Combined analysis of power spectra

- In astronomy, it is often necessary to compare the power spectra of two or more time series, e.g.:
  - X-ray binaries are often observed simultaneously in X-rays, UV and optical wavelengths (and γ-rays).
  - the Sun has been observed more or less routinely for many years and in a variety of modes (sunspots, radio, UV, X-ray, irradiance, etc.), so one may need to compare two or more solar data sets.
- One might also wish to estimate the significance of a particular peak that shows up in two or more power spectra.

Combined analysis of power spectra

- Assuming, that each power spectrum is distributed **exponentially** (e.g., Lomb-Scargle periodogram), Sturrock et al. (2005) proposed three such statistics, that are useful for the combined study of two or more time-series:
  - Combined Power Statistic
  - Minimum Power Statistic
  - Joint Power Statistic

The paper is on the course web-page

Combined analysis of power spectra

- **Combined Power Statistic**
  - If we wish to combine information from n independent power spectra, the combination that would correspond to the chi-square statistic is the sum of the powers, which we write as

$$\boldsymbol{Z = S_1 + S_2 + \cdots + S_n}.$$

  - The following function of Z ("**combined power statistic**") is distributed exponentially:

$$G_n(Z) = Z - \ln\left(1 + Z + \frac{1}{2}Z^2 + \cdots + \frac{1}{(n-1)!}Z^{n-1}\right).$$

Combined analysis of power spectra

- **Minimum Power Statistic**
  - We may wish to determine the frequency for which the minimum power among two or more power spectra has the maximum value. Let's consider the following quantity, formed from the independent variables $x_1, x_2, \ldots, x_n$, each of which is distributed exponentially:

$$\boldsymbol{U(x_1, x_2, \ldots, x_n) = \mathrm{Min}(x_1, x_2, \ldots, x_n)}$$

  - It can be shown that the following function of U ("**minimum power statistic**") is distributed exponentially:

$$\boldsymbol{K_n(U) = nU}$$

Combined analysis of power spectra

- **Joint Power Statistic**

  - Let’s now consider the need to compare spectra from two quite different times series. If one of the time-series has very strong peaks and the other has comparatively weak peaks, then simply adding the powers would not be very revealing, since the sum would be dominated by the stronger spectrum.

  - In this situation, it is more useful to form something resembling a “correlation function" by forming the product of the two powers. It proves convenient to work with the square root of the product (the geometric mean):

$$\boldsymbol{X = (S_1 S_2)^{1/2}}$$

Combined analysis of power spectra

- **Joint Power Statistic (cont.)**
  - The following function of *X* is distributed exponentially:

$$J_2 = -\ln\left(2X\,K_1(2X)\right)$$

where $K1$ is the Bessel function of the second kind.

  - A good approximation to J_2 is found to be:

$$J_{2A} = \frac{1.943\,X^2}{0.650 + X}.$$

Combined analysis of power spectra

- **Joint Power Statistic (cont.)**
  - Let's now consider joint power statistics of higher orders, and consider the following geometric mean of *n* powers:

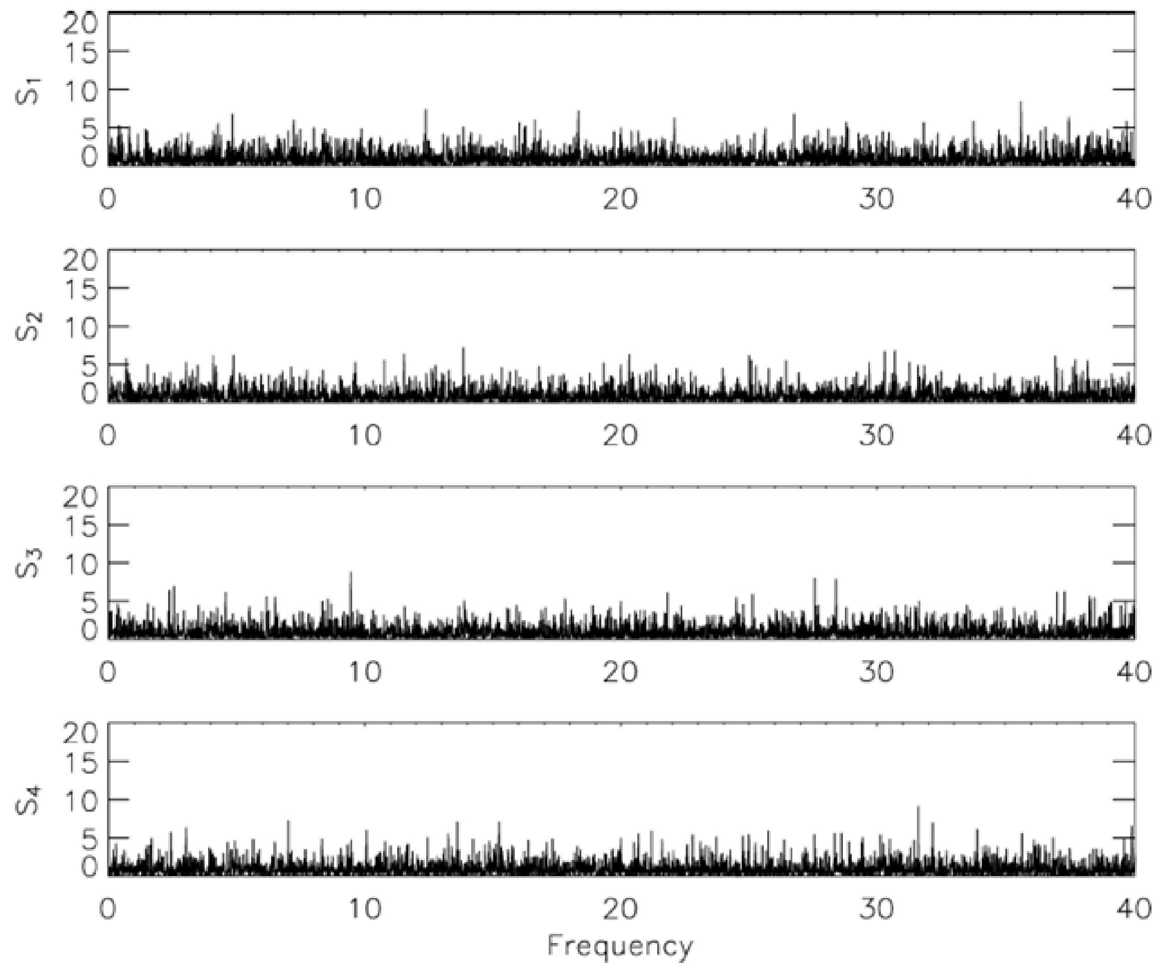
$$X = (S_1 \ldots S_n)^{1/n}$$

  - There is no useful analytical functions of *X* that are distributed exponentially, but there are very good approximations:

$$J_{3A} = \frac{2.916X^2}{1.022 + X}, \qquad J_{4A} = \frac{3.881X^2}{1.269 + X}.$$

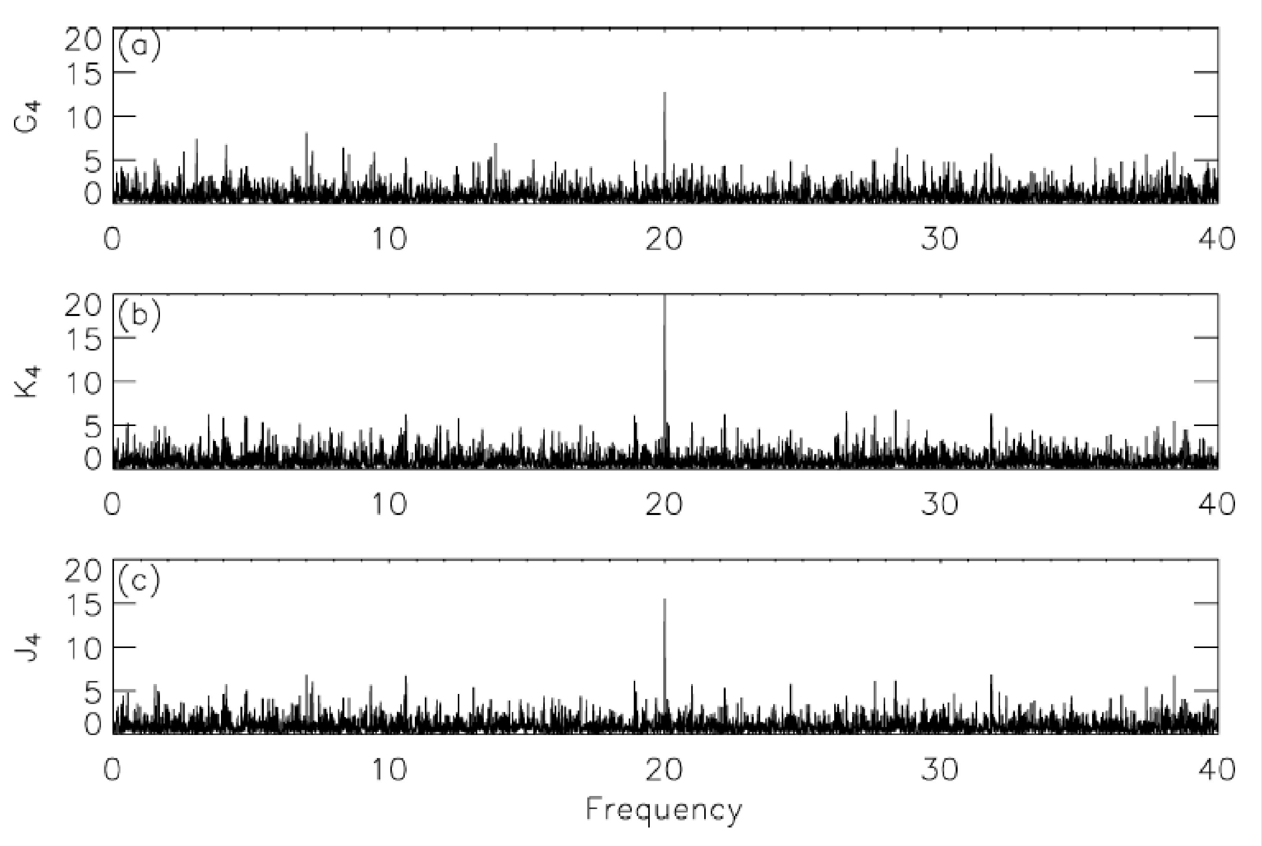
Combined analysis of power spectra

Four synthetic spectra, each with a signal of power 5 at $\nu = 20$.



Combined analysis of power spectra

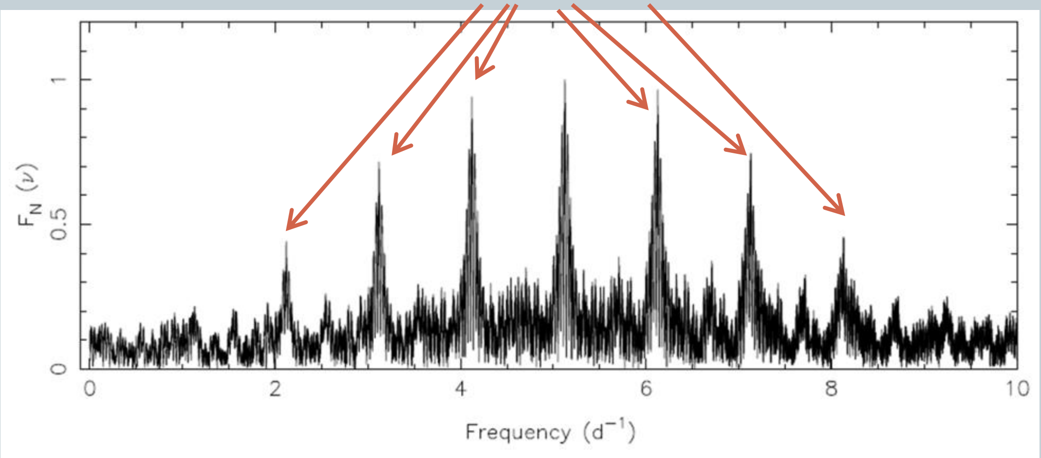
The combined power statistic, minimum power statistic, and joint power statistic, formed from the four synthetic spectra.



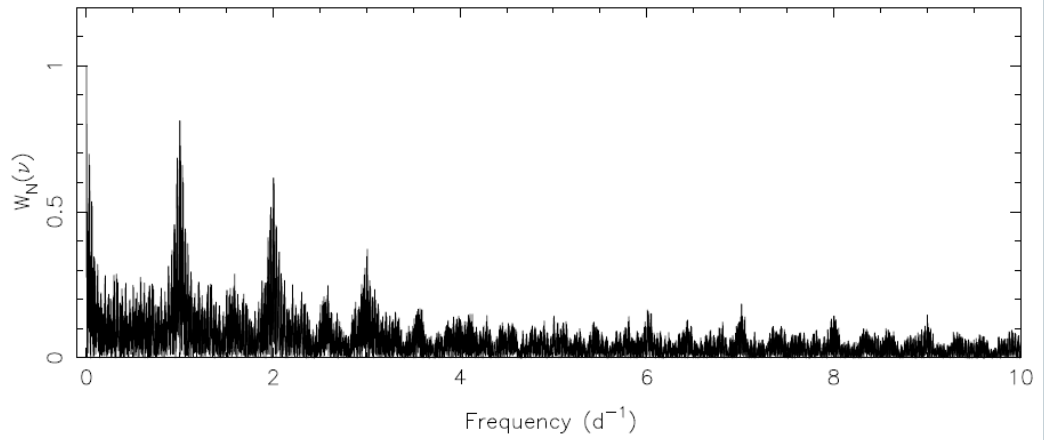
Dealing with Aliases. CLEAN algorithm.

1-day aliases

Power Spectrum:



Spectral Window:

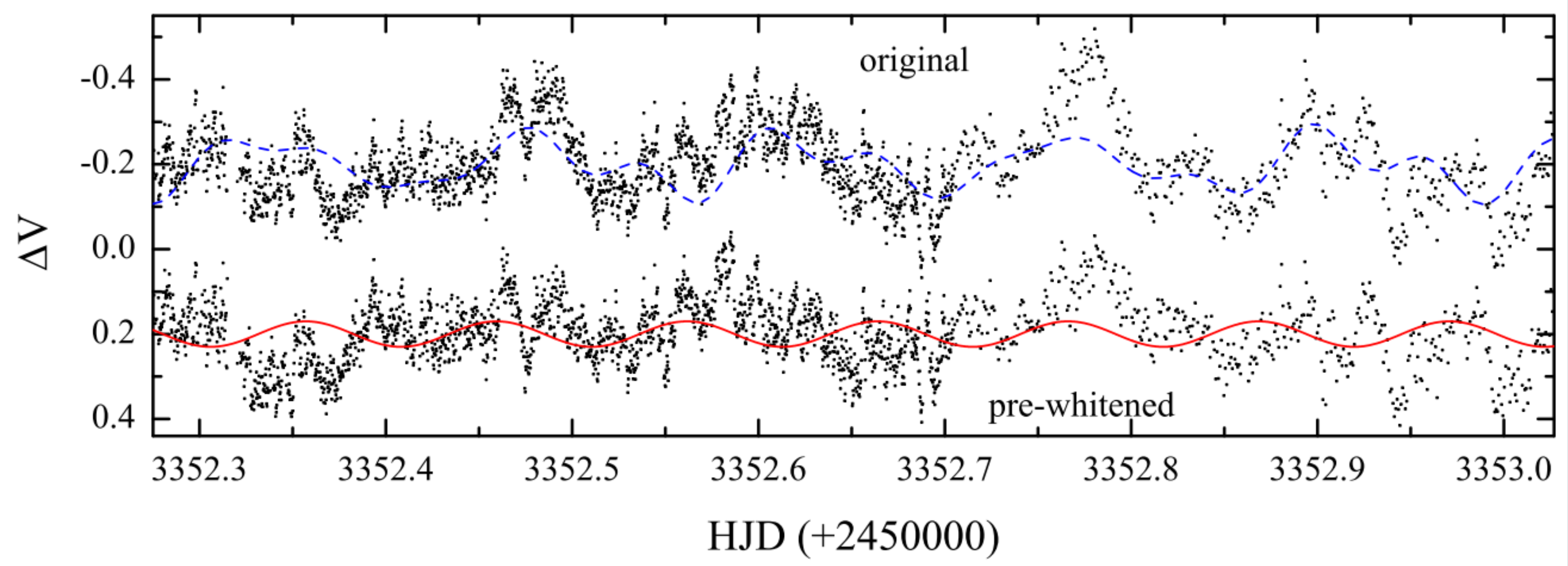


Dealing with Aliases

- How to deal with aliases?
  - Pre-whitening
  - Cleaning (Clean algorithm)

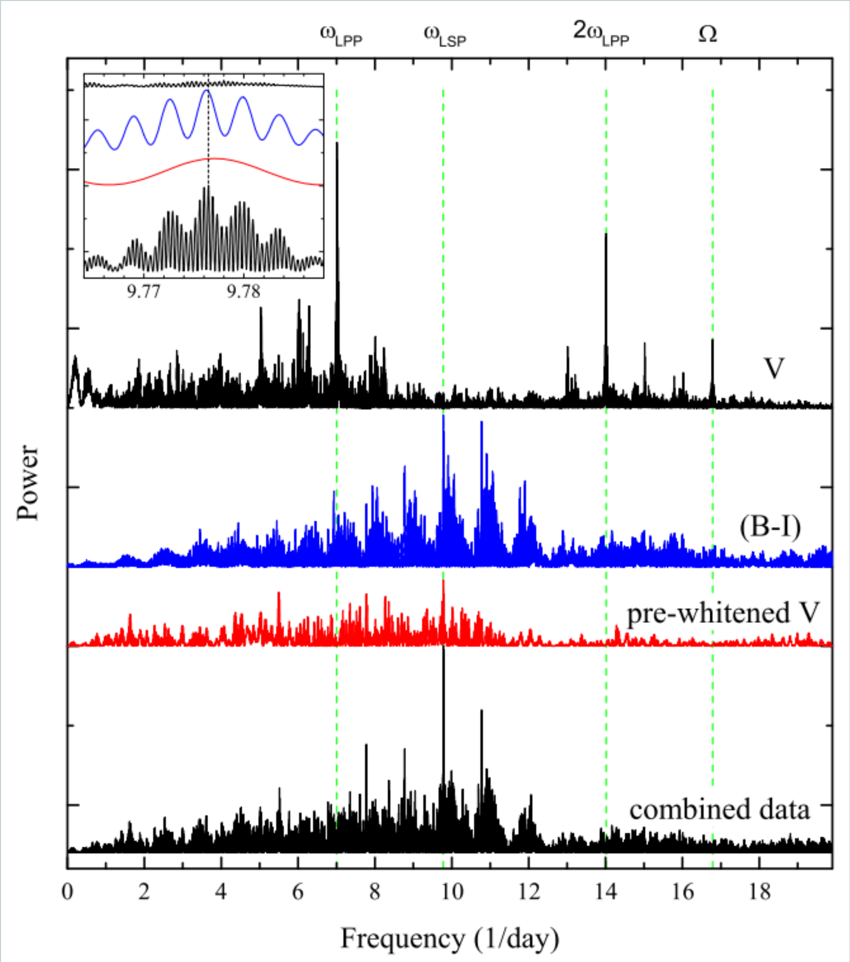
Pre-whitening

- If the light curve contains more than one periodic modulations, and the signal of interest with an unknown frequency is weak and hidden in noise, then one can try to remove the strongest signal of known frequency from the light curve:
  fit the light curve with a sine-wave (and its harmonics) and subtract it.



Pre-whitening

- If the light curve contains more than one periodic modulations, and the signal of interest with an unknown frequency is weak and hidden in noise, then one can try to remove the strongest signal of known frequency from the light curve.



Cleaning

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TIME SERIES ANALYSIS WITH CLEAN. I. DERIVATION OF A SPECTRUM

DAVID H. ROBERTS
Physics Department, Brandeis University, Waltham, Massachusetts 02254

JOSEPH LEHÁR AND JOHN W. DREHER
Physics Department, 26-315, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
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ABSTRACT

We present a method of time-series spectral analysis which is especially useful for unequally spaced data. Based on a complex, one-dimensional version of the CLEAN deconvolution algorithm widely used in two-dimensional image reconstruction, this technique provides a simple way to understand and remove the artifacts introduced by missing data. We describe the method, give several examples, and point out various analogies with the conventional use of CLEAN.

Clean Algorithm

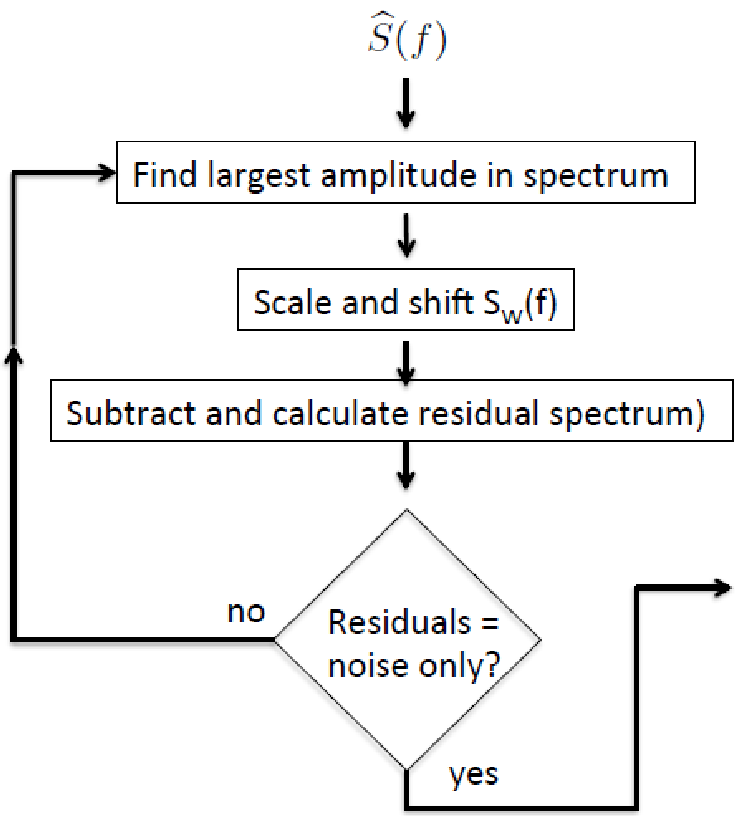
- The premise of CLEAN is that our data consist not only of the data amplitudes but also the detailed sampling in time.
- We therefore know that the true spectrum is convolved with a known window function.
- The actual algorithm is based on the fact that any function can be represented as a sum or integral over delta functions.

Spectral Analysis
S(f) = spectral estimate
$S_W(f) = \|FT\{W(t)\}\|^2$ = spectral window (calculated as the FT of sample times)
$S_C(f) = \Sigma\, C_j\, \Delta(f - f_j)$ CLEANed Spectrum Sum over CLEAN components $\Delta(f)$ = restoration function that represents the inherent frequency resolution

The restoring function is needed to fairly represent the resolution imposed by Fourier transform properties (uncertainty principle)

Clean Algorithm

Data yield $\widehat{S}(f)$ and $S_W(f) = |\tilde{W}(f)|^2$



$\widehat{S}_i(f_i)_{\max}\ @f_i \quad i = \text{iteration count}$

$\gamma_i S_W(f - f_i)$

$\widehat{S}_i(f) \rightarrow \widehat{S}_{i+1}(f) = \widehat{S}_i(f) - \gamma_i S_W(f - f_i)$

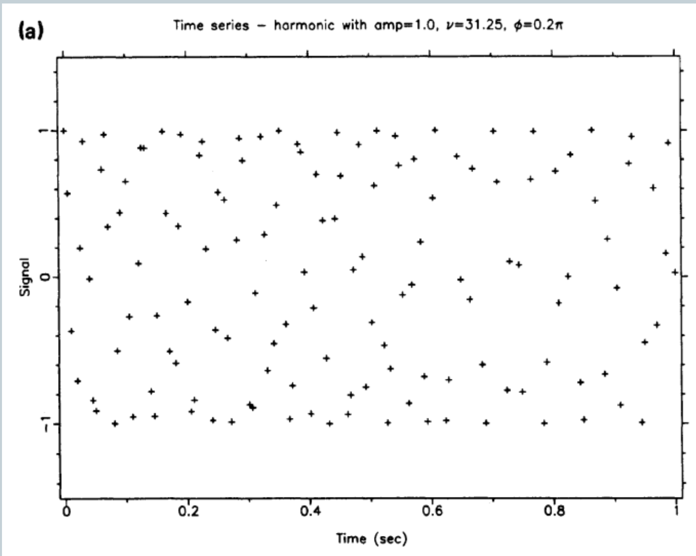
Collect CLEAN components

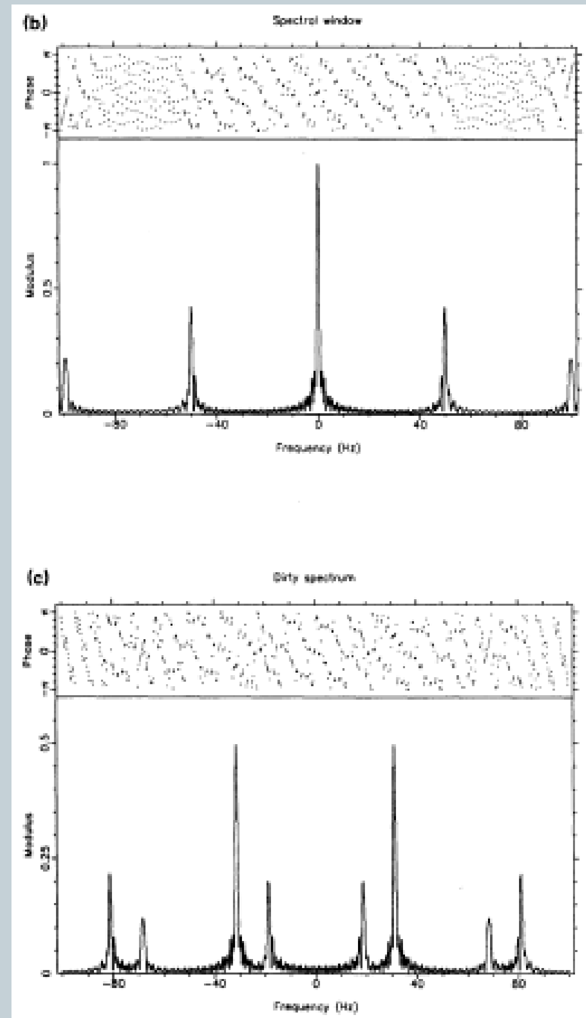
$\{\gamma_i, f_i,\ i = 1, N_c\}$

Form CLEAN spectrum

$$S_c(f) = \sum_{i=1}^{N_c} \gamma_i \Delta(f - f_i)$$

Clean Algorithm





a) Time-Series
b) The window function
c) The dirty spectrum

Clean Algorithm

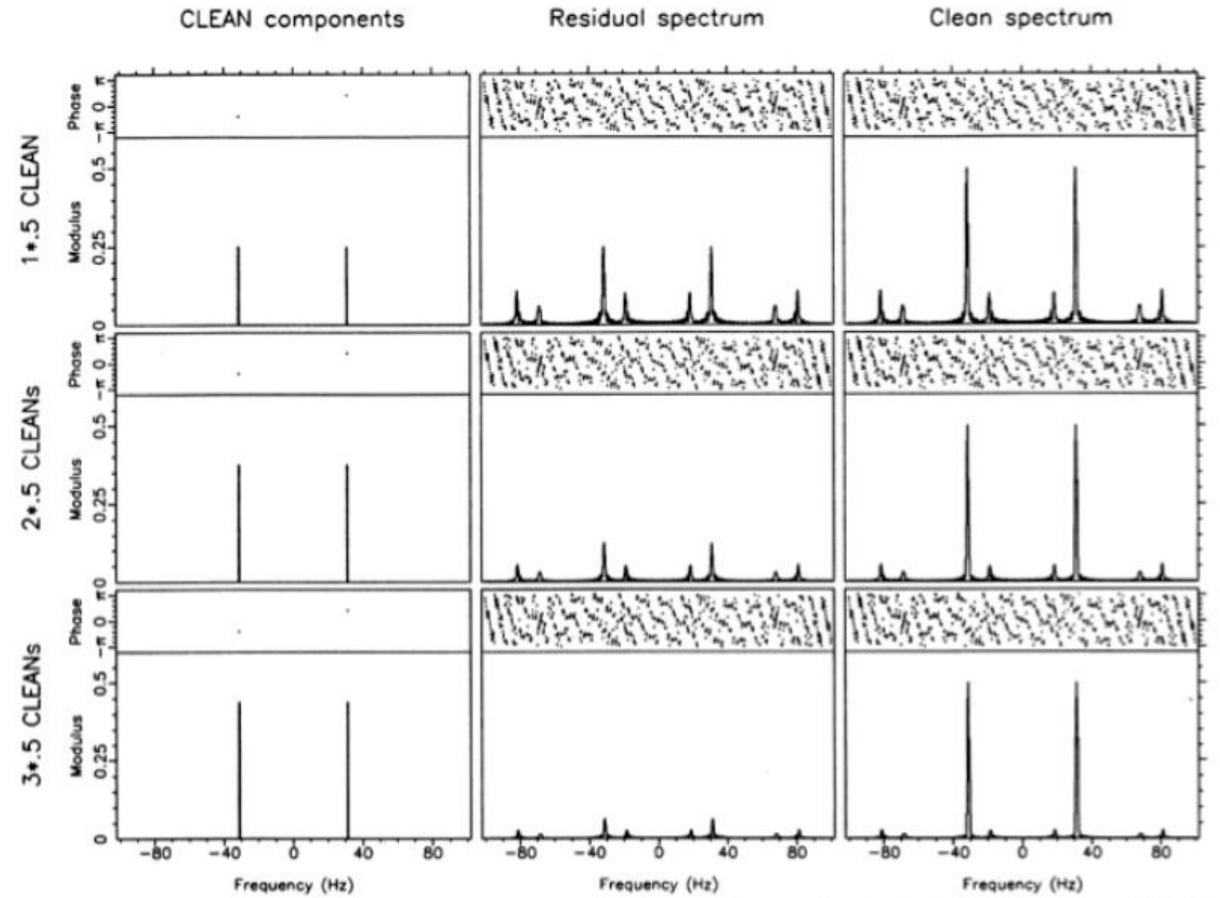


FIG. 2. Analysis of the time series in Fig. 1. (a)–(e). The clean components, residual spectra, and clean spectra after one, two, three, five and one hundred iterations with gain $g = 0.5$, and (f) after one iteration with $g = 1$. Note the change to a logarithmic scale for (d)–(f).

Clean Algorithm

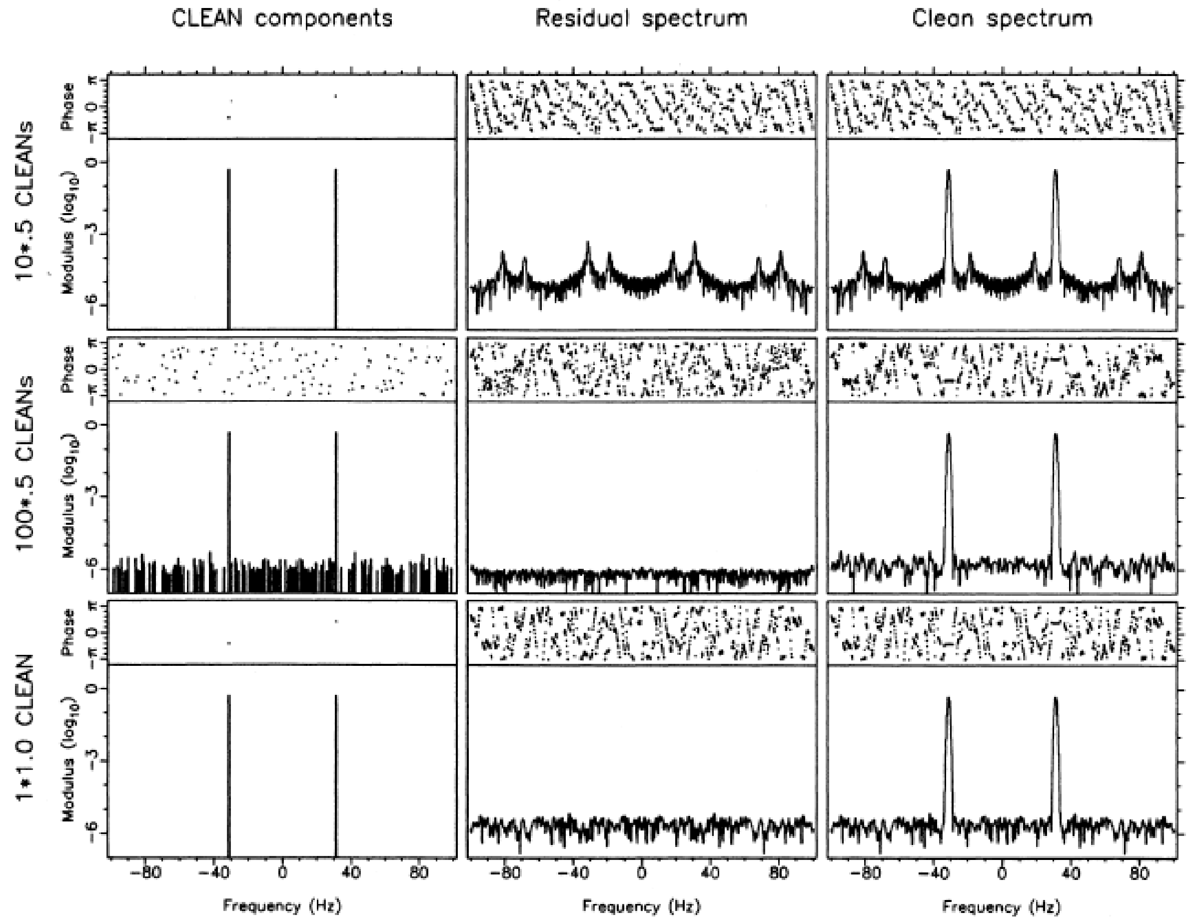


FIG. 2. (continued)