OBSERVATIONAL ASTROPHYSICS AND DATA ANALYSIS

Lecture 12

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Photometry

- **Colour indices:** this is the difference between magnitudes at two separate wavelengths:

$$C_{BV} = B - V;\ C_{VR} = V - R, \text{ and so on.}$$

- International colour index based upon photographic and photovisual magnitudes:

$$m_p - m_{pv} = C = B - V - 0.11$$

Photometry

- The B – V colour index is closely related to the spectral type with an almost linear relationship for main sequence stars.
- For most stars, the B and V regions are located on the long wavelength side of the maximum spectral intensity.
- If we assume that the effective wavelengths of the B and V filters are 4400 and 5500 Å, then using the Planck equation:

$$L_\lambda(T) = \frac{2\,h\,c_0^2}{\lambda^5}\left[\exp\left(\frac{h\,c_0}{\lambda\,k_B T}\right) - 1\right]^{-1}$$

we obtain:

$$B - V \approx -2.5 \log\left[3.05\,\frac{\exp(2.617 \times 10^4/K)}{\exp(3.27 \times 10^4/K)}\right]$$

Photometry

- For T < 10000 K this is approximately

$$B - V \approx -2.5 \log \left[3.05 \frac{\exp(2.617 \times 10^4/K)}{\exp(3.27 \times 10^4/K)} \right] = -1.21 + \frac{7090}{T}$$

The magnitude scale is an arbitrary one. For T = 10000 K, B – V = 0.0, but we have obtained ~0.5. Using this correction we get:

$$T = \frac{7090}{(B - V) + 0.71} K$$

Photometry

- More distant stars are affected by interstellar absorption, and since this is strongly inversely dependent upon wavelength.
- The colour excess measure the degree to which the spectrum is reddened:

$$E_{U-B} = (U - B) - (U - B)_0$$

$$E_{B-V} = (B - V) - (B - V)_0$$

where the subscript 0 denotes unreddened quantities – intrinsic colour indices.

Photometry

- In the optical spectrum, interstellar absorption varies with both wavelength and the distance like this semi-empirical relationship:

$$A_\lambda = 6.5\times10^{-10}/\lambda - 2.0\times10^{-4}\ \text{mag pc}^{-1}$$

where λ is in nanometers

Photometry

- Simple UBV photometry for hot stars results in determinations of temperature, Balmer discontinuity, spectral type, and reddening. From the latter we can estimate distance.
- Thus, we have a very high return of information for a small amount of observational effort. This is why the relatively crude methods of wideband photometry is so popular.

Photometry

Absolute flux from a zero-magnitude star like Vega

Symbol	λ (μm)	ν (Hz)	F_λ (W cm^{-2} μm^{-1})	F_ν (Jy)*
U	0.36	8.3×10^{14}	4.35×10^{-12}	1,880
B	0.43	7.0×10^{14}	7.20×10^{-12}	4,440
V	0.54	5.6×10^{14}	3.92×10^{-12}	3,810
R	0.70	4.3×10^{14}	1.76×10^{-12}	2,880
I	0.80	3.7×10^{14}	1.20×10^{-12}	2,500
J	1.25	2.4×10^{14}	2.90×10^{-13}	1,520
H	1.65	1.8×10^{14}	1.08×10^{-13}	980
K	2.2	1.36×10^{14}	3.8×10^{-14}	620
L	3.5	8.6×10^{13}	6.9×10^{-15}	280
M	4.8	6.3×10^{13}	2.0×10^{-15}	153
N	9.1	3.0×10^{13}	1.09×10^{-16}	37

Photometry: Fun with Units

- **Why do we continue to use magnitudes?**
  - Historical reasons: astronomers have built up a vast literature of catalogues and measurements in the magnitude system
  - The magnitude system is logarithmic, which turns the huge range in brightness ratios into a much smaller range in magnitude differences: the difference between the Sun and the faintest star visible to the naked eye is only 32 magnitudes.
  - Simplicity: Astronomers have figured out how to use magnitudes in some practical ways which turn out to be easier to compute than the corresponding brightness ratios.

Photometry: Fun with Units

- Astronomers who study objects outside the optical wavelengths do not have any historical measurements to incorporate into their work
- In those regimes, measurements are almost always quoted in "more rational" systems: units which are linear with intensity (rather than logarithmic) and which become larger for brighter objects:
  - erg $s^{-1} cm^{-2}$ $Å^{-1}$
  - erg $s^{-1} cm^{-2}$ Hz^{-1}

  - 1 Jansky [Jy] = 10^{-26} W m^{-2} Hz^{-1} = 10^{-23} erg $s^{-1} cm^{-2}$ Hz^{-1} = 3×10^{-16} λ^{-1} W cm^{-2} μm^{-1} (if λ is expressed in microns)

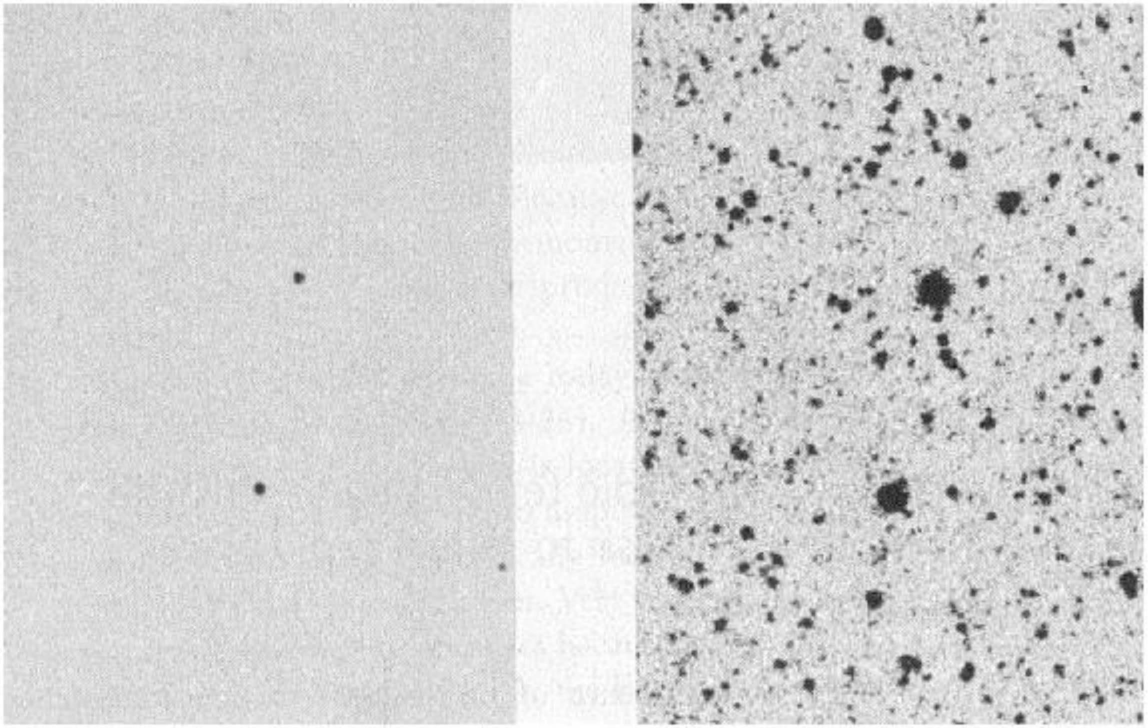
Photometry: Fun with Units

- Fluxes for a V = 0 star of spectral type A0 V at 5450 Å:
  - $f^0_\lambda = 3.92 \times 10^{-9}$ erg $s^{-1} cm^{-2}$ $Å^{-1}$, or
  - $\varphi^0_\lambda = f^0_\lambda / h\nu$ =1055 photons $s^{-1} cm^{-2}$ $Å^{-1}$
  - Useful:
    - 1 Jy = 1.51×10^3 photons $s^{-1} cm^{-2} (\Delta\lambda / \lambda)^{-1}$
    - $\Delta\lambda / \lambda$ = 0.15 (U), 0.22 (B), 0.16 (V), 0.23 (R), 0.19 (I)

Night Sky Brightnesses

Lunar Age (days)	U	B	V	R	I
0	22.0	22.7	21.8	20.9	19.9
3	21.5	22.4	21.7	20.8	19.9
7	19.9	21.6	21.4	20.6	19.7
10	18.5	20.7	20.7	20.3	19.5
14	17.0	19.5	20.0	19.9	19.2

Practical Photometry: S/N (2)



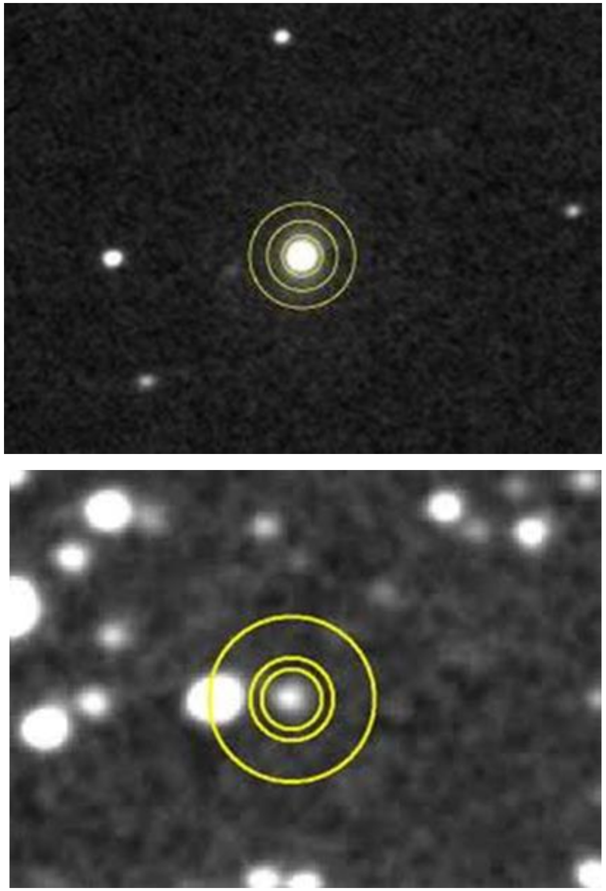
Physical limitations on the precision of photometric measurements

- To calculate the *Output* Signal-To-Noise Ratio of an observation we need to know the signal, and all sources of noise. These are:
  - Photon noise (shot noise) from the signal;
  - Photon noise from the sky background under the signal;
  - Photon noise from the sky background measurement to be subtracted off;
  - Readout noise from all sources;
  - Fixed pattern noise;
  - Bias noise;
  - Dark current noise.

Physical limitations on the precision of photometric measurements

- Detective quantum efficiency= DQE $=[\frac{SNR_{out}}{SNR_{in}}]^2$
- We observe a star on a CCD detector, and process the data in the simplest way possible.
- $Reduced\ Frame = \frac{Object\ Frame - Bias\ Frame}{Flat\ Frame - Bias\ Frame}$

Physical limitations on the precision of photometric measurements



The upper is good, the bottom is bad

- An area centred on the star is defined to be the object area, and is large enough to contain all of the photons from that star.
- An equal area some distance away, which is found to be free of stars, is defined as the sky background area, and the sky background is measured from that.

Aperture photometry

- There are a number of parameters we need to take into account to calculate the signal which reaches the detector:
  - **t** – exposure time
  - **D** – diameter of the telescope
  - S_{sky} [photons / (cm^2 arcsec^2 second)] – brightness of the sky
  - **η** – quantum efficiency of a detector (QE)
  - φ_* [photons / (cm^2 second Å)] – the source flux to be measured
  - Star is observed in a circular aperture of area **α** square arcseconds which covers $\mathbf{n_{pix}}$ pixels
  - Sky background is determined from a circular aperture of the same size
  - Readout noise is $\boldsymbol{\sigma_R}$ electrons
  - We observe a star of magnitude V in the V filter

Signal calculation

- We start from the number of photons incident upon the top of the atmosphere of the Earth from this star:

$$N_* = \varphi_* \Delta\lambda \ A \text{ photons/second}$$

incident upon the top of the atmosphere in photometric (clear) conditions

$\Delta\lambda$ is the filter passband in Å

φ_* is the flux from a star in photons $s^{-1} cm^{-2}$ Å^{-1}

A is the telescope collecting area in centimetres^2

Signal calculation

- That's at the top of the atmosphere. There are a number of efficiency factors we need to multiply by to calculate the signal which reaches the detector:
  - Atmospheric transmission ε_{atm} (~0.88 for a star at the zenith, in the V filter).
  - Telescope reflection efficiency ε_{tel} (~0.92 per mirror = 0.846 for a Cassegrain telescope)
  - Filter transmission ε_{filt} (~0.85 for a broadband filter)
  - CCD Responsive Quantum Efficiency η (~0.75)
  - Cryostat entrance window efficiency ε_{win} (~0.95)

Signal calculation

- There is also a geometric efficiency factor as part of the aperture of the telescope is blocked by the secondary mirror.

For a ***D*** metre aperture telescope with a ***d*** metre secondary mirror:

$$\varepsilon_{\text{geom}} = \frac{\pi D^2 - \pi d^2}{\pi D^2} = \frac{D^2 - d^2}{D^2}$$

For example, if ***D***=2.0m and ***d***=0.6m, then $\varepsilon = 0.91$

Signal calculation

- For a star the number of photons which is detected is given by:

$$N_{star} = \eta \, \varepsilon_{atm} \, \varepsilon_{tel} \, \varepsilon_{filt} \, \varepsilon_{win} \, \varepsilon_{geom} \, \varphi_{*} \, \Delta\lambda \, A \, t$$

t is the exposure time in seconds,

$\Delta\lambda$ = 870 Å for the V band.

For example for a star of magnitude V=23 on a 2 metre telescope with the efficiencies we have quoted:

$$N_{star} = 2.5 \, t$$

Signal calculation

- In the absence of sky background and readout noise it would be simple, we would integrate for 1000 seconds, detect 2500 photons, and have a signal to noise ratio of 50. But sky and readout noise are significant.
- Every square arcsecond of sky gives:

$$N_{sky} = \eta\, \varepsilon_{atm}\, \varepsilon_{tel}\, \varepsilon_{filt}\, \varepsilon_{win}\, \varepsilon_{geom}\, \varphi_{sky}\, \Delta\lambda\, A\, t$$

φ_{sky} is the flux from the sky in photons $s^{-1} cm^{-2}$ Å^{-1} $arcsec^{-1}$

$\approx 10\, t$ photons from the dark sky ($V \approx 21.5$)

Aperture photometry

- Assume we have two apertures, one on the star and one on sky. Star aperture includes sky as well, and our estimate of the star intensity is the difference between the two.
- Signal in the sky aperture is:

$$n_{sky} = \alpha\, N_{sky}$$

- Signal in the star aperture is:

$$n_{*+sky} = \alpha\, N_{sky} + N_{star}$$

Noise on the measurements

- Noise on the measurements has two components, photon noise which is given by the square root of the number of photons, and readout noise, which is determined by the readout noise and by the number of pixels in the aperture. The noise components add in quadrature:

$$\sigma_{sky}^2 = n_{sky} + n_{pix}\,\sigma_R^2$$

$$\sigma_{*+sky}^2 = n_{*+sky} + n_{pix}\,\sigma_R^2$$

$$n_* \approx n_{*+sky} - n_{sky}$$

$$\sigma_*^2 = n_{*+sky} + n_{sky} + 2\,n_{pix}\,\sigma_R^2$$

$$\sigma_*^2 = 2\,\alpha\,N_{sky} + N_{star} + 2\,n_{pix}\,\sigma_R^2$$

Signal to Noise ratio

$$S/N = n_* / \sigma_* = N_{star} / \sigma_*$$

$$\frac{S}{N} = \frac{N_{star}}{\sqrt{2\ \alpha\ Nsky + N_{star} + 2\ npix\ \sigma_R^2}}$$

If exposure time is short then readout noise ($\sigma_R \sim 10$) will **dominate.**

Improving the Signal to Noise

- Larger Sky Aperture – Increasing the sky aperture and scaling it to the size of the object aperture, or using several sky apertures and averaging them, reduces the noise to:

$$\sigma_*^2 = \zeta\, \alpha\, N_{sky} + N_{star} + \zeta\, n_{pix}\, \sigma_R^2$$

where $\zeta = (1 + n_{pix1} / n_{pix2})$, and n_{pix1} and n_{pix2} are the number of pixels in the star and sky apertures respectively. In practice the sky aperture is often an annulus around the star aperture. Must be careful that stars do not get in the sky aperture!

Improving the Signal to Noise

- Smaller object aperture – reducing the object aperture reduces both sky noise and readout noise. However you lose signal. The problem is if you are comparing the signal in different images, and fluctuations in image size (seeing) cause the amount of signal you lose to vary, then this introduces systematic errors in the brightness measured (photometry).

Solution – Aperture Correction