OBSERVATIONAL ASTROPHYSICS AND DATA ANALYSIS

Lecture 7

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Physical limitations on the precision of photometric measurements

- What we get out of our detectors?
- Have we taken enough data?
- How much longer should we observe?

The important quantity that compares the level of a desired signal to the level of background noise is

the Output Signal-to-Noise Ratio (S/N)

Physical limitations on the precision of photometric measurements

- Let's consider a stationary source of light (a star) with an average photon flux at the detector of $\mathbf{N}_*$ photons per second.
- The intensity of a source will produce the **average** number of photons, but the actual number collected will be more than, equal to, or less than the average, and their distribution about that average will be **a Poisson distribution**.
- The "counts" accumulated in a CCD pixel (or similar detectors) have **a Poisson distribution**.

Physical limitations on the precision of photometric measurements

- The standard deviation of the photon noise is equal to the square root of the average number of photons. The ***Input* Signal-to-Noise Ratio** is then

$$S/N = \frac{N_*}{\sqrt{N_*}} = \sqrt{N_*}$$

where N_* is the average number of photons collected.

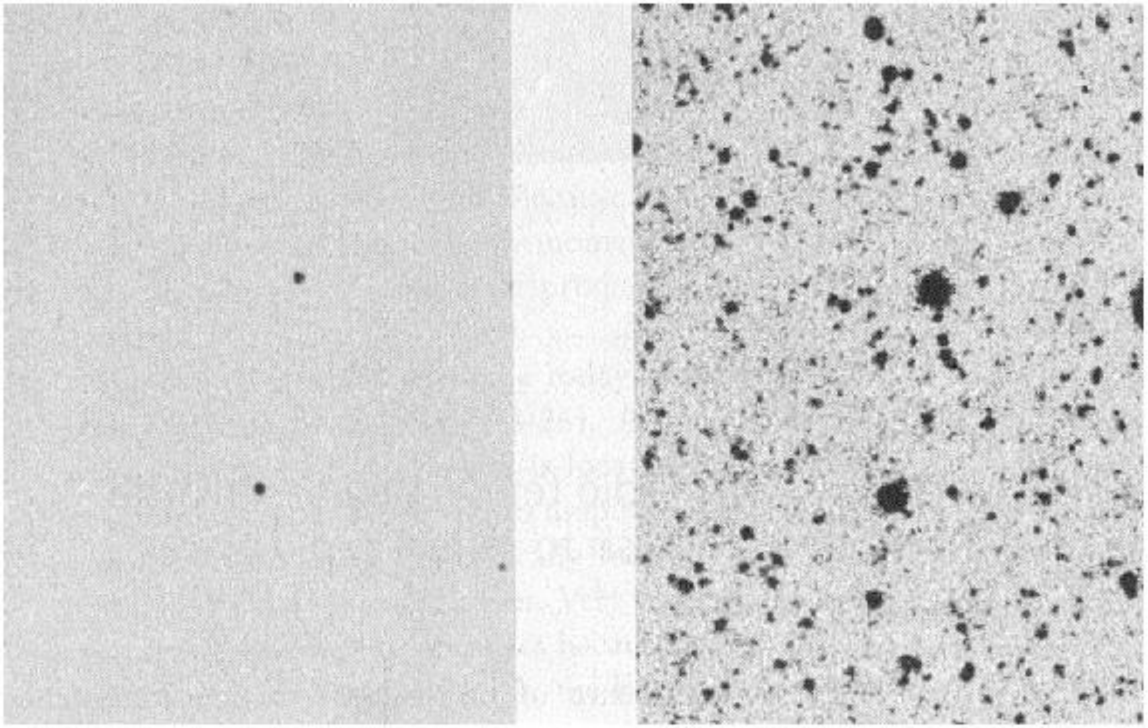
- When N_* is very large, the signal-to-noise ratio is very large as well. It can be seen that photon noise becomes more important when the number of photons collected is small.

Physical limitations on the precision of photometric measurements

- In the absence of sky background and readout noise it would be simple to calculate S/N, we would integrate for some time, detect some 10000 photons, and have a signal to noise ratio of 100 or so.
- So if we ignore the sky background it's a simple calculation, but we can't.

Physical limitations on the precision of photometric measurements

Sky and readout noise are significant.



Physical limitations on the precision of photometric measurements

- In dark sky at a dark site (no moon, no reflected street light), the magnitude of a 1 arcsecond patch of sky in the V band is approximately V_{SKY}=21.5 mag.

Thus, every square arcsecond of sky gives:

$$S_{sky} = 2.5\times10^{-3} \text{ photons / (cm}^2 \text{ arcsec}^2 \text{ second)}$$

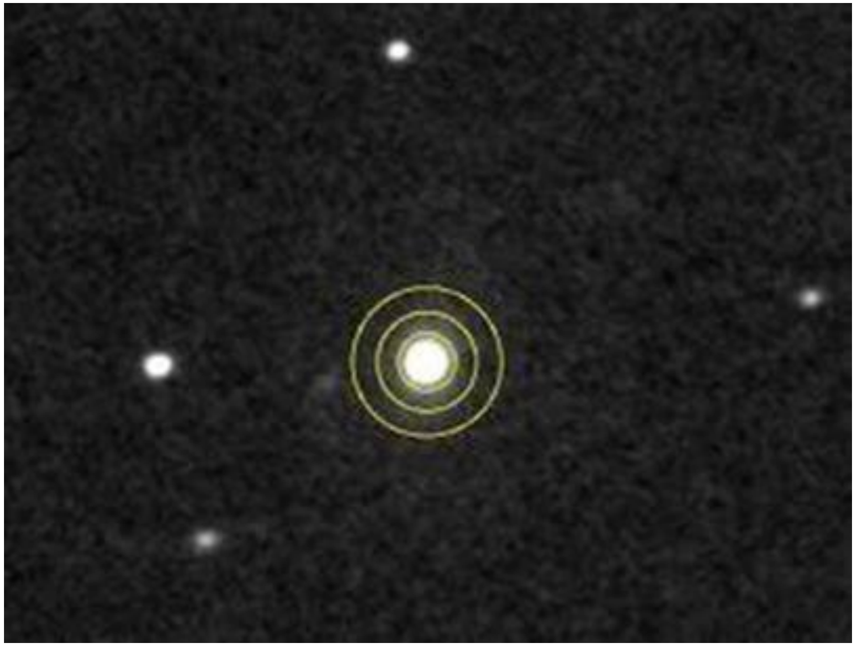
Physical limitations on the precision of photometric measurements

- To calculate the *Output* Signal-To-Noise Ratio of an observation we need to know the signal, and all sources of noise. These are:
  - Photon noise (shot noise) from the signal;
  - Photon noise from the sky background under the signal;
  - Photon noise from the sky background measurement to be subtracted off;
  - Readout noise from all sources;
  - Fixed pattern noise;
  - Bias noise;
  - Dark current noise.

A case study of simple aperture photometry

- We observe a star on a CCD detector, and process the data in the simplest way possible.
- An area centred on the star is defined to be the object area, and is large enough to contain all of the photons from that star.
- An equal area some distance away, which is found to be free of stars, is defined as the sky background area, and the sky background is measured from that.

A case study of simple aperture photometry



- We define circular apertures of equal size are around the star, and in a region of the image with no stars in, which we call the sky aperture.

A case study of simple aperture photometry

- We will make some assumptions:
  - We have eliminated fixed pattern noise by dividing the image by a normalised long exposure of a uniform light source, this is called *a flat field*.
  - Bias noise and dark current noise are negligible, as this is a cryogenically cooled, buried channel CCD.

Aperture photometry

- There are a number of parameters we need to take into account to calculate the signal which reaches the detector:
  - t – exposure time
  - β – angular size of a source (defined by the seeing)
  - D – diameter of the telescope
  - S_{sky} [photons / (cm^2 arcsec^2 second)] – brightness of the sky
  - η – quantum efficiency of a detector (QE)
  - f_* [photons / (cm^2 arcsec^2 second)] – the source flux to be measured

Signal calculation

- We start from the number of photons incident from this star, from the sky, and from the star + the sky:
  - $A \sim D^2$ is the telescope collecting area [cm^2]
  - $B \sim \beta^2$ is the source area on the sky [arcsec^2]
- $n_* \approx \eta\, D^2\, t\, f_*$ – an average number of photons from the source
- $n_{sky} \approx \eta\, D^2\, t\, \beta^2\, S$ – an average number of photons from the sky
- $n_{*+sky} \approx \eta\, D^2\, t\, (f_* + \beta^2\, S)$ – an average number of photons from the source and the sky

Signal calculation

- That is without the Readout noise and other detector noises. If we want to take them into account – we must to add $N_d = n_d\, t$ to the right side of equations.
- There is also a geometric efficiency factor as part of the aperture of the telescope is blocked by the secondary mirror.

Noise on the measurements

- Noise on the measurements is given by the square root of the number of photons:

$$\sigma_{*+sky} = \sqrt{n_{*+sky}}$$

$$n_* \approx n_{*+sky} - n_{sky}$$

$$\sigma_* = \sqrt{n_{*+sky} + n_{sky}} = \sqrt{n_* + 2n_{sky}}$$

(If x and y have independent random errors σ_x and σ_y , then the error in $z = x \pm y$ is $\sigma_z^2 = \sigma_x^2 + \sigma_y^2$)

Signal to Noise ratio

$$S/N = \frac{n_*}{\sigma_*} = \frac{n_*}{\sqrt{n_* + 2n_{sky}}} = \frac{\eta\, D\, t\, f_*}{\sqrt{\eta\, t\,(f_* + 2\,\beta^2\, S)}}$$

Let's now consider two special cases:

- If the Source dominates over the Sky: $\mathbf{n_* \gg n_{sky}}$
- If the Sky noise dominates: $\mathbf{n_{sky} \gg n_*}$

Signal to Noise ratio

If the Source dominates over the Sky: $\mathbf{n_* \gg n_{sky}}$

$$\mathrm{S/N} \cong \frac{n_*}{\sqrt{n_*}} = \sqrt{n_*} = D\sqrt{\eta\, t\, f_*}$$

$\mathbf{f_{min} \sim 1 / (D^2\, t)}$ for the given S/N

Signal to Noise ratio

If the Sky noise dominates: $\mathbf{n_{sky}} \gg \mathbf{n_*}$

$$S/N \cong \frac{n_*}{\sqrt{2n_{sky}}} = \frac{\eta\, D\, t\, f_*}{\sqrt{2\,\eta\ t\ \beta^2\, S}} = \frac{D\, f_*}{\beta}\sqrt{\frac{\eta\, t}{2\, S}}$$

$$f_{min} \sim \frac{\beta}{D}\sqrt{\frac{S}{\eta\, t}}$$ for the given S/N

Signal to Noise ratio

the Source dominates over the Sky	the Sky noise dominates
$$S/N \cong D\sqrt{\eta\, t\, f_*}$$	$$S/N \cong \frac{D\, f_*}{\beta}\sqrt{\frac{\eta\, t}{2\, S}}$$
$$f_{min} \sim \frac{1}{D^2 t}$$	$$f_{min} \sim \frac{\beta}{D}\sqrt{\frac{S}{\eta t}}$$
for the given S/N	for the given S/N