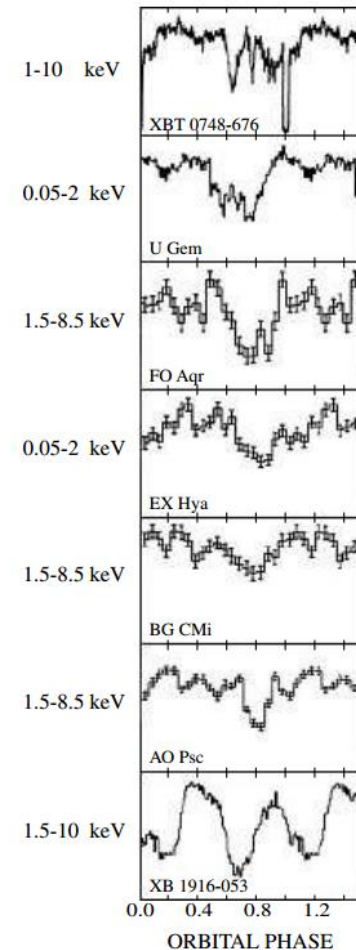
Stream-disk impact region  
(from Kunze, Speith and Hessman, 2001)

# Stream-disk overflow

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- The stream overflow can cause the X-ray absorption dips observed in cataclysmic variables (CVs) and low-mass X-ray binaries (LMXBs) around orbital phase 0.7, if the inclination is at least  $65^\circ$ .

Orbital phase-folded X-ray light curves  
(Hellier et al, 1993)



# Stream-disk simulations

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- If the impact region is optically thick, so that the energy of impact is not quickly radiated, then part of the stream bounces off the disk and is sprayed into the Roche lobe;
- The denser core of the stream can penetrate into the edge of the disk releasing its kinetic energy at optical depths greater than unity, thus locally heating the rim, increasing its scale height and causing a bulge that runs around the edge of the disk for typically half the perimeter;
- The stream overflow.

# Stream-disk simulations

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- **However!** Bisikalo et al.:
- *"The interaction between the stream from the inner Lagrange point and the disk is shockless"*
- *"A region of enhanced energy release is formed due to the interaction between the circum-disk halo and the stream and is located beyond the disk, and the resulting shock is fairly extended."*
- *Instead of "a bright spot" – "a hot line"*

# Accretion Disks

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- Accretion disks are important in astrophysics as they efficiently transform gravitational potential energy into radiation.
- The gas accreted in a mass transfer binary must lose the gravitational potential energy liberated as it falls toward the mass gaining star. If this energy is radiated, luminosity is:

$$L \approx \frac{GM\dot{M}}{R}$$

where  $M$  and  $R$  are the mass and radius of the accreting star.

# Energetics of accretion

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- The luminosity of an accretion disk is the bigger the larger the mass flow rate is, the higher the mass of the accretor is, and the more compact the accretor is.
- Compare to the rest mass energy of the gas accreted per unit time:

$$\dot{M}c^2$$

- Efficiency of the accretion process (fraction of the rest mass energy that is radiated):

$$\varepsilon \approx \frac{GM\dot{M}}{R} \times \frac{1}{\dot{M}c^2} = \frac{GM}{Rc^2}$$

# Energetics of accretion

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- Accretion onto a **main sequence star**: as template we take the Sun:  $M=1.99 \cdot 10^{33}$ g,  $R=6.96 \cdot 10^{10}$ cm:

$$\varepsilon \approx 2 \cdot 10^{-6}$$

- Compare to nuclear fusion of hydrogen to helium. Energy release is  $6 \cdot 10^{18}$  erg per gram of hydrogen:

$$\varepsilon_{H \rightarrow He} \approx 7 \cdot 10^{-3} \quad (0.7\%)$$

- Thus, the specific energy output of accretion onto a main sequence star is more than three orders of magnitude less efficient than the hydrogen-helium fusion.
- The absolute values of energy output, i.e., the luminosity, however, depend also on the amount of mass involved in the fusion and the accretion process.

# Energetics of accretion

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- Accretion onto a **white dwarf**: typical mass  $10^{33}$ g,  $R=10^9$ cm:

$$\varepsilon \approx 10^{-4}$$

- Accretion energy is still much smaller - if the accreted hydrogen burns on the surface of the white dwarf can release a lot more energy.
- However, it becomes an interesting energy source in such systems by the sheer fact that nuclear fusion does no longer happen in white dwarfs.



# Energetics of accretion

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- **A neutron star:** typical mass  $3 \cdot 10^{33}$ g,  $R=10^6$ cm:

$$\varepsilon \approx 0.2$$

- **A black hole:**

$$\varepsilon \approx 1/12$$

(More detailed relativistic calculation gives  $\varepsilon \approx 0.06$  to  $0.4$ )

- **Very high efficiency - accreting neutron stars and black holes in binaries are luminous sources, normally in X-ray radiation**

# Accretion disk properties

## Order-of-magnitude estimates

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### □ Mass flow rates in interacting close binary systems.

As template we take a cataclysmic variable (CV)- a binary star system that has a *white dwarf* and a normal star companion. It is typically small – the entire binary system is usually the size of the Earth-Moon system – with an orbital period of 1 to 10 hours. The observed disk luminosity is of order of one solar luminosity  $L=3.9 \cdot 10^{33}$  erg/s. Then

$$\begin{aligned}\dot{M} &= \frac{LR_{WD}}{GM_{WD}} \\ &= 2.0 \cdot 10^{16} \frac{\text{g}}{\text{s}} \left( \frac{L}{L_{\odot}} \right) \left( \frac{R_{WD}}{10^{-2} R_{\odot}} \right) \left( \frac{M_{WD}}{M_{\odot}} \right)^{-1} = \\ &= \underline{\underline{3 \cdot 10^{-10} \frac{M_{\odot}}{\text{yr}}}} \left( \frac{L}{L_{\odot}} \right) \left( \frac{R_{WD}}{10^{-2} R_{\odot}} \right) \left( \frac{M_{WD}}{M_{\odot}} \right)^{-1}\end{aligned}$$

# Accretion disk properties

## Order-of-magnitude estimates

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### □ Temperatures of accretion disks.

When one knows the outer radius of a disk and its luminosity one can estimate an average effective temperature of a disk. The luminosity is proportional the radiating area times the forth power of the (average) temperature (assuming the black body approximation):

$$L_d \approx 2R_d^2 \pi \sigma T_{eff}^4$$

The factor of 2 takes care of the two surfaces of an accretion disk (“top” and “bottom” surface).

# Accretion disk properties

## Order-of-magnitude estimates

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### □ Temperatures of accretion disks (cont).

With the total disk luminosity of  $L \approx GMM\dot{M}/R_{WD}$  we get

$$T_{eff} \approx 10^4 K \left[ \frac{\left( \frac{M_{WD}}{M_{\odot}} \right) \left( \frac{\dot{M}}{10^{-9} M_{\odot} \text{ yr}^{-1}} \right)}{\left( \frac{R_{WD}}{10^{-2} R_{\odot}} \right) \left( \frac{R_d}{R_{\odot}} \right)^2} \right]^{1/4}$$

# Accretion disk properties

## Order-of-magnitude estimates

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### □ **Temperatures of accretion disks (cont).**

The average temperature depends only weakly (by its fourth root) on accretor's mass and size and on the accretion rate, but somewhat stronger (by its root) on the disk size.

□ The dependence on the available area shows that the effective temperature cannot be constant throughout the disk. As long as  $R_d \gg R_{WD}$ , a change of  $R_d$  does not alter the luminosity considerably while, at the same time, it strongly changes the available radiating area and thus the effective temperature.

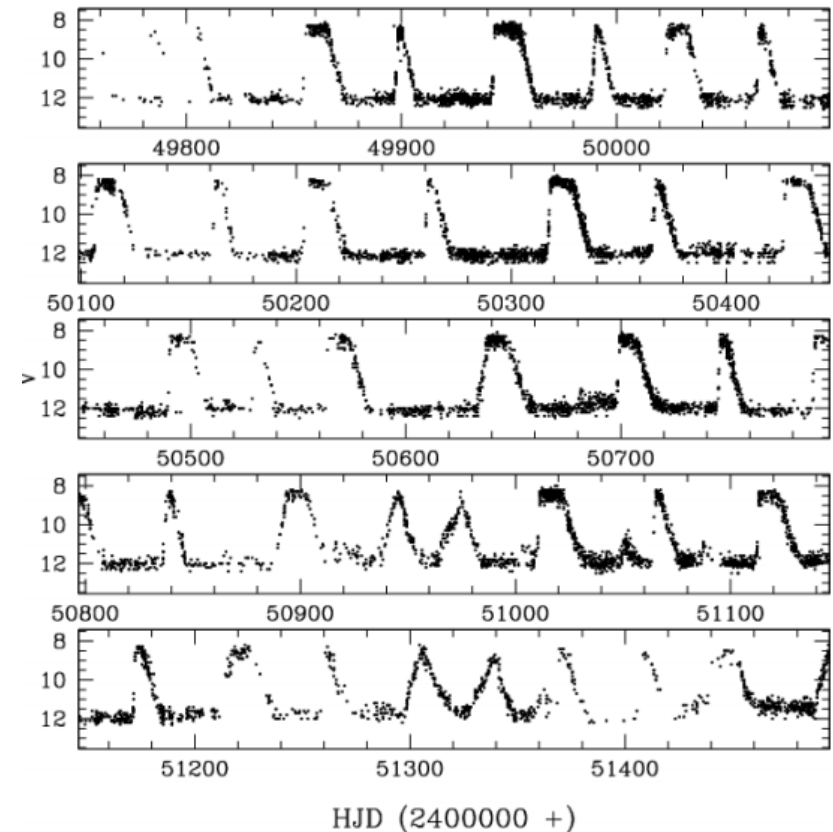
□ This leads to the suspicion that the effective temperature in accretion disks decreases with increasing radius. This is the case, indeed.

# Accretion disk properties

## Order-of-magnitude estimates

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- **Disk masses in dwarf novae.**  
Dwarf novae - a subtype of CVs which exhibits outbursts with a characteristic (though not strict) repetition period of a few ten days.
- The outbursts are caused when the accretion disk reaches a critical temperature, the disk becomes unstable and the gas collapses onto the white dwarf.



# Accretion disk properties

## Order-of-magnitude estimates

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### □ Disk masses in dwarf novae (cont).

The repetition time scale of the outbursts defines the average evolution time scale, i.e. the time scale it takes a particle on average to move through the entire extent of the disk.

- Let us assume an evolution time scale  $\tau_{\text{disk}} \approx 10 \text{ days} \approx 10^{-1.5} \text{ yr}$ . Together with the mass flow rate we get a disk mass in a dwarf

$$M_{\text{disk,DN}} \approx \dot{M}_{\text{DN}} \tau_{\text{DN}} \approx 10^{-9} M_{\odot} \text{yr}^{-1} 10^{-1.5} \text{yr} \approx 10^{-10.5} M_{\odot} \ll M_{\text{WD}}$$

- Dwarf nova accretion disks have a much smaller mass than the white dwarf about which they move and onto which they accrete. In this respect DN disks are practically **massless**.

# Accretion disk properties

## Order-of-magnitude estimates

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### □ Velocities in dwarf nova disks.

If the disk is practically massless, the particles in the disk move about the centre according to Kepler's third law as long as they move on closed orbits. The azimuthal velocity

$$V_{\phi} = \sqrt{\frac{GM}{R}} = 4.4 \times 10^7 \frac{cm}{s} \left( \frac{M_{WD}}{M_{\odot}} \right)^{1/2} \left( \frac{R}{R_{\odot}} \right)^{-1/2}$$

- One can estimate the radial velocity  $V_R$  through the disk with which the mass moves

$$V_R \approx \frac{R}{\tau_{\text{disk}}} \approx 7 \times 10^4 \frac{cm}{s} \left( \frac{R}{R_{\odot}} \right)$$



# Accretion disk properties

## Order-of-magnitude estimates

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### □ Velocities in dwarf nova disks (cont.).

Comparing the azimuthal and radial velocities, one finds

$$\frac{V_R}{V_\phi} = 1.6 \times 10^{-3} \left( \frac{M_{WD}}{M_\odot} \right)^{-1/2} \left( \frac{R}{R_\odot} \right)^{3/2} \ll 1$$

- This shows that in accretion disks the motion is almost that of test particles orbiting the centre of the disk in circles, superposed by a slow radial inward drift.